

Closed Curvilinear Surfaces

A decorative graphic consisting of a solid red horizontal bar that transitions into a series of white horizontal stripes of varying lengths on a dark gray background.

Dada

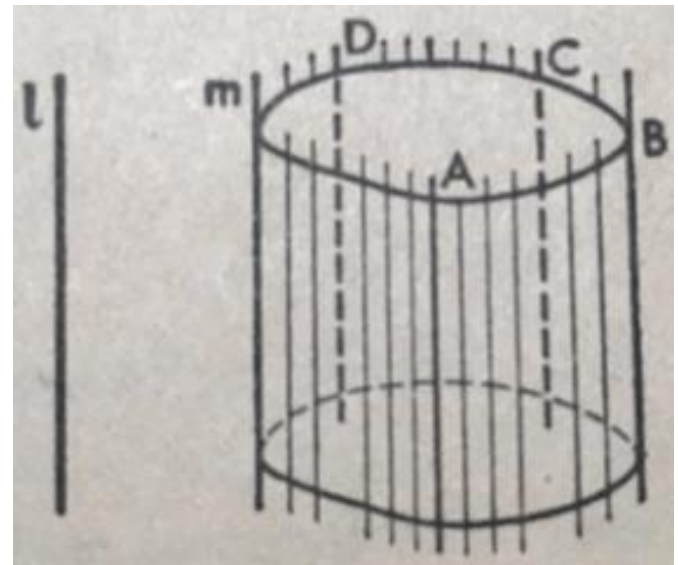
PNU-Math

CYLINDER



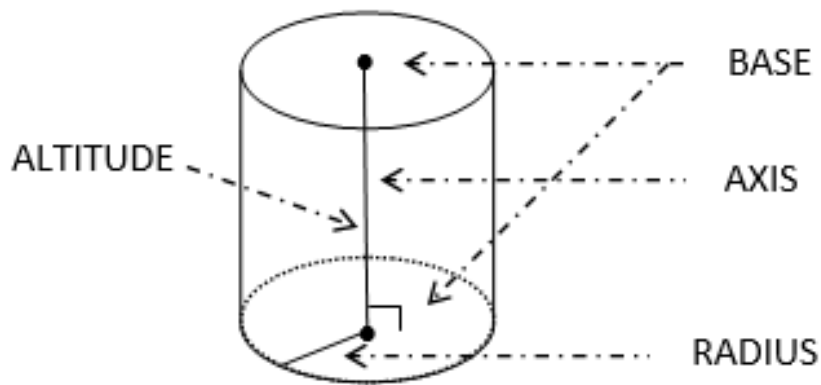
Closed Curve Cylindrical Surface

is a surface generated by a line that moves so that it is always parallel to a given line and continuously touches a fixed plane curve, not in a plane of the given line.

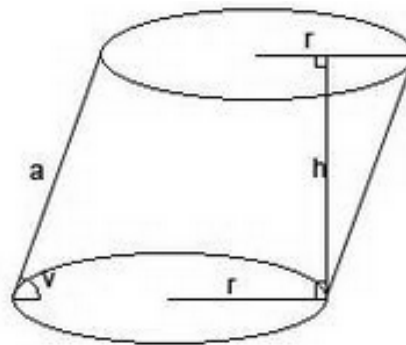


CYLINDER

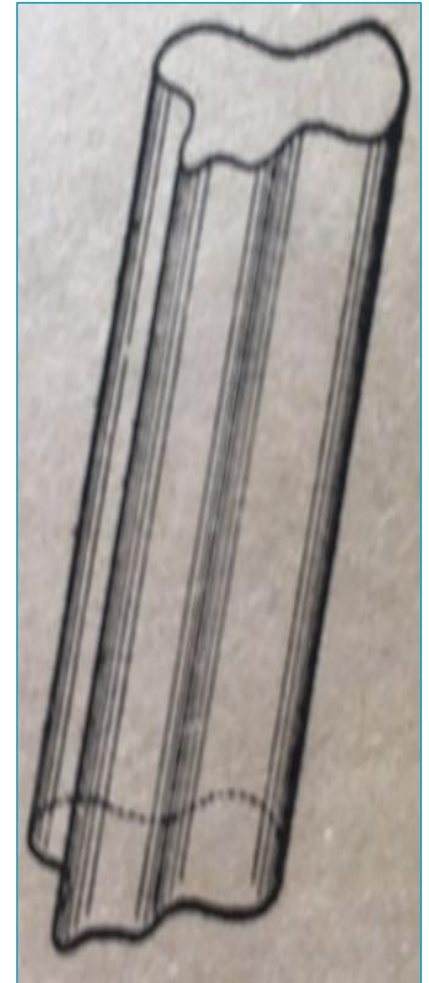
is a closed curvilinear surface formed when a closed curve cylindrical surface is cut by two parallel planes which intersect all the elements.



Right Circular Cylinder



Oblique Cylinder



Formulas for Circular Cylinder

Lateral Area (LA) – area of the lateral surface
– $LA = 2\pi rh$

Total Surface Area (TA) – sum of the LA and the areas of the bases.
– $TA = 2\pi rh + 2\pi r^2$

Volume – product of the area of the base and the height of the circular cylinder
– $V = \pi r^2 h$

Exercises

1. If a right circular cylinder has lateral area of 132π sq. yd and volume of 396π cu. yd, what is its total area?
2. A can in the shape of a right circular cylinder with altitude 9 in and base radius $3\frac{1}{2}$ in is made of sheet steel. Disregarding the thickness of the steel, find the number of square inches of steel necessary to construct the can. Let $\pi = \frac{22}{7}$.
3. The diameter of the base of a circular cylinder is 8 ft and the elements are inclined to the base at an angle of 30° . If an element is 7 ft long, find the:
(3.1) LA, **(3.2)** TA, and **(3.3)** volume of the cylinder.
4. The element and the 4-cm radius of an oblique circular cylinder form a 60° angle. Its altitude joins the center of the upper base and any point of the lower base. Find the:
(4.1) LA, **(4.2)** TA, and **(4.3)** volume of the cylinder.

Exercises

5. A regular hexagonal prism whose altitude is 150 cm and whose apothem is $4\sqrt{6}$ cm is inscribed in a right circular cylinder. Find the: **(5.1)** LA, **(5.2)** TA, and **(5.3)** volume of the cylinder and the prism.

A PRISM INSCRIBED IN A CIRCULAR CYLINDER

is a prism whose lateral edges are elements of the cylinder and whose bases are inscribed in the bases of the cylinder.

6. A regular pentagonal prism whose altitude is 100 cm and each side of whose base is 8 cm is circumscribed about a right circular cylinder. Find the: **(6.1)** LA, **(6.2)** TA, and **(6.3)** volume of the cylinder and the prism.

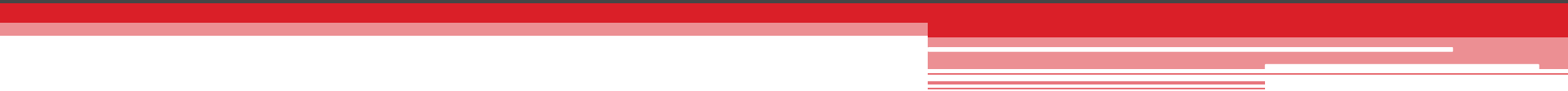
A PRISM CIRCUMSCRIBED ABOUT A CIRCULAR CYLINDER

is a prism all of whose lateral faces are tangent to the cylinder and whose bases are circumscribed about the bases of the cylinder.

Seatwork: *by group; 1 whole sheet; show your complete solutions*

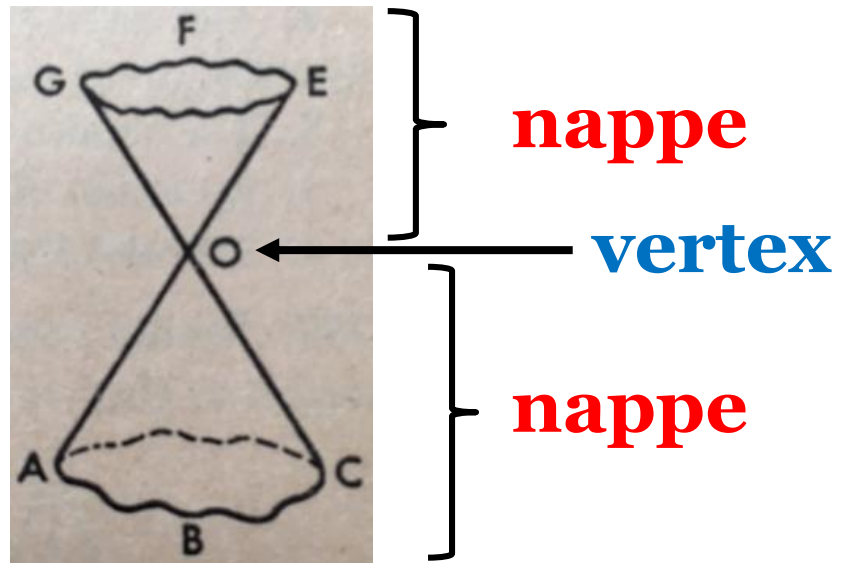
- 1.** A regular triangular prism whose altitude is 2.5 ft and each side of whose base is $2\sqrt{3}$ inches is inscribed in a right circular cylinder. Find the total area of the cylinder in square inches.
- 2.** Find the total areas and volumes of the rectangular prism and the cylinder if the prism with square bases and 6-cm diagonal is circumscribed about a right circular cylinder of height 12 cm.
- 3.** An element of an oblique circular cylinder intersects the base at an angle of 45° . If the element is $2\sqrt{6}$ in and the circumference of the base is 18π in, find the total area and volume of the cylinder.
- 4.** An iron pipe 10 ft long has an internal diameter of 1 foot. If the iron is one-half inch thick, find the volume of metal in the pipe. Express the answer in cubic feet.

CONE



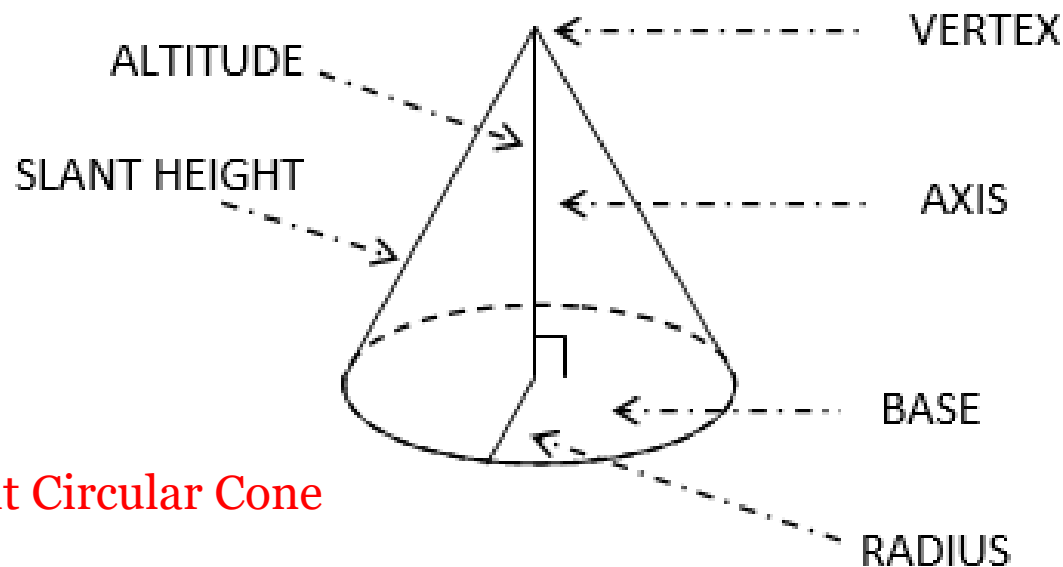
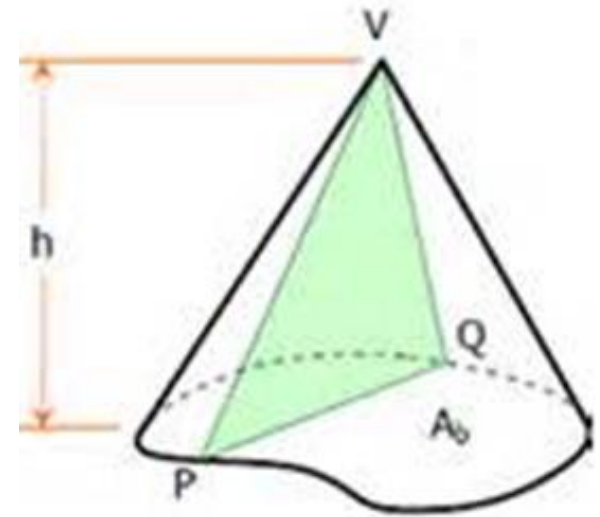
Closed Curve Conical Surface

is a surface generated by a moving line which constantly touches a closed plane curve and which passes through a fixed point outside the plane of the fixed curve.



CONE

is a closed curvilinear surface formed by one nappe of a closed curve conical surface and a section of a plane which intersects all the elements.



Right Circular Cone

Formulas for Right Circular Cone

Lateral Area (LA) – area of the lateral surface

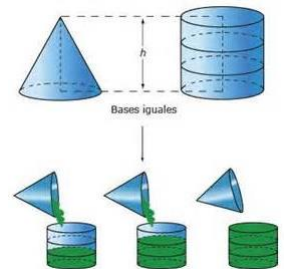
$$- \mathbf{LA} = \pi r \ell$$

Total Surface Area (TA) – sum of the LA and the areas of the bases.

$$- \mathbf{TA} = \pi r \ell + \pi r^2$$

Volume – product of the area of the base and the height of the circular cylinder

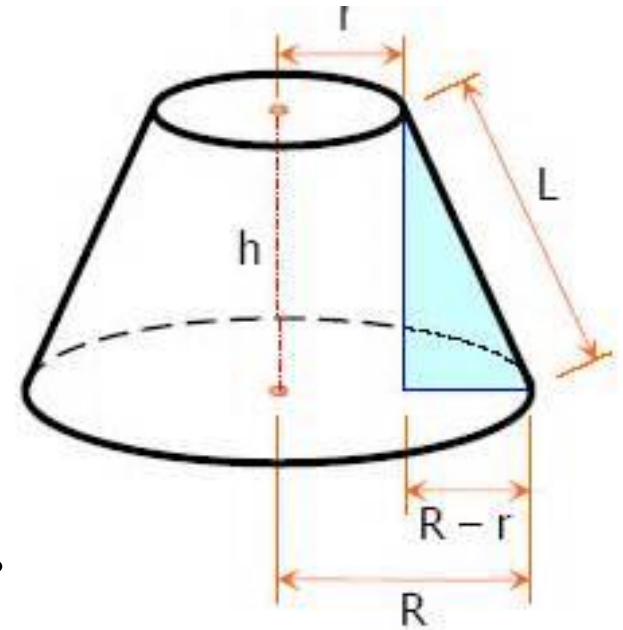
$$- \mathbf{V} = \frac{1}{3} \pi r^2 h$$



Exercises

- 1.** A right triangle, whose legs are 8 ft and 15 ft, is revolved about its longer leg as an axis. Find the lateral area generated in one complete revolution.
- 2.** Find the LA and V of a right circular cone whose slant height is 15-cm long and whose TA is 100π sq. cm.
- 3.** A right circular cone is inscribed in a cube having diagonal length of $10\sqrt{15}$ cm.
 - (3.1)** Find the total area and volume of the cone.
 - (3.2)** Find the volume of the region between the cone and the cube.
- 4.** The volume of a right circular cone is 243π cu. ft. and the slant height makes an angle of 45° with the base. Find the altitude of the cone.

Frustum of a Cone is a closed curvilinear surface included between the base of the cone and a section formed by a plane parallel to the base and cutting all the lateral edges.



Frustum of a Right Circular Cone

Formulas

$$\text{Lateral Area (LA)} = \frac{1}{2} l(C_1 + C_2)$$

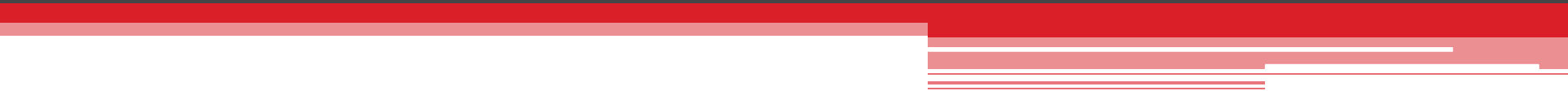
$$\text{Total Surface Area (TA)} = \text{LA} + A_1 + A_2$$

$$\text{Volume} = \frac{\pi h}{3} \left((r_1)^2 + (r_2)^2 + r_1 r_2 \right)$$

Exercises

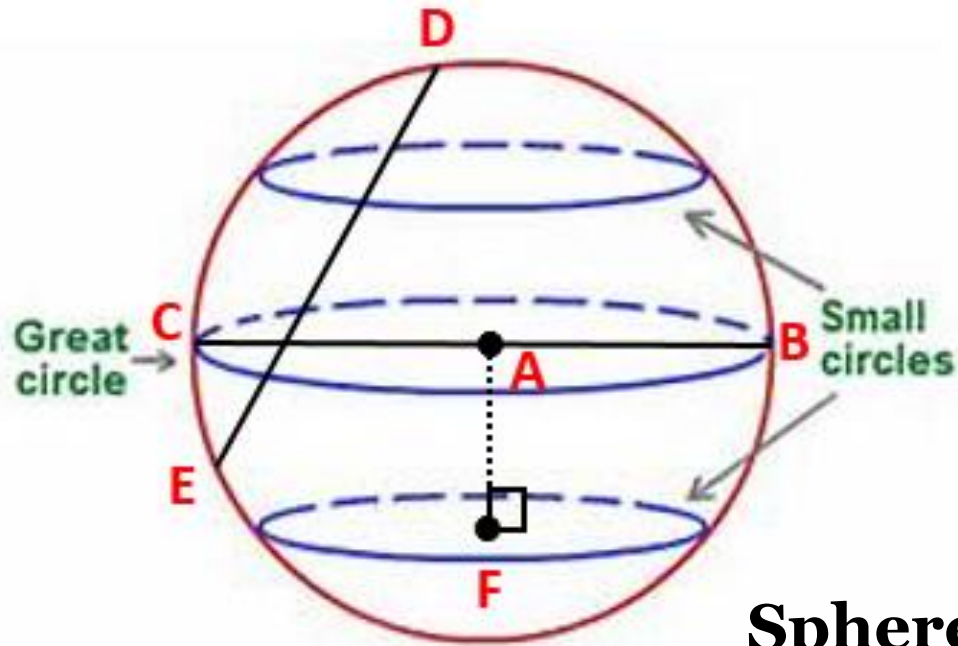
5. The diameter of the lower base of a frustum of a right circular cone is 24 ft., the radius of the upper base is 8 ft., and the altitude is 15 ft. Find the TA of the frustum.
6. The radii of the bases of a frustum of a right circular cone are 5 in. and 3 in., respectively, and the slant height is $2\sqrt{11}$. Find the volume.
7. The shorter leg and longer leg of a right triangle are respectively the 6-cm axis and the radius of the upper base of the frustum of a right circular cone. If the hypotenuse of the triangle is $2\sqrt{21}$ -cm long and the slant height of the frustum makes an angle of 60° with the lower base, what is the (7.1) volume of the frustum? (7.2) TA of the frustum?

SPHERE



SPHERE

is a closed curvilinear surface all points of which are equidistant from a point within called the center.



Sphere A

Radius of a sphere is the line segment whose endpoints are ____ and ____.

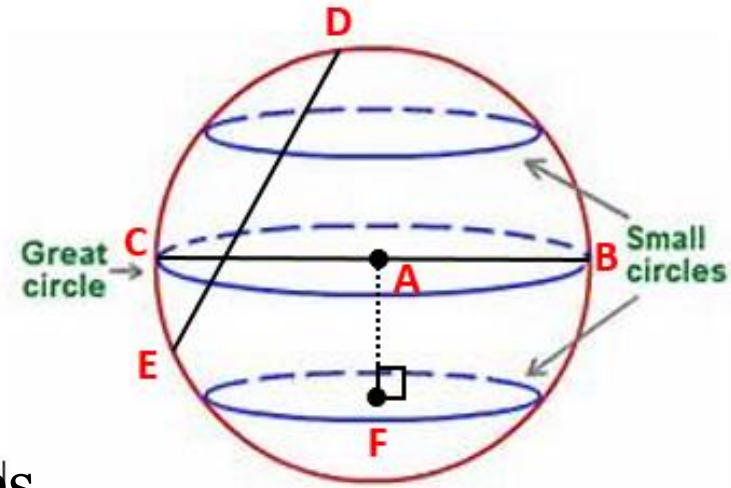
Chord of a sphere is the line segment whose endpoints are ____.

Diameter of a sphere is a ____ the passes through the ____ of the sphere.

Circle of a sphere is the plane section of the sphere. The circles could be *small circles* or ____.

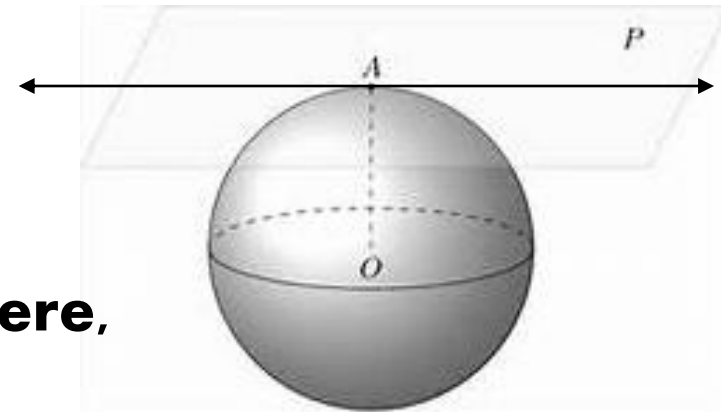
Great Circle of a sphere is the circle that contains the ____ of the sphere. While **Small Circle** is a circle that does not ____.

- **All *great circles* are congruent.**
- **The *great circles* bisect the sphere and each other.**



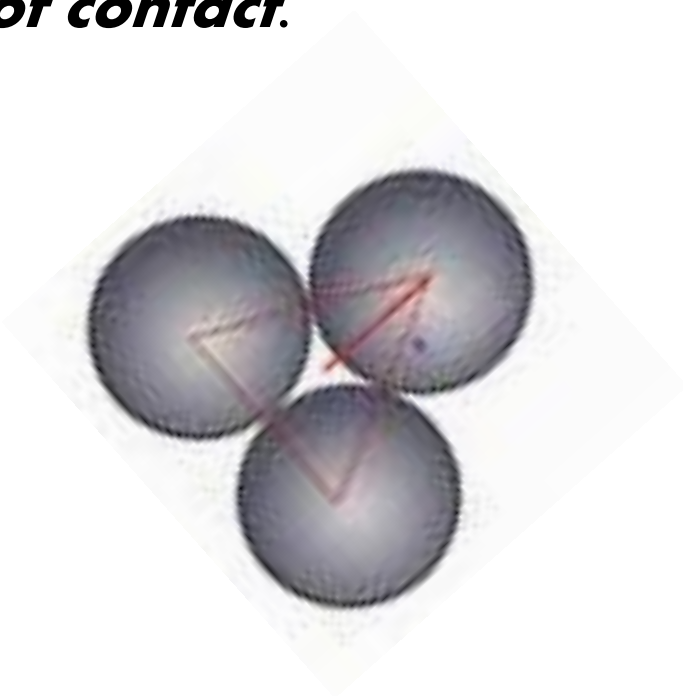
A line or a plane is tangent to a sphere if it passes through the sphere at _____. This point is called *point of contact*.

If a line or a plane is tangent to a sphere, then the _____ of the sphere is *perpendicular* to the given line or plane at the *point of contact*.



Example

Three spheres are arranged so that each is tangent to the other two. If the distances between their centers are 11 cm, 14 cm, and 13 cm, respectively, what is the length of the radius of each sphere?



Exercises

A line from the center of the sphere to the center of a circle of the sphere is perpendicular to the plane of the circle.

1. A circle of a sphere has a radius of 5 in. If the diameter of the sphere is 26 in., how far is the plane of the circle from the center of the sphere?
2. In a sphere of radius 9 in., an equilateral triangle is inscribed in a circle of the sphere. If the side of the triangle is 6 in., how far from the center of the sphere is the plane of the triangle?

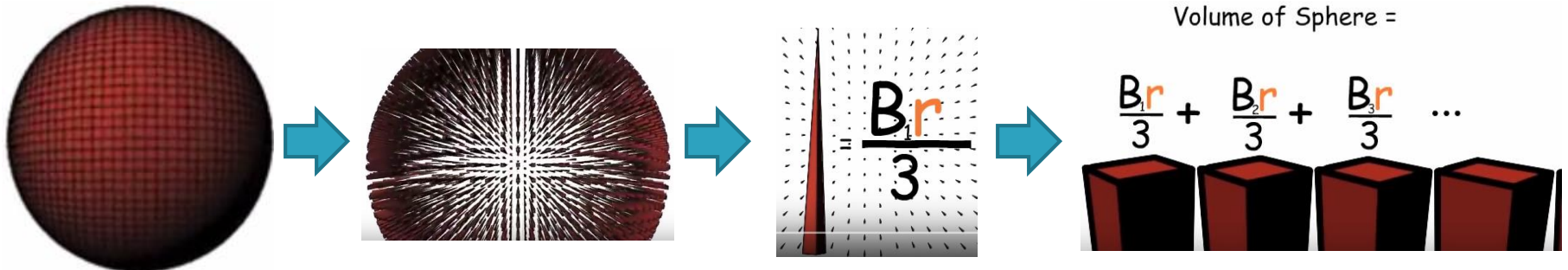
Formulas for the Sphere

Surface Area The surface area (SA) of a sphere of radius r is four times the area of a great circle. That is,

$$SA = 4\pi r^2$$

Volume The volume (V) of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$



Exercises

A sphere is *inscribed in* a polyhedron if it is tangent to each of the faces of the polyhedron.

A sphere is *circumscribed about* a polyhedron if all vertices of the polyhedron lie on the sphere.

1. A sphere of radius r is inscribed in a cube of edge e . Find the volume of the sphere if: **(1.1)** the diagonal of the cube is $6\sqrt{6}$ in. **(1.2)** The area of the sphere is $192\pi \text{ cm}^2$.
2. The radius of a sphere circumscribed about a regular tetrahedron is 9 in. Find the volume between the sphere and the tetrahedron.

A sphere is *inscribed in* a regular tetrahedron: $r = \frac{h}{4}$

A sphere is *circumscribed about* a regular tetrahedron: $r = \frac{3}{4}h$