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Effect of Problem Posing on Students' Conceptual Understanding and Procedural Knowledge in Geometry

Ramon A. Raza
Philippine Normal University

EFFECT OF PROBLEM POSING ON STUDENTS' CONCEPTUAL UNDERSTANDING AND PROCEDURAL KNOWLEDGE IN GEOMETRY

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by
RAMON A. RAZA
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APPROVAL SHEET

This thesis entitled **"EFFECT OF PROBLEM POSING ON STUDENTS' CONCEPTUAL UNDERSTANDING AND PROCEDURAL KNOWLEDGE IN GEOMETRY"** prepared and submitted by **RAMON A. RAZA** in partial fulfillment of the requirements for the degree Master of Arts in Education with specialization in Mathematics Education is hereby recommended for approval and acceptance.

RENE R. BELECINA, Ph.D.
Adviser

GLADYS C. NIVERA, Ph.D.
Adviser

Approved in partial fulfillment of the requirements for the degree Master of Arts in Education with specialization in Mathematics Education by the Oral Examination Committee.

ZENAIDA Q. REYES, Ph.D.
Chairperson

WILMER G. MARQUEZ, M.A.
Member

ALLAN S. REYES, Ph.D.
Member

ELISA S. BACCAY, Ed.D.
Member

Accepted in partial fulfillment of the requirements for the degree Master of Arts in Education with specialization in Mathematics Education.

____
Date

ZENAIDA Q. REYES, Ph.D.
Dean
College of Graduate Studies and
Teacher Education Research

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DEDICATION

For our Heavenly Father who has blessed me with knowledge and skills

For my family with their boundless support and unparalleled love

For my girlfriend who has been a source of inspiration

For my close friends with their constant smiles

For every colleague who aims to use their talents with purpose.

And for every student who deserves to learn the wonderful world of mathematics.

ABSTRACT

NAME: Ramon A. Raza

TITLE: Effect of Problem Posing on Students' Conceptual Understanding and Procedural Knowledge in Geometry

KEY CONCEPTS: problem posing, conceptual understanding, procedural knowledge

DEGREE: Master of Arts in Education

SPECIALIZATION: Mathematics

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Gladys C. Nivera, Ph.D.

The main purpose of the study was to determine the effect of problem posing on students' conceptual understanding and procedural knowledge. The study utilized a quasi-experimental design specifically the Pre-test Post-test Control Group Design. The instruments used were conceptual understanding and procedural knowledge tests, and students' perception of problem posing. The participants of the study were two groups of junior high school students. The control group was taught using the traditional method while the experimental group was taught using problem posing.

The effect of problem posing on student's conceptual understanding and procedural knowledge was determined by comparing the students' pre-test and post- test mean scores. The results revealed that the students in the experimental group obtained the same results with the control group in conceptual understanding and procedural knowledge. Thus, there was no significant difference between the experimental group and the control group in terms of conceptual understanding and procedural knowledge posttests. This implies that problem posing has the same effect as the usual method on students' conceptual understanding and procedural knowledge. Further, the students perceived that problem posing was helpful in improving their problem solving skills.

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The Problem and Its Background

"Let us value every adversity and challenge that comes our way. Let it be the fire that forges the steel in our character, the rubbing that polishes the gem in our souls."

– Vicente Hao-Chin, Jr. (2008)

Introduction

With the continuous advancement of today's education, mathematics teachers are facing major changes not only in the content they teach, but also on the manner how they teach. The students at present need an advanced education, more than the kind offered them the past few years. Students' mathematical ability needs to be cultivated and developed through a good teaching approach. Numerous strategies, models, and approaches were studied in the past to meet various student needs for learning, particularly in mathematics, but still students see the world of mathematics as a complex series of problems and equations to be solved, detached from their own reality. For instance, teachers usually present mathematics as prearranged sets of formula to lecture, followed by mathematical problems to be solved by the students, thus separating the role of students in creating mathematics. Further, students view mathematics as a difficult subject, which inadvertently hinders their learning development and academic achievement. For example, the result of the National

Achievement Test school year 2014 – 2015 showed that Filipino students attain on the average a mean percentage score of below 50% in Mathematics. The institutional study of Gapo (2012) revealed that from school year 2009 – 2011, the students' average in mathematics achievement test was 76.56 %, indicating poor performance. Such alarming results serve as a challenge to all mathematical educators and researchers to further improve students' mathematical understanding. To address the challenge, teachers should prepare their students through increased use of appropriate strategies or processes in teaching mathematics (Gapo, 2012).

Word problem-solving approach in teaching mathematics is vastly utilized by many teachers. Although correct calculation for solution is required in mathematical word problems (Leung, 2009), mathematical word problems differ from calculation problems because of the addition of linguistic information. Whereas, a calculation problem is already set up for solution, a word problem requires students to construct a problem model by identifying what information is missing, determining the number sentence that incorporates the given and the missing information, and deriving the calculation problem for finding the missing information. As might be expected therefore, word problems are source of difficulty for many students because they often experience the language deficits that specifically account for mathematical problem-solving difficulty (Leung, 2009).

In Naval Architecture and Marine Engineering Institute (NAMEI) Polytechnic, high school students have difficulty in problem solving which consequently leads to a poor performance in mathematics subjects (Gapo, 2012). Solving word problems, which is a major component of the high school mathematics curriculum, is a form of mathematical cognition that is distinct from calculation skill (Fuchs, Stuebing, Fletcher, Hamlett, & Lambert, 2006). There appears to be a need to identify effective strategies to improve word problem solving early among the secondary students.

Mathematical problem solving is a cognitive processes involving intricate procedure in which the students must first understand the problem, find a suitable strategy, use that strategy, and then evaluate and reflect on the final answer. The challenge is to translate this complex mental process into reality by understanding the students' world of mathematics and their perception of the subject. Priest's (2009) study enumerated such perceptions reflecting misconceptions on mathematics. First in the student perception is the idea that mathematical problems should be computed quickly and in a few steps. Next to this is their view that mathematics means following rules and memorizing and it is computation. Third is the perception that what matters in a problem is NOT the meaning, but the size of the numbers. Following this is their idea that non-routine problems should only be for the "bonus". Also, they believe that only geniuses can generate problems in mathematics. They claim that to solve problems, they should look for one "key" word, resulting in a single procedure for problem solution. Finally, they view that the teacher should provide all the information to learn, requiring students to merely regurgitate it at the appropriate time.

The teacher could correct these flawed beliefs of students by providing a clear distinction between the roles of the teacher and students in creating mathematics. Typically in a classroom setting the role of the teacher is to give a problem, and for the student to solve. But problem posing is recognized as an important component in the nature of mathematical thinking and problem-solving task (Priest, 2009) since the former takes a complementary role in further portraying mathematics as an empirical subject, one that is integrated and creatable.

With purposeful and consistent planning, teachers can give opportunities for students to reflect on their own thinking and make connections between mathematics and the real world. These learner-led opportunities spring from such creative experiences teachers offer in the classroom (Kwek & Lye 2015). With the importance of problem posing being recognized, the students are given more chance to create their own mathematical problems as it involves generating new problems and reformatting a given problem (Leung, 2009).

Studies suggest that student problem posing foster student creativity (Yuan & Sriraman, 2011) and it is more likely to connect mathematics to students' own interests, something that is often not the case with traditional textbook problems. In addition, problem posing might facilitate students' independence from their teachers and textbooks.

Cunningham (2004) averred that posing problems improve students’ reasoning and reflection when they are given the opportunity to create mathematical problems rather than the teacher doing it for them. Further, it fosters a sense of ownership for their own learning since they have a role in constructing their own knowledge. This ownership of the problems results in enthusiasm towards the process of learning mathematics as well as a high level of engagement and curiosity (Lavy & Shriki, 2007). When students begin posing their own original mathematical questions and see these questions become the focus of discussion, their perception of the subject is profoundly altered. When they get to spend time working on these questions, their ownership of the experience produces excitement and motivation. Hence, the discussions and activities that follow help students expand their problem-posing repertoire and promote the habit of creating new problems.

However, problem-posing requires more than the mere tweaking of a pre-existing question. With structured coaching, students will also develop a greater ability to assess how interesting and productive their new questions are likely to be. As students gain sophistication in this activity, their problems will move from aimless variations toward some clearer mathematical purpose (Priest, 2009).

Problem posing activities being a highly cognitively demanding task has a positive impact on students' mathematical development. In his research, Pittalis (2004) concluded that the four cognitive processes (filtering, editing, comprehending and translating) mediate the ability to pose problems. Lavy and Shriki (2007) also stated in their research that involvement in problem posing has the potential to develop the mathematical knowledge and consolidate basic concepts. Problem posing activities can promote students' conceptual understanding, foster their ability to reason and communicate mathematically (procedural knowledge), and capture their interest and curiosity (Cai, Moyer, Wang, Hwang, Nie, and Garber 2010).

It is a widely accepted norm that a teacher's role is as ally that guides and instructs the students. Less attention is given to the complementary possibility that teachers as an adversary can be just as effective. By problem posing, teachers challenge the students and question the mathematical problem students pose, and by doing these the teachers compel students to make a defensible argument that would train their conceptual understanding and procedural knowledge.

The problem poser would usually need to consider the nature of the context and possible solution paths to the problem posed. This process of problem generation and consideration for multiple solution paths provides excellent opportunities for fostering divergent and flexible thinking. Such thinking habits not only enhance problem-solving skills, but also help to reinforce and enrich basic mathematics concepts (Kwek & Lye, 2015). Initial problem exploration fits with the recommendation from the mathematics education literature that students have opportunities to struggle—to figure out something that is not immediately apparent (Hiebert & Grouws, 2009). Comparison, self-explanation, and exploration are all promising instructional methods for promoting conceptual understanding and procedural knowledge, as are sequencing problems so that conceptual relations are more apparent (Canobi, 2009). The analysis of the posed problems makes the participants map the level of their own notions and concepts, understanding, various interpretations and makes them realize possible misconceptions and erroneous reasoning. It is an impulse for work on themselves, a reeducation as they rethink their own knowledge.

It was confirmed that the result of inclusion of problem posing into the curricula is a better approach to problem solving. It stimulates the use of various representations, construction of knowledge nets, development of creative thinking, improvement of attitude to mathematics, and increase in self-confidence (Ticha, 2010).

Overall, there is an extensive evidence from a variety of mathematical domains indicating that the development of conceptual understanding and procedural knowledge of mathematics is often iterative, with one type of knowledge supporting gains in the other knowledge, which in turn supports gains in the other type of knowledge. Conceptual understanding may help with the construction, selection, and appropriate execution of problem-solving procedures. At the same time, practice on implementing procedures may help students develop and deepen understanding of concepts, especially if the practice is designed to make underlying concepts more apparent. Both kinds of knowledge are intertwined and can strengthen each other over time (Johnson, and Schneider 2013).

As Einstein (1938) once said, posing an interesting problem is more important than solving the problem. Problem posing requires creativity, imagination and exploration, encouraging students to write their own word problems, and it results with their increasing participation in classroom environments. Hence, with these concepts in mind, the recent study dwells on trying out an instructional method in mathematics with the aim of helping the students learn the concepts and appreciate the process of doing so. Concomitantly, it examined the effectiveness of problem posing on increasing the mastery level of students in mathematics.

Considering the studies done in the Philippines, problem posing has long been under the shadow of problem solving, which should not be the case as the literature suggest. Further, many international studies have been done about problem posing but no study had focused on the effect of problem posing on the conceptual understanding and procedural knowledge of the students.

Conceptual Framework

Task theories and constructivism provide the theoretical foundations for problem posing. Dickerson (1999) stated that “if educators accept the constructivist theory of learning as a progressive organization and reorganization of ideas under the stimulus of a dynamic environment, then problem posing which seeks to build meaning, relevance, and logic through a learner’s own language and experiences seems a promising pedagogical approach”. “According to constructivists, students actively build mathematical knowledge when they strive to make a sensible pattern out of the confusion of the world around them” (Cobb, Yackel, & Wood, 1992; Battista, 1999).

Further, learning in the social area between students or between teachers and students is firstly constructed, one which is termed as the inter-psychological arena. Students learn and gain behaviors in this area with reflections of the social interactions taking place. The social nature of problem posing facilitates learning and cognitive change where students write and share problems together (Winograd, 1990).

Pajares (1998) mentioned, “Collaboration in learning is valued as students learn to establish and defend their own positions while respecting the positions of others”. “As students listen to others’ problems, they are able to clarify and refine their own concept understanding, promoting their own ability to go from solution of specific problems to production of generalized problem-solving strategies” (Cappo & Osterman, 1991). Students feel more equal in status with their peers, than with teachers, so they may be more likely to ask questions to seek classifications within their small groups (Dickerson,1999).

The inscription feature of problem posing takes full advantage of learning as it forces students to take an idea, interact with it cognitively, something likened to thinking, analysis, and synthesis. Writing problems advances students’ reasoning and communication skills, at the same time, expounding their understanding of mathematical procedures and arranging for relation across the curriculum (Burton, 1992; Richards, 1990; Matz & Leier, 1992).

“When students write their own problems, many of the linguistic and reading difficulties of solving textbook word problems may be reduced, if not eliminated, as their familiarity with language maximizes success” (Burns & Richards, 1981; Wirtz & Kahn, 1982; Wright & Stevens, 1983; Kilpatrick, 1987; Resnick & Resnick, 1996). When students create their own mathematical problems, they practice their own ideas, giving them time to reason, and thus increasing their understanding and procedural knowledge (Geeslin, 1997; Cappo & Osterman, 1991).

Bell and Bell (1985) concluded that writing problems may stimulate learning because the writing procedure itself is self-paced and result-oriented. When students create their own problems, they can increase an appreciation and understanding of the fundamental component of problems, developing their skills to sense and reason number relationships and simplify these concepts to the real world (Dickerson, 1999). “When students use their own language, vocabulary, grammar, interests and contexts, the connections between the old and new are made strongly, quickly, and easily” (Lesh, 1981; Walter, 1992; Winograd, 1990).

The usage of problem posing diminishes dependence on the one correct-response syndrome of most textbook problem-solving presentations, and encourages the development of realistic curiosity of learners (Brown, 1976, 1983; Walter & Brown, 1977; Burton, 1984; Cromwell & Sasser, 1987; Lerman, 1987; Silver & Mamona, 1989a; Silver & Mamona, 1989b; Silver et al., 1990). As a result, learners are motivated to improve their own mathematical skills and knowledge to simplify, specify and investigate, while at the same time refining their mathematical skills in terms of accuracy, clarity, and organization (Polya, 1945; Wright & Stevens, 1980; Petersen & Jungck, 1988; Solorzano, 1989). “Problem posing encourages students to clarify, refine, and consolidate their thinking” (NCTM, 1989, p.9; Kliman&Richards, 1992).

Dickerson (1999) stated that,

> *"Problem posing, which allows students to write problems using their own language, grammar, syntax, and context, may provide a viable alternative to traditional problem-solving instruction. It encourages students to use mathematics to make sense out of their world by building connections between previous and new knowledge through authentic, personally meaningful experiences."*

He discovered that problem-posing instruction tends to be an effective approach to improve the problem-solving achievement of students. He found that "problem posing suggests a better instructional strategy that meets the goals of NCTM for reasoning, communication, and connections within mathematics".

Further, problem posing has endless roots traced back to Polya's (1957) four phase model of problem solving, which states that the problem solver must "(1) understand the problem, (2) devise a plan, (3) carry out the plan, (4) and then look back at his/her action. The looking back stage involves checking for correctness and determines the best solution as well as asks the problem solvers to pose or formulate original problems that are in some way related to the problem just solved".

Problem posing, lengthways with problem solving, is centered to the discipline of mathematics and the nature of mathematical knowledge (Silver, 1994). In mathematics education study, problem posing has been practiced both as an instruction technique and an activity. For example, "in her one-year study involved designing and implementing a problem posing program for fifth grade student’s number sense and novel problem solving skills, English (1999) found substantial developments in (a) student’s recognition and utilization of problem structures, (b) their perceptions of, and preferences for, different problem types, and (c) their development of diverse mathematical thinking in contrast to those who did not participate in problem posing program”.

In problem posing, sequences of process queries or elements of problem posing, are asked to the learners to come up with a new created problem. Its process starts with selection, an act of choosing or picking mathematical objects from a number or group topics which are appropriate to their interests (Amato, 2004).

The object selection is the first action. The students analyze what type of mathematical materials are appropriate ones so as to associate them. This means that students are the ones who have the absolute plan and decide for what they learn. In turn, taking decisions is regularly conditioned by interests of the students.

Once the material has been chosen, the classification of the problem components takes place. In this action, the process to be followed is to breakdown of its components (analysis), and then the acquired data is systematized and associated in accordance to certain criteria. In the same manner, there is a respective integration of components (synthesis) in a way that students can make up some other more complex components of the material. “A flat figure can be taken as an example, which is made up by segments, forming triangles by certain trios which are part of the figure too" (Cruz, 2002). In the process of selection, classification is adapted by needs of pedagogical order such as how to assess, stimulate, illustrate, among other processes, so that it depends on a sensible material to the students. "An objective scanning in the process of problem–posing demands to clear out not only its shape, but its content as well. Such clearing out is possible if the problem–formulating act is seen as a problem itself". According to Pehkonen (1995) "object scanning is about an open problem, so that it starts from an initial situation which can be a precise one or not; and it also deals with knowing the final goal, which is essentially not precise, as well as of the certainty that the process of obtaining the new problem is unknown as priori".

Classification is the act or process of integration of components – also known as synthesis, a process that students can make up more complex components of the material in groups or categories in accordance to a certain criteria (Cruz, 2002). Classification of components takes place after the selection of material. Through this procedure, the process to be followed is taking its components separately – commonly known as analysis, and then the gained data will be systematized and matched with accordance to identified standards.

Transformation is the process of shifting (as by rotation or mapping) one configuration or expression into another according to mathematical rule, like "a modification of variables or coordinates in which a function of new variable or coordinate is substituted for each original variable or coordinates".

Object transformation can be partial, total or identical. After the generalization of certain emerging elements during classification, changes may take place. Such logical operation is very complex and may have a synthetic or analytic nature. "Basically, the generalization facilitates the change from a specific concept to a generic one by removing from its content the clues specified" (Priest, 2009).

Association is a process of forming mental connections or bonds between ideas and concepts that include the elements which are separated by abstraction and then move on to be related with a joint of the mathematical concepts in an recognized topic (Cruz, 2002).

Searching is a process of looking into or over carefully or thoroughly in an effort to find or discover something, or to examine in seeking something. Once the information is synthesized, the questions formed by students are valued with the aim of selecting one or several of the sets of questions (Leung, 2009).

These changes may take place after the generalization of certain emerging elements during classification. Such logical operation is very complex and may have a synthetic or analytic nature. Essentially, the generalization facilitates the change from a specific concept to a generic one by removing from its content the clues specifying it. It is also possible to transform the object by using analogies, in this case it is about a reasoning on the pertaining of a given sign with another object departing from the homology of substantial sign with another object. According to the character of the information transferred from the modal to the prototype, the analogy can be of properties or relations. After having transformed or not the object, the subsequent action comprises the association of concepts. Therewith, the elements obtained during the classification are separated by abstraction and then moved on to be related with a joint of the mathematical concepts. Such elements may link with certain properties (area, perimeter, monotony …) or with a certain relationship (similarity, parallelism, congruence).

Decision making is again necessary since the subject must select a subset of such associated concepts.

In this study, conceptual understanding was assumed to have been affected by integrating problem posing. It involved a situation where students were able to recreate formulas and proofs without the rote process. Moreover, students were allowed to make choices and apply their understanding through active engagement (Silver, 1994). Conceptual understanding enabled the students to solve mathematical problems in various forms and novel settings. Students with high levels of conceptual knowledge were capable of solving problems that they have never come across before (Walter, 2007).

The National Research Council in its review of the mathematics education research literature defines conceptual knowledge as ‘comprehension of mathematical concepts, operations, and relations’ (Kilpatrick, Swafford, & Findell, 2001). This type of knowledge is sometimes called conceptual understanding or principled knowledge. A number of mathematics education researchers (Baroody, Feil, & Johnson, 2007) suggested that conceptual knowledge should be defined as ‘knowledge about facts, generalizations, and principles’ without requiring that the knowledge be richly connected.

On the other hand, procedural knowledge in mathematics is knowing what steps and action should be done to solve a problem using conceptual understanding. The procedures can be (1) algorithms—a predetermined sequence of actions that will lead to the correct answer when executed correctly, or (2) possible actions that must be sequenced appropriately to solve a given problem (e.g. equation-solving steps). This knowledge develops through problem-solving practice, and thus is tied to particular problem types. Further, ‘It is the clearly sequential nature of procedures that probably sets them most apart from other forms of knowledge’ (Bethany Rittle-Johnson, and Michael Schneider 2013).

Moreover, procedural knowledge is another variable theorized to be greatly affected when integrating problem posing in teaching. Procedural knowledge can aid in the understanding of conceptual understanding (Silver, 1994).

Procedural knowledge is the ability to execute action sequences to solve mathematical problem. To assess procedural knowledge, researchers typically use routine tasks such as counting a row of objects or solving standard arithmetic computations because students are likely to use previously learned step-by-step solution methods to solve the problems (Hiebert & Wearne, 1996).

Problem posing was used in the study and viewed as a learning activity, where the student posed questions in response to different circumstances and real-life situations, which are considered another mathematical problem (Leung, 2009). On the other hand, conceptual understanding and procedural knowledge are both mathematical knowledge that may be affected using problem posing.

Figure 1 shows the conceptual paradigm of this study. It can be seen from the figure that the main objective of the study was to find out the effect of problem posing on conceptual understanding and procedural knowledge of students towards geometry.

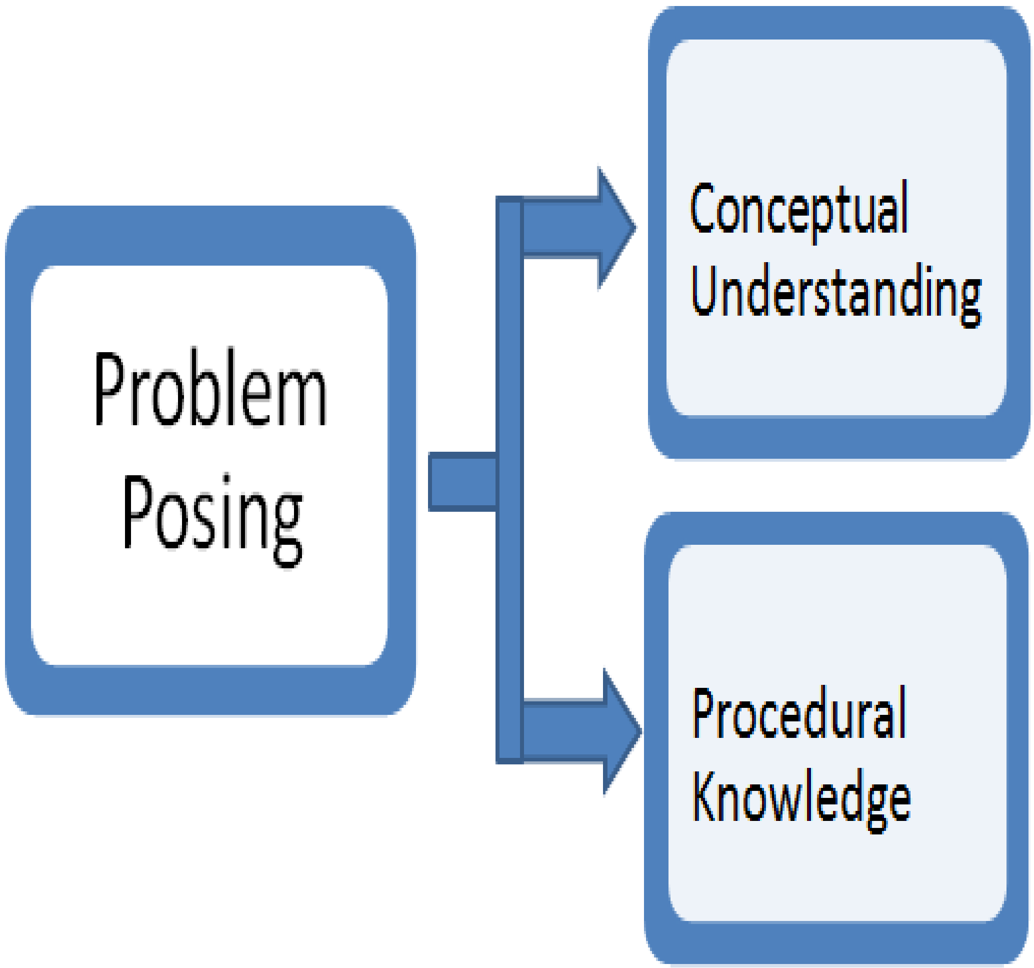

Figure 1. Conceptual Paradigm

Statement of the Problem

The purpose of this study was to determine the effect of problem posing on students' conceptual understanding and procedural knowledge in Geometry.

Specifically, the study sought to answer the following questions:

1. How may the students in the control group and experimental group be described in terms of the following?
   1.1. conceptual understanding
   1.2. procedural knowledge
2. To what extent does problem posing affect students'
   2.1. conceptual understanding?
   2.2. procedural knowledge?
3. What are the students' perceptions on the use of problem posing?

Hypotheses of the Study

In this study, the following hypotheses were statistically tested:

1. There is no significant difference in the conceptual understanding of students who receive instruction with problem posing and those who receive traditional method of teaching.

2. There is no significant difference in the procedural knowledge of students who receive instruction with problem posing and those who receive traditional method of teaching.

Significance of the Study

This study is deemed significant for the benefits it offers the following people:

Teachers. The study was initiated with hope that problem posing may be used as an alternative option by adding a feature or task in mathematics class. The findings could also help teachers and future educators to develop their instructional methods that will promote the growth of their student's performance in problem solving. The study may also render information regarding student's conceptual understanding and procedural knowledge when challenged with problem posing.

School Administrators. This study may provide reference material for school administrators needed in preparing mathematics curriculum, instructional materials, and additional feature or task in class that can improve mathematics instruction.

Textbook Writers. The findings of the study will provide and recommend further data for creating textbooks and instructional materials that may help students develop their conceptual understanding and procedural knowledge through problem posing. Such texts may consider the elements and structure of problem posing as an additional feature or task in math class.

Policymakers. The facts and information derived from this study about problem posing could serve as a baseline data for the development of future curricula, instruction, and assessment materials.

Future Researchers. The result of the study may encourage researchers especially mathematics educators to further investigate and expand the presented research materials on problem posing. Further, the findings of this study will provide them baseline information in exploring more knowledge about problem posing.

Scope and Delimitation of the Study

This study was limited to the investigation of the effect of problem posing on student's conceptual understanding and procedural knowledge in Geometry. Specifically, the participants of this study were Grade 9 junior high school students of a Maritime Institution located at Mandaluyong City, during the School Year 2015-2016. The study was delimited on the topic of Triangle Similarity in geometry. Further, this study was conducted for a period of one month (approximately four weeks) of the school year. Finally, conceptual understanding and procedural knowledge were measured through the utilization of tests on the said domains. Said tests were validated by the experts prior to actual administration. Thus, the study was limited to the results gathered using these instruments.

Definition of Terms

For better and clearer understanding of this study, the following terminologies were defined as used in this study:

Conceptual Understanding. This refers to the knowledge that involves a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics (Hope, 2006). In this study, it is the students' knowledge on how they understand the geometrical terms, theorems and concepts in Geometry. Students' knowledge on conceptual understanding was measured using the conceptual understanding test.

Control Group. It refers to the group that does not receive treatment and is then used as a benchmark to measure how the other tested subjects do. In this study, it was the group of students taught using traditional way of teaching.

Experimental Group. It refers to a group that receives the variable being tested. The experimental group was compared to a control group that did not receive the tested variable. In this study, this refers to the group of students taught with the use of problem posing. Further, this group was used to answer the questions raised in the experiment.

Problem Posing. It refers to the process of reformulation of a given problem, creation of a new problem, or generation of a problem (English & Grove, 1998). In this study, it is defined as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as a meaningful mathematical problem. Further, it is a free problem-posing situation where students in experimental group were asked to create a new problem. Students were asked to write a new question about a specific topic on the materials selected and were used as a guide to formulate the new problem. It involves the activities like selection, classification, transformation, association, and posing.

Procedural Knowledge. This refers to the familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols (Priest, 2009). In this study, procedural knowledge refers to the students' skills on applying the theorems and concepts in solving problems in Geometry. It was measured using the procedural knowledge test.

CHAPTER II

Review of Related Literature

In this chapter, the literature related to the present study were reviewed and discussed thematically such as the problem posing instruction, conceptual understanding, and procedural knowledge.

Problem Posing

In the literature of problem posing from diverse viewpoints, researchers have identified categories for the problem-posing conditions. Stoyanova and Ellerton (1998) presented three main categories of problem posing namely *“free, semi-structured,* and *structured”* for the problem-posing situations.

In a *“free problem-posing situation”*, students are merely asked to create an absolutely new problem on a basis of artificial or realistic situations. Students may be prompted to write a new question about a specific topic of a given content. For example, “a teacher may ask learners to write a problem about quadratic function without giving any starting point or data”. At this phase, students are required to be prepared to collect all of the main parts of a problem.

Semi-structured situation is a problem-posing condition where students are exposed to an agreed condition and are invited to discover the arrangement or to finish the problem using knowledge, skills, concepts and connection from their prior mathematical experiences. This includes inciting students to pose problems from the collected data or from the given response, such as posing a class of problems connected to a detailed solution method; posing sequences of interrelated problems to a specific concept(s); and posing problems focused on pictures, equations, and inequalities, to name a few. For example, “a teacher may ask students, *try this: Consider that you have 2 coins to toss*, and want them to complete the sentence in order to pose a problem”.

In a *“structured problem-posing situation”*, a well-designed problem or problem solution is prearranged, and the goal is to create new problems which transmit, someway, to the agreed solution or problem.

In the field of mathematics education, “problem posing has long been under the shadow of problem solving, yet researchers started to be aware of the potential of problem posing. In fact, there has been a growing recognition of the need to incorporate problem-posing activities into the mathematics classrooms” (Stoyanova, 1998). For instance, it was indicated that “problem posing is a significant component of the mathematics curriculum and is considered to lie in the heart of mathematical activities” (Brown &Walter, 1983, 1993; Kilpatrick, 1987; Moses, Bjork & Goldenberg, 1990; National Council of Teachers of Mathematics [NCTM], 1989, 1991; Silver, 1990, 1994).

The existence of problem-posing task in the curriculum can raise more various and flexible thinking, boost students’ problem-solving abilities, widen their awareness of mathematics, and enhance and associate essential mathematical theories (Brown & Walter, 1993;English, 1996, in press a; Silver & Burkett, 1993; Simon, 1993). Further, problem posing activities may offer significant comprehension as students understand mathematical concepts and procedures, as well as their insight of, and attitudes towards problem solving and mathematics as a whole (Brown &Walter, 1993; English, 1996; Van den Heuvel-Panhuizen, Middleton, & Streefland,1995).

The significance of an ability to pose noteworthy problems was acknowledged by Einstein and Infeld (1938), who inscribed:

> *"The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, require creative imagination and real advance in science"*
>
> *(Ellerton&Clarkson, p.1010).*

One means of providing students with such chances that kindle higher-order thinking is to let the students transfer out investigations, particularly open-ended investigations, where students pose the problem to be investigated and plan the actions to respond on the inquiry (Chin&Kayalvizhi, 2002). Subsequently, investigations can provide students with the chance to investigate problems of specific applications. Students reassure proprietorship while also involving the combined procedures which are usually established to be most tough to acquire (Arena, 1996).

Problem posing can similarly stimulate an inner self of inquisitiveness and more varied and flexible thinking (English, 1997). Students who are involved in problem posing undertakings developed innovative, imaginative, and enthusiastic learners. Students have the chance of directing the problems they pose to their fields of curiosity in accordance to their cognitive capabilities (Goldenberg, 1993; Mason, 2000; Moses, Bjork,& Goldenberg, 1990). Studies confirmed that "problem posing might reduce common fears and anxieties about mathematics" (Brown & Walter, 1993; English, 1997; Moses et al.,1990; Silver, 1994).

The presence of problem posing activities benefits the students' advance and enhanced outlooks towards mathematics, decrease mistaken interpretations on mathematics, and become more accountable for their learning (Brown & Walter, 1993; English, 1997; Silver, Mamona-Downs, Leung, & Kenney, 1996).

Learners read and scrutinize the information and reason critically about problem creation and organization. "Students are actively engaged, minimizing inattention and off task behavior" (Davidson & Pearce 1988). When the learners inscribe their specific problems, "they become actively engaged, eager, involved in the art of thought, and motivated to solve the more realistic, meaningful problems that they wrote which reflect their own life" (Rudnitsky 1995; Ford, 1990).

Moreover, Lavy and Bershadsky (2003) experimented the types of problems posed by pre-service educators on the foundation of complex solid geometry activities by means of the “what if not?” strategy and the scholastic value of such task. Twenty-eight (28) pre-service educators participated in two workshops in which they had to pose problems on the source of prearranged information. An investigation of the modeled problems discovered a wide range of problems comprising those having a change of one of the statistical data to another specific proofed problem. Various kinds of posed problems revealed some occurrences such as a larger frequency of posed problems with another statistical value and a lack of posed problems comprising formal generality. They also deliberated the scholastic strengths of problem posing in solid geometry by means of the “what if not?” strategy (Brown & Walter, 1983,1993), which made the students reconsider the geometrical concepts used while generating new problems and making relations concerning the given and the new concepts. Doing this had an effect on students learning and developed their understanding.

The study conducted by Cai and Hwang (2002) observed US and Chinese sixth grade pupils’ simplification skills in answering pattern-based problems, “their generative thinking in problem posing, and the relationships between students’ performance on problem solving and problem posing tasks”. Across the problem solving activities,

> *“Chinese students had higher success rates than US students. The disparities appear to be related to students’ use of differing strategies. Chinese students tend to choose abstract strategies and symbolic representations while US students favor concrete strategies and drawing representations. If the analysis is limited to those students who used concrete strategies, the*

success rates between the two samples become almost identical. With regard to problem posing, the US and Chinese samples both produce problems of various types, though the types occur in differing sequences. There was a much stronger link between problem solving and problem posing for the Chinese sample than there was for the US sample".

Nicolaou and Philippou (2002) studied the relation between effectiveness in problem posing, problem-posing activity, and mathematical achievement. They discovered a strong correlation between students' ability in problem posing and the student's mathematical achievements. They also found significant differences in problem posing ability between sixth and fifth grade students. Moreover, preceding effectiveness theories in problem posing were a strong predictor of students' performance in problem posing and in mathematics achievement.

Chin and Kayalvizhi (2002) conducted a study to examine students' responses for a particular open-ended problems given the two kinds of examinations. The problems for the first investigation were created independently, while the other problems were created in groups. "Among the questions that were posed individually, only 11.7% could be answered by performing hands-on investigations". Majority of the questions inquired were constructed on general knowledge and enclosed a wide range of topics. However, "when questions were generated in groups after examples were shown, there was a significant increase in the number of questions that were amenable to science investigations (71,4%), but they related to fewer topics".

The study, conducted by Cai (2002), in which Singaporean fourth, fifth, and sixth grade pupils' mathematical thinking in problem posing and problem solving were explored showed that, "the majority of Singaporean participants are able to select appropriate solution strategies to solve these problems, and choose appropriate solution representations to clearly communicate their solution processes. Most Singaporean students are able to pose problems beyond the initial figures in the pattern".

The outcomes of this study also presented that across the four tasks, a higher percentage of students' grade level show indication of ensuring correct answers as the grade level advances. Remarkably, the overall statistically significant differences across the three grade levels are mainly between fourth and fifth grade students, yet "between fifth and sixth grade students, there are no statistically significant differences in most of the analyses".

Comparing the results on US and Chinese students' mathematical thinking, Singaporean students appeared to be much more alike to Chinese students than to US students.

Stover (1982) explored the concerns of having students make format modifications to mathematics problems. He noticed "substantial improvement in students' ability to solve problems of the type they had learned to modify". In like manner, the study conducted by Silver and Cai (1993) found a "strong positive relationship between problem posing based on a brief story with an unstated question and the problem-solving performance of the middle school students on open-ended mathematical problems”.

A related study by Stoyanova (1998) on the effects of a variety of problem posing conditions on students’ problem-solving and problem-posing mathematical performances reported that, “on a range of predefined performing categories, students exposed to problem-posing and problem-solving activities outperformed students exposed only to problem-solving activities”. This study pointed the positive effect of problem posing as an alternative instructional method on increasing learning among students.

“Stressing self-perception, problem-posing education bases its philosophy on creativity and stimulates true reflection and action upon reality” (Milner, 2003). In relation to this, Owens (1999) discovered that students taught with problem posing attained a significant achievement than those trained in traditional instruction in algebra. Commonly, instructors using problem posing lessons begin instruction by presenting students with non-concrete learning traits by providing extra guidance and assistance in problem posing lessons to make the most of their achievement advances.

Another related study was one on pre-service teachers by Crespo (2003) who observed the variations in the problem posing strategies of a group of elementary pre-service teachers as they posed problems to pupils. According to her,

> *"Learning to pose mathematical tasks is one of the challenges of learning to teach mathematics. How and when pre-service teachers may learn this essential practice, however, is not at all clear".*

This observation hinted the need to probe on the construct; hence, she did a study which revealed that their (pre-service teachers) far along problem posing proved to be significantly different from their earlier problems. Accordingly, instead of posing habitual single steps and computational problems, these pre-service teachers utilized posing problems that had numerous methods and explanations, were open-ended and investigative, and were cognitively more complex. The respondents' problem posing style also transformed. The pupils' edition became less leading and less focused on avoiding pupils' errors, but more of making editions that made students' work easier or more narrowed on the mathematical scope of the problem instead. Posing problems to genuine viewers, engaging in combined posing, and having access and chances to discover new kinds of problems are emphasized as important factors in upholding and supporting the stated changes.

The research of Craig (1999) discovered that,

> *"Pre-service teachers cannot be expected to teach differently than they are taught. Problem posing should be modeled and implemented in the university mathematics classroom as well in kindergarten through high school. Problem posing as a method of teaching a mathematical concept is difficult for pre-service teachers".*

Problem solving and problem posing have been much in use to improve the mathematical skills of students, and these are coupled with the use of metacognitive planning and organization approaches, like the study of Montague and Bos (1986) that investigated the effects of an eight-step metacognitive treatment with 6 adolescents.

> *"Students were taught to read problems, paraphrase the problems aloud, graphically display known and unknown information, state the known and unknown information, hypothesize solution methods, estimate answers, calculate answers, and check answers. Using a single-subject design, the researchers showed that this metacognitive treatment promoted word-problem skill. This approach has been extended for students, but with less evidence of benefits".*

Aside from metacognition, teachers need to create a problematic situation by focusing on the introduction of the concept for students to elaborate a mathematical problem (practical or not) in relation to this concept. In the first stage, the teacher formulates a problem and figures out how to solve it.

> *"At this moment there may occur regressions, changes of hypothesis, even the teacher may select new objects for his analysis. Nevertheless, after a fruitless work, a flexible thought may begin the searching of a practical situation. Once the problem has been solved it is necessary to appeal to a second stage which includes the problem correction and the variation of complexity degree. At this time, a new interaction arises where the teacher analyzes if his elaborated problem satisfies the outlined goal."*
>
> *(Montague and Bos, 1986)*

Conceptual Understanding

Studies on the effectiveness of teaching methods to improve students' level of conceptual understanding have also been conducted. For example, a study by Zulnaidi & Zakaria (2009) indicated that there were significant differences in mathematics conceptual understanding between students who were taught the information mapping strategy group and the traditional group, showing that there was a positive relationship between conceptual understanding and mathematics achievement.

There are also studies conducted on students' conceptual understanding in relation to mathematics (Dede, 2004; Amato, 2005; Abd. Rahman, 2006; Zakaria *et al*., 2010). For instance, Zakaria *et al*. (2010) conducted a study on the level of conceptual mathematical knowledge from topic sequences and series. The study also examined the relationship between conceptual understanding and student achievement in mathematics. It was conducted through a survey of 250 college students from one matriculation center in Kedah. The results showed that conceptual understanding of accounting students was at a low level when compared to that of physical and life science students; and that there was a significant relationship between conceptual understanding and mathematics achievement of students.

Another study determined the relationship between students' mastery of mathematical concepts and their attitudes and anxiety, the result of which suggested that students' concepts of algebra were very low (Abd. Rahman, 2006). Conducted among 200 students from three schools in Seremban, Negeri Sembilan, it showed that the relationship between students' mastery of concepts and attitudes was low, while anxiety did not correlate significantly with students' mastery of concepts. Another study was conducted on algebraic concepts of 120 students from grade 8. Using 17 open-ended questions answered by students followed by an interview, the study found that the students had difficulties in understanding the concept for various reasons, such as lack of information about operations, errors prior to transferring knowledge to new situations, and uncertainties about their prior knowledge (Dede, 2004).

Procedural Knowledge

A study was conducted on the concept of perimeter, area, the relationship between them, and volume. Respondents consisted of 110 teacher trainees from the Faculty of Education at Lakehead University, Ontario, and three groups of students of the Primary/Junior (P/J), Junior/Intermediate (J/I), and Intermediate/Senior (I/S) categories. After the respondents were given a test and interviewed, the study found that regardless of group or background on mathematics, teachers generally had less understanding of the concepts in the given questions except for the four broad concepts. Most of the respondents used their procedural understanding to enable them to memorize the formula (Walter, 2007).

Students should be exposed to procedural and conceptual understanding because both understandings are highly correlated (Amato, 2004). Nevertheless, even until today, mathematics education researchers continue their search to understand the balance between the two understandings. Researchers agree that both are important and that integrating of both understandings is important to increase students' mathematical problem solving skills (Mary & Heather, 2006).

Recent analysis conducted in Malaysia by Zakaria and Zaini (2009) focused on the level of procedural knowledge and conceptual understanding of 105 pre-service teachers in three teacher training colleges. Using the instrument by Faulkenberry (2003) involving 17 subjective items on fractions, the study showed that the level of conceptual and procedural knowledge was high-average with a mean score of 44.72 points from the overall score of 68 points. This suggested that the respondents displayed competence in representing a fraction as part of a set, a region, and a ratio.

The studies carried out abroad discussed specifically the effects of problem posing instruction or activities on number sense, solid geometry, mathematical skills and thinking of pre-service teachers as well as the students. Furthermore, the literature on Mathematics and problem posing claims that the latter has significant effects on students’ problem solving ability. Thus, the researcher would like to probe the findings further by testing the effect of problem posing activities on the students' achievement in Mathematics subject.

Mainly, the last level in problem posing is closely related to Pòlya's Looking Back stage, which confirms the problem formulating conception as a problem itself. In order to embed this statement into the problem framework, the researcher carried out certain variations, taking into account the Campistrous & Rizo's point of reference. The current research conceived level I, II, IV, and V of the said reference as actions of one strategy, while level III can be seen as a complex technique. Overall, generalization, particularization, the use of analogies, considering as unknown some elements of a problem, applying an algorithmic procedure, among others, may be found among the possible techniques.

Baki and Kartal (2004) developed a scale for characterization of conceptual and procedural knowledge in the light of definitions and classification of conceptual and procedural knowledge from the literature. Although this scale was developed to evaluate students' answer about the topic on algebra, it can be applied to other mathematical topics such as fractions. The criteria consist of general features of conceptual and procedural knowledge, as shown in the matrix below (Please see Appendix V, page 157).

Synthesis

The use of problem posing in Mathematics classrooms provides students the opportunities to link their own interest with all aspects of their Mathematics education, for it is thought to play an important role for linking students' personal interests with their education.

Mathematics is a way of thinking, while problem posing activities provide opportunities that engage students in a natural way of reflective Mathematical thinking. Such activities nurture students' attempts to explore problem and solution structures rather than to focus only on finding solutions. Therefore, developing students' ability to pose and explore problems, and to apply these in their everyday experiences should be seen as a vital component of Mathematical instruction at all levels.

Throughout these reviews, problem posing was found to form conceptual understanding and procedural knowledge for the learning of mathematical topics. Conceptual understanding and procedural knowledge are critical to the construct of fundamental knowledge of the topics by linking relevant mathematical concepts and making meaningful learning.

For example, Geometry is one topic in mathematics education that represents frameworks for various important concepts such as triangle similarities and proportions, which many students find difficult for problem posing, especially on understanding and solving mathematical word problem. This shows that the gap between students' conceptual understanding and procedural knowledge may cause a challenge in their problem posing.

The study encompasses the existing body of research regarding the use of problem posing instruction to develop and enhance the conceptual and procedural understanding of the students. Although foreign studies abound in relation to problem posing, local research on the effective use of problem posing seems wanting, and this prompts the present study to investigate the mathematical skills of students on the conceptual understanding and procedural knowledge when exposed to problem posing instruction. It is believed that teaching is a vigorous process, one that translates into looking for innovative strategies and methods in teaching mathematics in order to develop students' higher order thinking skills.

CHAPTER III

Methodology

This chapter covers the research design, participants of the study, research instruments, data gathering procedures, and the statistical treatments used in the study.

Research Design

This study used a quasi-experimental method to find out the effect of problem posing on students' conceptual understanding and procedural knowledge. Quasi-experimental method deals with the test under controlled conditions with the aid of measuring instruments (Santiago, 2009). This type of research usually determines whether a variable affects one situation by determining the difference, whether it is significant or not.

The study used the pretest-posttest control group design, ensuring that all conditions were the same for both the experimental and control groups, but with the exception that the experimental group was exposed to problem posing, whereas the control group was not.

The diagram below shows the design of the experiment.

Group	Pre-Test	Treatment	Post -Test
E	**O_1**	**X**	**O_2**
C	**O_3**		**O_4**

Where:

E	=	Experimental Group
C	=	Control Group
O_1, O_3	=	Pretests in Conceptual Understanding and Procedural Knowledge
X	=	Problem Posing Instruction
O_2, O_4	=	Posttests in Conceptual Understanding and Procedural Knowledge

The experimental group (Grade 9 – Ruby) was taught using problem posing while the control group (Grade – 9 Garnet) was taught using the usual way. In other words, the two groups of students were taught the same lessons and topics with the same method of assessment, but were taught using different teaching method and task.

Setting of the Study

NAMEI Polytechnic Institute was inaugurated as the Naval Architecture and Marine Engineering Institute in 1947 at Sales St. corner Ronquillo, Sta. Cruz, Manila. The late Don Felix Padilla, its founder and president, is also the first Filipino graduate of Naval Architecture and Marine Engineering from the Massachusetts Institute of Technology.

In 1960, the High School Department was formally established. Since the student population was rapidly growing and the premises could hold no more than what was possible on the original site, growth of the High School was held in abeyance and development of a new school campus in Mandaluyong was begun.

Finally, in 1962, coinciding with the incorporation of the Institute under the name of NAMEI Polytechnic Institute, the entire school was transferred to its new campus where it is presently situated.

Participants of the Study

The participants of the study were seventy (70) grade nine junior high school students enrolled in NAME Institute during the school year 2015 – 2016.

Two intact groups each consisted of thirty-five (35) students served as the control and experimental groups of the study, totaling to 70 students. They took pretests for conceptual understanding and procedural knowledge, the resulting mean scores of which were compared to insure that the groups were comparable in terms of conceptual understanding and procedural knowledge in geometry.

Table 1. Comparison between the Pretest Mean Scores on Conceptual Understanding and Procedural Knowledge of the Control and Experimental Groups

Variable	Group	Mean	SD	df	*t* value	*p* value	Interpretation
Conceptual Understanding	Control	6.63	2.47	68	0.15	0.88	Not Significant
	Experimental	6.71	2.43				
Procedural Knowledge	Control	6.29	2.37	68	-0.12	0.91	Not Significant
	Experimental	6.23	1.66				

Table 1 shows the comparison of the pretest mean scores of control and experimental groups. The table shows that there was no significant difference on the conceptual understanding pretest between control and experimental group.

Correspondingly, the difference between control and experimental group in terms of procedural knowledge pretest was also not significant. These results credited the fact that the two groups of student had the same initial or prior knowledge in terms of conceptual understanding and procedural knowledge.

The schedule of instruction of two groups follows:

Groups	Time	Days	Time Allotment
Experimental Group	7:30 – 8:30	Monday – Friday	60 minutes
Control Group	8:30 – 9:30	Monday – Friday	60 minutes

Four weeks were allotted in delivering the problem posing instruction in the experimental group on the selected topics. The participants underwent the elements of problem posing on the topics.

Research Instrument

The research instruments utilized in the study were lessons plans integrating problem posing, conceptual understanding test, procedural knowledge test, and questionnaire on students' perception of problem posing.

Lesson Plans Integrating Problem Posing

Review was conducted on the course syllabus of Grade 9 students based on the curriculum guide from the Department of Education (DepEd). Consequently, the result of the review guided the selection of the topics for the lessons.

Each lesson lasted for one hour per session in five school days. The first day of the lesson focused on the presentation of topic, allowing the students to select materials like books, magazines, and journals to come up with the concept of the topic. A set of questions was posted and answered by students to guide them in classifying the information gathered from the material. Correspondingly, the teacher posted a problem for the students to answer as they associated the information gathered to create their own original problem. Finally, the students attempted to pose and present the generated problem for explanation and approval. Initial revisions were done by the students as the teacher clarified some details or part of a problem subject for final revision, retention, and rejection of the information integrated in the problem.

The lessons on similar triangles were developed, ensuring that problem posing was used and integrated on the learning process of the students in the experimental group. The lesson topics included were: (a) ratio and proportion; (b) theorems on proportionality; (c) similarity on triangles; and (d) right triangle similarity (Please see Appendix A, page 78). Same topic and competency on lesson plans for control group were prepared (Please see Appendix B, page 86).

Further, the quizzes and assignment feedback on individual student's progress and group discussions were done for each topic on the weekly lesson plans.

Week 1 Lesson: Ratio and Proportion. This lesson was designed to enable the students to look for the comparison and equality of the two or more quantities. Students attempted to pose a related problem through the guide questions of the teacher.

Week 2 Lesson: Theorems on Proportionality. This lesson was designed to determine the relationships, theorems and axioms of every quantity on the given proportion. Students were expected to generate a problem using the teacher's problem posing.

Week 3 and 4 Lessons: Similarity on Triangles and Right Triangle Similarity Theorem. These lessons were developed to allow the students to apply the theorems of proportionality in similar triangles and use the concept of right triangle similarity theorem.

Conceptual Understanding and Procedural Knowledge Tests

Conceptual understanding and procedural knowledge tests were developed, consisting of the topics on ratio and proportion, theorems on proportionality, similarity on triangles, and right triangle similarity. These test materials underwent item analysis and revisions prior to their actual use. Further, these tests were administered to both control and experimental groups to measure the conceptual understanding and procedural knowledge of the students before and after the instruction.

The development, validation, and test for reliability of the two sets of test underwent these stages.

Stage 1. Preparation of Initial Draft. A table of specification for the first draft was constructed to ensure that all topics were presented in the tests. Based on the table of specifications, the items on conceptual understanding test and procedural knowledge test were drafted. (Please see Appendix C, page 89).

A total of 80 multiple-choice items were constructed, in which the number of items was divided equally into two parts, the conceptual understanding test and procedural knowledge test. (Please see Appendix D, page 90).

Each item was classified according to cognitive level: knowledge, comprehension, application, analysis, synthesis, and evaluation.

Stage 2. Content Validation and Face Validation. Initial drafts of the conceptual understanding and procedural knowledge tests were prepared and submitted to mathematics experts for validation. The initial drafts were presented to the researcher's advisers for their evaluation and suggestions. An evaluation form was provided for the objectivity of the results. (Please see Appendix E, page 103).

The items were improved based on the suggestions and recommendations of the mathematics experts (Please see Appendix F, page 107).

Stage 3. First Try-Out and Item Analysis. The developed tests were tried out to Grade 10 students. These students were selected because they had just learned the topics the previous school year. After the first try-out, the test items were analyzed using the upper and lower index method. (Please see Appendix G, page 113).

The item analysis served as the basis for decision making on the item to retain, revise, or reject. Out of 40 items on each type of test constructed, ten (10) items were rejected and one (1) item was revised.

Stage 4. Second Try-Out and Item Analysis. The second drafts of the tests were administered to another group of Grade 10 students. After the second try-out, item analysis was done to find out the indices of discrimination and difficulty of the revised items. The test results of the second try-out were used to determine the final form of the said tests. (Please see Appendix I, page 127).

Stage 5. Establishing the Reliability of the Tests. After the second try-out, revisions were made based on the analysis of the results. The final draft of the tests were administered to another group of fifty-one (51) Grade 10 students from Ilaya Barangka Integrated School upon approval of the request to the School Principal to do so (please see Appendix M) to establish the reliability of the instrument using Kudar Richardson Formula – 21.The results of the test were used to determine the consistency of the item scores on the conceptual and procedural understanding tests. (Please see Appendix J, page 129).

Thus, the final drafts and the corresponding answer key for conceptual understanding and procedural knowledge tests were formed with 25 items for each test. (Please see Appendices K and L, pages 130-137).

Questionnaire on Students' Perception of Problem Posing

The questionnaire was developed to gather information about the perception of students on problem posing. It featured both Likert-type and open-ended questions that provided information on each participant's evaluation about problem posing and its usefulness. The open-ended questions required the participants to report what they like the most and like the least about problem posing.

In addition, the questionnaire included open-ended questions that required the students to answer whether they would continue using problem posing approach and would recommend it to other students. (Please see Appendix M, page 138).

The scale shown below was used to interpret the responses of the students on the questionnaire:

Mean	**Verbal Interpretation**
4.51 – 5.00	Strongly Agree
3.51 – 4.50	Agree
2.51 – 3.50	Unsure
1.51 – 2.50	Disagree
1.00 – 1.50	Strongly Disagree

Data Gathering Procedure

The following phases of data-gathering procedure were undertaken in the conduct of the study:

Phase 1. Pre-Experimental Phase

Permission of the School Director and High School Principal of NAMEI Polytechnic Institute, Mandaluyong City was sought to conduct the study in the school and allow the Grade 9 students to participate in the study (Please see Appendix N, page 139).

Letters of request to evaluate, to review, and to check the research instruments were sent to mathematics experts and other consultants of the study. (Please see Appendix O, page 143).

A two-day orientation on the conduct of the study was done with the experimental group in order to familiarize them on problem posing. A developed calendar of activities was prepared, which served as guide on teaching the experimental group. (Please see Appendix P, page 144).

The 25-item test on conceptual understanding and another 25-item test on procedural knowledge were administered to the participants of the study.

Phase 2. Experimental Phase

Teaching the Experimental Group. The experimental group was taught using problem posing. Students were asked to gather information from the materials they used as sources and references. The students were asked to construct a problem on the topic presented. This was done either individually or by group. In addition, the students had to present their work in the class to defend their work. The purpose of this was to check and verify if the students came up with a correct construction of problem. Assignments were also given to the students requiring them to make any revisions (if needed) to improve the posed problem (Please see Appendix A, page 78).

A calendar of activities (Please see Appendix P, page 144) was shown to describe how the lessons were discussed and done on the experimental group.

Teaching the Control Group. The control group was taught using Lecture-discussion, group activities, assignment, and seatwork. Quizzes, unit test, and other paper-pencil tests were also part of the usual activities in the control group. Individual tasks were given weekly, while group work and problem solving were given three times during the experimental period (Please see Appendix B, page 86).

The teacher introduced the topic using lecture-discussion method or deductive method. Students were asked to solve the problem taken from the textbook individually. At the end of the session, the teacher summarized the lesson and gave assignments based on textbook.

Attendance of the participants of the study was monitored through the attendance sheet log, and this was used to summarize the number of times they were present during instruction on problem posing. (Please see Appendix Q, page 146).

Phase 3. Post-Experimental Phase

At the end of the experimentation on problem posing lessons, the conceptual understanding and procedural knowledge tests were administered for one hour to the two groups of participants.

Same scoring with the pretest was utilized to assign score for the response of the students in the test. Posttest papers were examined to determine how the experimental group effectively used problem posing approach. The scores in the conceptual understanding and procedural knowledge tests were tallied and summarized. (Please see Appendices S and T, pages 149-150).

The participants of the study were told to answer the evaluation form on problem posing. After the experiment, it was ensured that the participants in the experimental group evaluated the problem posing individually. Moreover, they were reminded not to leave any items unanswered.

Data Analysis Procedure

The following statistical tools were employed to analyze the data gathered:

Mean. This was used to describe the level of conceptual understanding and procedural knowledge of the participants of the study.

Percentage. This was used to describe the responses of the students on the survey questionnaire statements about the problem posing instruction.

Standard Deviation. This was used to determine the spread of the scores in conceptual understanding and procedural knowledge tests results.

Independent Sample t-test. This was used to determine the significant difference between the pretest mean sores of students in the two groups.

t-test for Dependent Samples. This was used to determine the significant difference between the pretest and posttest mean scores of the students given the two tests.

CHAPTER IV

Presentation, Analysis and Interpretation of Data

This chapter deals with the presentation and analysis of data. Corresponding interpretation and discussion are based on the order of the statements of the problem and the hypotheses of the study.

Problem 1: How may the students in the control and experimental groups be described in terms of the following?

1.1. Conceptual Understanding

1.2. Procedural Knowledge

Table 2 shows the pretest and posttest mean scores of the control and experimental groups on conceptual understanding and procedural knowledge. As can be gleaned from the table, the control and experimental groups obtained a significant increase in the posttests in terms of conceptual understanding and procedural understanding. The results in conceptual understanding test revealed that the experimental group achieved a higher mean score in both pretest and posttest as compared with the control group. Similarly, the experimental group also attained a higher mean score in procedural knowledge posttest than the control group. However, the control group obtained a higher score on the procedural knowledge pretest than the experimental group.

This implies that the experimental group remained consistently higher on the pretest and posttest mean scores than the control group in terms of conceptual understanding. Likewise, the procedural knowledge test revealed that the experimental group continued to be higher on posttest than the control group, although the control group registered a better result than the experimental group on the pretest.

Table 2.Students' Pretest and Posttest Mean Scores on Conceptual Understanding and Procedural Knowledge

Group	Test	Conceptual Understanding			Procedural Knowledge		
		Mean	SD	Interpretation	Mean	SD	Interpretation
Control	Pretest	6.63	2.47	Below Average	6.29	2.37	Below Average
	Posttest	10.66	2.89	Average	9.94	3.40	Average
Experimental	Pretest	6.71	2.43	Below Average	6.23	1.66	Below Average
	Posttest	11.17	2.05	Average	10.14	1.67	Average

In general, the results imply that both experimental and control groups added an increment in the mean scores on the posttest results as compared with the pretest.

Problem 2: To what extent does problem posing affect the students'

2.1. Conceptual Understanding?

2.2. Procedural Knowledge?

Table 3 shows the comparison of the pretest and posttest mean scores of the control group on the conceptual understanding and procedural knowledge. It can be seen in the table that there is a significant difference between the pretest and posttest mean scores of the control group. This implies that the conceptual understanding of the students in control group increased significantly after teaching them using the traditional method.

Likewise, the difference between the pretest and posttest mean scores of the control group on procedural knowledge is also significant. This implies that the students' procedural knowledge improved significantly after teaching them the traditional method. The results can be attributed to the fact that pretest scores were remarkably low pretest scores could be due to the fact that they did not have enough knowledge of the topics prior to instruction.

Table 3. Comparison Between Pretest and Posttest Mean Scores on Conceptual Understanding and Procedural Knowledge of the Control Group

Variable	Test	Mean	SD	df	*t* value	*p* value	Interpretation
Conceptual Understanding	Pretest	6.63	2.47	34	-5.66	0.00	Significant
	Posttest	10.66	2.89				
Procedural Knowledge	Pretest	6.29	2.37	34	-6.84	0.00	Significant
	Posttest	9.94	3.40				

Table 4 shows the comparison of the pretest and posttest mean scores of the experimental group on the two tests. As can be gleaned from the table, there is a significant difference between the pretest and posttest mean scores of the experimental group. It indicates that the conceptual understanding of the students in experimental group increased significantly after teaching them using the problem posing.

Similarly, the difference between the pretest and posttest mean scores of the experimental group on procedural knowledge is also significant. It indicates that the students' procedural knowledge improved significantly after teaching them using the problem posing. The remarkably low pretest scores could be due to the fact that the students did not have enough knowledge of the topics prior to instruction.

Table 4. Comparison Between Pretest and Posttest Mean Scores on Conceptual Understanding and Procedural Knowledge of the Experimental Group

Variable	Test	Mean	SD	df	*t* value	*p* value	Interpretation
Conceptual Understanding	Pretest	6.71	2.43	34	9.45	0.00	Significant
	Posttest	11.17	2.05				
Procedural Knowledge	Pretest	6.23	1.66	34	10.78	0.00	Significant
	Posttest	10.14	1.67				

Table 5 shows the comparison between the posttest mean scores of the experimental and control groups on conceptual understanding and procedural knowledge.

It is evident from the table that there is no significant difference between the posttest mean scores of the experimental and control groups on conceptual understanding and procedural knowledge. While experimental group performed higher than the control group, the difference is not significant. This implies that the use of problem posing produced the same effect as the usual method on conceptual understanding and procedural knowledge of the students. This can perhaps be explained by the fact that the experimental group was not yet immersed or familiar with problem posing, as revealed during the interview where all students admitted that they encountered problem posing only for the first time.

Table 5. Comparison Between the Posttest Mean Scores on Conceptual Understanding and Procedural Knowledge of the Control and Experimental Groups

Variable	Group	Mean	SD	df	*t* value	*p* value	Interpretation
Conceptual Understanding	Control	10.66	2.89	68	0.86	0.39	Not Significant
	Experimental	11.17	2.05				
Procedural Knowledge	Control	9.94	3.40	68	0.31	0.76	Not Significant
	Experimental	10.14	1.67				

The sample problems posted by the students were recorded (please see Appendix R) and analyzed. It revealed that the students shown understanding of the concepts on the lesson since they were able to pose problem as follows:

> *"When CJ cooks omelet, she puts 1 cup of milk and 2 eggs in every serving. If she cooks 10 serving, how many cups of milk and eggs will be used?"*

The understanding of concepts on the discussed topic was evidenced since the students posed a problem that can be solve using the concept of the discussed lesson. Further, the mathematical task enhances the ability of the students to share their knowledge and relate to their prior skills. Thus, it supports the study of Stoyanova and Ellerton (1996) that problem posing encourages students to reflect on their specific previous experiences. (Please see Appendix R, page 148).

However, some students find difficulty on communication skills in posing the problem. The medium of language serve as a hindrance for the learner in expressing their thoughts through problem posing. Here is a sample posed problem:

> *"If only five basketball players are in a game, how would they divide themselves to play in a four-quarter game?"*

Conceptual understanding of the students was not clearly evident on the problem. The thought of student on the posed problem can mislead to other interpretation. Thus, communication skills must be considered in posing a problem.

Problem 3: What are the students' perceptions on the use of problem posing?

The students' perception of problem posing instruction was obtained using the evaluation instrument. The evaluation instrument included five statements about the process involved in the problem posing. Table 6 presents the perception of the students about the usefulness of problem posing approach. As the table shows, the students reported that they agreed on the use of problem posing for mathematics instruction.

Moreover, the students rated the problem posing with a mean score of 4.03, interpreted as helpful in understanding the concept of the lesson. On the other hand, the students evaluated the problem posing approach with a mean score of 3.91 rating, descriptively interpreted as aiding them to easily attain the procedural understanding of the lesson, especially the adversarial problem posing instruction.

Table 6. Students' Perception of Problem Posing

	Statement	Mean	Interpretation
1.	Problem posing is helpful in understanding the concept in the lesson.	4.03	Agree
2.	The procedural knowledge on the lesson is easily attained through problem posing.	3.91	Agree
3.	The use of systematic procedures in lesson was attained using problem posing.	4.17	Agree
4.	The length of time used in problem posing was effective and efficient.	3.80	Agree
5.	The teacher applied the problem posing clearly.	4.46	Agree
	Average	**4.07**	**Agree**

The students also reported positive responses about the problem posing approach as reflected in their written responses revealing what they liked most about the approach.

The analysis on the students' personal reflections about problem posing instruction points to three main themes of most likeable features of problem posing.

Mostly, students' responses were associated with the inclination to the process involved in the problem posing. Most of the students showed positive responses in posing problem. The students felt independency in the learning process with the use of problem posing. Here are some responses:

> *"I like the most about problem posing is when it awakens the mind of students of what is being talked about in the class. We don't just depend on the teacher in deepening our knowledge."*

The student perceived that problem posing elicits a positive attitude towards the lesson in mathematics. Moreover, they acknowledged a greater responsibility in learning the topic. This supports the study of Priest (2009) that problem-posing is recognized as an important component in the nature of mathematical thinking.

Another student explains the feeling on the use of problem posing:

> *"I like the most is that all groups in the class are given a chance to show and share their own knowledge about the topic through posing the created problems."*

Showing and sharing the students' own idea on their posed problem implies an authentic and creative way in learning the lesson. Further, the social interaction with their co-learners also boosts their self-confidence and creates positive attitude on their environment and on the subject as well.

Another student shared experience in problem posing:

> *"I like that problem posing approach helps us in understanding the concept of the lesson."*

Even though results on the use of problem posing were positive, some students expressed their negative feelings about it. Some mentioned that their difficulties on the use of problem posing are anchored on discomforts and clustered around the tiresome steps and process of creating the problem for posing. Less than half of the students explicitly cited that they least liked performing the problem posing because they do not know the kind of problem to ask and the level of problem's difficulty and complexity to pose.

> *"The least that I like about problem posing is that some topics are very difficult to create questions and we need to carefully revise it most of the time."*

The perceptions of students on problem posing is related to Leung's (2009) finding, saying that word problems are source of difficulty for many students because they often experience the language deficits that specifically account for mathematical problem-posing difficulty.

The students commented:

> *"I don't like the part that the problems are getting harder to be created."*
>
> *"I don't like most is the revision of the problem."*

In Song's (2007) study, problem posing undergoes developing problems through revisions, adding new data, eliminating some data, changing variables, or constructing a new problem based on the original idea.

When students were asked if they would continue on the use of problem posing in mathematics, they all answered favorably, giving a resounding "yes". Students also commented that they would recommend the problem posing to others because as cited in their responses, problem posing "will help others to improve their understanding skills. It will also help them know how to open up their opinion to others in creating a problem."

To sum up, the students showed favorable response toward posing a problem as instructional approach in mathematics. Apparently, when students began posing their own original mathematical problems and saw these problems becoming the focus of discussion, their perception of the subject was deeply transformed. When they got time working on posing problem, their ownership of the experience produced excitement and motivation.

CHAPTER V

Summary, Findings, Conclusion and Recommendation

This chapter includes the summary of the study, the findings, based on the information and data obtained from the study, the conclusions derived from the answers sought in this study, and the recommendations based on the findings and conclusions.

Summary

To reiterate, the study was undertaken to investigate the effect of adversarial problem posing on the students' conceptual understanding and procedural knowledge in geometry.

The study used experimental method specifically the pretest posttest control group design with two intact groups of student participants.

One group was taught using problem posing while the other group was taught using the traditional method of teaching. Both groups were given the pretest and posttest in conceptual understanding and procedural understanding, but only the experimental group answered the questionnaire on perception of problem posing.

Findings

The information and data obtained from the study revealed the following findings:

1. The students in both experimental and control groups were remarkably below average in the conceptual understanding and procedural knowledge pretests.

2. There was no significant difference between the posttest mean scores of the experimental and control groups on conceptual understanding and procedural knowledge.

3. Majority of the students in the experimental group perceived the use of problem posing useful in learning Geometry. They liked to pose problems and share their own idea with their co-learners. They also expressed the idea that problem posing helped them in understanding the concepts of the lesson. They added that problem posing gave them a chance not to depend too much on the teacher.

4. The students reported that they would continue using problem posing and even claimed that they would recommend it to others, especially those who have difficulty in understanding the concepts and procedures on the topics in Geometry.

5. Students noted few disadvantages of problem posing. One disadvantage in the use of problem posing was the time taken to revise the problem posed.

Conclusion

Based on the findings derived from this study, the following conclusions were formulated:

1. The use of problem posing produces the same effect as the usual method on conceptual understanding and procedural knowledge.

2. Problem posing instruction helps students understand and appreciate better the concepts and procedures in understanding topics in mathematics.

3. Problem posing may make sense as an alternative and useful feature or task in math class in developing procedural knowledge of the students as it underscores the elements that lead to successful understanding.

4. It challenges mathematics teachers to find sound instructional activities or intervention to ensure the development of students' conceptual understanding and procedural knowledge.

Recommendation

Based on the findings and conclusions of this study, the following are hereby recommended:

1. Teachers should start using problem posing in their classes in a gradual manner. Problem posing can be used in just one unit only for a specific purpose. It may be tried to particular class only. It may also be used as a start with usual method of teaching. The idea, hence, is to start one step at a time and not to attempt so many things at the same time.

2. Mathematics teachers should model and develop problem posing lessons as an alternative method in teaching mathematics, and to investigate the effectiveness of problem posing instruction in teaching other branches of mathematics.

3. Problem posing may be introduced and integrated with other methods to provide enriching opportunity for students to develop competence, conceptual understanding and procedural knowledge.

4. Mathematics curriculum developers should invest in problem posing that will enhance the conceptual understanding and procedural knowledge of students in Mathematics.

5. School administrators or professional organization for mathematics teachers may sponsor seminar-workshop on problem posing in

mathematics so that teachers will be equipped with knowledge and skills in employing problem posing in mathematics class.

6. Future researchers are encouraged to investigate the effect of problem posing on the conceptual understanding and procedural knowledge in mathematics using different experimental design and considering other skills in mathematics. They may also explore other variables relevant to problem posing instruction that may influence conceptual understanding and procedural knowledge, such as reading comprehension, background knowledge and/or other learning styles.

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APPENDIX A

Problem Posing Lesson Plans

WEEK 1 LESSON
RATIO AND PROPORTION

I. Learning Competencies

The students will be able to:

- describe ratio and proportion
- apply the fundamental theorems of proportionality
- solve problems involving proportion

II. Learning Concepts and Materials

A. Major Concept: Triangle Similarity

B. Sub Topic: Ratio and Proportion

C. References: Dildao, S. J., De Leon C. M. and Bernabe J. G. 2002. *Geometry.* Published and printed by JTW All rights reserved. 189 – 190 pp.

Fortes, E.,Garcia, T., Legaspi, M., Alcabedas, E., Bayle M., Alano, C., 2007. *Geometry*. ABIVA Publishing

Orines, F. B., and C. B. Manalo, 2004. *Next Century Mathematics, Advanced Algebra, Trigonometry and Statistics,* 2nd Edition, Phoenix Publishing House

D. Teaching Approach: Problem Posing

E. Materials: PowerPoint presentation, manila paper, cartolina, colored chalks and chalkboard, graphing board and duplicated materials.

III. Learning Strategies and Activities

TEACHER'S ACTIVITY	*STUDENT'S ACTIVITY*

A. Preliminary Activities

1. ***Greetings***
2. ***Opening Prayer***
3. **Cleanliness and Orderliness**
4. ***Checking of Attendance***

B. Lesson Proper

Class, your goal for this lesson is to pose a problem involving ratio and proportion. You should develop a thorough plan to pose the problem. The approach and answers should be explicitly detailed and reasonable throughout.

1. ***Selection***

The students will choose among the sets *of materials to come up with the concept of ratio.*

♦ *Ratio is a comparison of two or more quantities having the same unit.*

2. ***Classification***

Class, try to do the following:

Direction: Determine if the given condition forms a ratio or not.

a. There are 30 students in a classroom → *It forms Ratio (30:1)*

b. Five students shares two computer sets for the lesson → *It forms Ratio (5:1)*

c. A car travels 444 km in 6 hours → *It does not form a Ratio but a rate instead*

3. ***Association****(the teacher will pose a sample problem)*

Cesar had 4 seedlings of squash and 6 seedlings of tomatoes, while Renato had 8 seedlings of squash and 12 seedlings of tomatoes.

Guide Questions:

♦ What is the ratio of squash seedlings to tomato seedlings each boy had?

♦ For Cesar, the ratio of squash to tomato seedlings is 4:6

♦ For Renato, the ratio of squash to tomato seedlings is 8:12

♦ Can we simplify the two ratios?

♦ Yes

♦ Do they have the same ratio?

♦ They have the same ratio of 2:3

♦ Correct!

(the teacher will present three problem sets to the students)

4. ***Searching*** *(the teacher will apply the concept of ratio in proportion and let the students prepare to pose a problem)*

♦ The equality of two ratios is called <u>proportion</u>.

♦ In a:b = c:d, <u>b</u> and <u>c</u> are means, while <u>a</u> and <u>d</u> are extremes.

♦ The law of proportion states that the product of the means is equal to the product of the extremes.

5. ***Posing***

♦ Class you may pose now your problem.

♦The ratio of boys to girls in the class is 2:3. If there are 23 girls in the class, how many are boys?

♦ I am so sorry class but there must be something wrong in the problem.

6. ***Searching***

♦ Class, we know that proportion is the equality of two ratios.

♦ If there are 23 girls, there would be no counting number that will satisfy the posed problem.

7. ***Transformation***

PROBLEM POSING:

♦ What if the number of girls is even?
♦ Will it be possible to obtain a counting number for the boys?
♦ *What if . . .?* *How about . . .?* *Do you think that. . .?*

8. ***Searching***

♦ Class try to do some changes in the given or you may add some questions in the problem posed. *(The students will reflect and do the necessary changes on the posed problem)*

♦ Are you now ready to pose the problem? ♦ Yes, sir.

9. ***Posing the corrected problem***

♦ Class pose now your revised problem. ***♦The ratio of boys to girls in the class is 2:3. If there are 36 girls in the class, how many are boys?***

♦ How many students are there in the class?

♦ *Congratulations, you were able to pose a problem correctly.*

♦ Do you understand now the lesson? ♦Yes, sir. We fully understand the lesson.

IV. Learning Assessment

Class prepare for the quiz. You may keep now all your notes.
Arrange your chairs and sit properly.
Do the following:

- In at least three sentences, define ratio and proportion.
- Create a problem using the concept of ratio and proportion.

Class, let us assess your work.
(The teacher will check the work of each student using the rubric)
(The teacher will record the score of students)

V. Assignment

Class, you may copy now your assignment on your notebook.

- How are ratio and proportion applied in similar triangles?
- How are the measurements of each corresponding angles of similar triangle?
- Show sample problems with solution.

PREPARED:

RAMON A. RAZA

WEEK 2
THEOREMS ON PROPORTIONALITY

I. Learning Competencies
The students will be able to:
- recall ratio and proportion
- prove theorems on proportionality
- apply the theorems of proportionality in problem solving

II. Learning Concepts and Materials
A. Major Concept: Triangle Similarity
B. Sub Topic: Theorems on Proportionality
C. References: Dildao, S. J., De Leon C. M. and Bernabe J. G. 2002. *Geometry.* Published and printed by JTW All rights reserved. 189 – 190 pp.
Fortes, E.,Garcia, T., Legaspi, M., Alcabedas, E., Bayle M., Alano, C., 2007. *Geometry*. ABIVA Publishing
Orines, F. B., and C. B. Manalo, 2004. *Next Century Mathematics, Advanced Algebra, Trigonometry and Statistics,* 2nd Edition, Phoenix Publishing House
D. Teaching Approach: Problem Posing
F. Materials: PowerPoint presentation, manila paper, cartolina, colored chalks and chalkboard, graphing board and duplicated materials.

III. LearningStrategies and Activities

TEACHER'S ACTIVITY *STUDENT'S ACTIVITY*

A. Preliminary Activities
1. ***a. Greetings***
2. ***b. Opening Prayer***
3. ***c.* Cleanliness and Orderliness**
4. ***d. Checking of Attendance***

B. Lesson Proper

Class, your goal for this lesson is to pose a problem involving theorems on proportionality.
You should develop a thorough plan to pose the problem. The approach and answers should be explicitly detailed and reasonable throughout.

a. Selection

The students will choose among the sets *of materials to come up with the concept of theorems on proportionality.*

♦ *If two triangles have equal altitudes, then the ratio of their areas is equal to the ratio of their bases.*
♦ *Two triangles on the same base and between two parallels are equal in area.*

b. Classification

Class, try to do the following:
Direction: Perform what is asked.

i. Prove: If two triangles have equal altitudes, then the ratio of their areas is equal to the ratio of their bases
ii. Prove: Triangles on the same base and between the same parallels are equal in area.

iii. Prove the converse of the Proportionality Theorem, “If a line divides two sides of a triangle proportionally, then the line is parallel to the third side.”

c. Association*(the teacher will pose a sample problem)*

(the teacher will present three more problem sets to the students)

D. ***Searching*** *(the teacher will apply the concept of ratio in proportion and let the students prepare to pose a problem)*

♦ **If two triangles have equal altitudes, then the ratio of their areas is equal to the ratio of their bases**

♦ **Triangles on the same base and between the same parallels are equal in area.**

♦**If a line divides two sides of a triangle proportionally, then the line is parallel to the third side**

e. ***Posing***

♦ Class you may pose now your problem.

♦ *I am so sorry class but there must be something wrong in the problem.*

F. ***Searching***

♦ *Class, we know that proportion is the equality of two ratios.*

♦ *Look back on your posed problem and try to review the given and situation.*

g. Transformation

PROBLEM POSING:

♦ What if the measurement of one side of triangle changed?

♦ Will it be possible to obtain the same or different answer?

♦ *What if . . .? How about . . .? Do you think that. . .?*

H. ***Searching***

♦ Class try to do some changes in the given or you may add some questions in the problem posed. *(The students will reflect and do the necessary changes on the posed problem)*

♦ Are you now ready to pose the problem? ♦ Yes, sir.

i. ***Posing the corrected problem***

(Students will pose the revised problem.)

♦ *Congratulations, you were able now to pose a problem.*

♦ Do you understand now the lesson? ♦Yes, sir. We fully understand the lesson.

IV. Learning Assessment

Class prepare for the quiz. You may keep now all your notes.

Arrange your chairs and sit properly.

Do the following:

- In at least three sentences, discuss the different proportionality theorem.
- Create a problem using the concept of theory on proportion.

Class, let us assess your work.

(The teacher will check the work of each student using the rubric)

(The teacher will record the score of students)

V. Assignment

Class, you may copy now your assignment on your notebook.

- How are ratio and proportion applied in similar triangles?
- How are the measurements of each corresponding angles of similar triangle?
- Show sample problems with solution.

WEEKS 3 AND 4
SIMILARITY ON TRIANGLE
AND RIGHT TRIANGLE SIMILARITY

I. Learning Competencies

The students will be able to:

- recall the theorems on proportionality
- apply theorems of proportionality in similar and right triangles
- solve problems on similar and right triangles

II. Learning Concepts and Materials

A. Major Concept: Triangle Similarity
B. Sub Topic: Similarity on Triangle and Right Triangle Similarity
C. References: Dildao, S. J., De Leon C. M. and Bernabe J. G. 2002. *Geometry.* Published and printed by JTW All rights reserved. 189 – 190 pp.
Fortes, E.,Garcia, T., Legaspi, M., Alcabedas, E., Bayle M., Alano, C., 2007. *Geometry*. ABIVA Publishing
Orines, F. B., and C. B. Manalo, 2004. *Next Century Mathematics, Advanced Algebra, Trigonometry and Statistics,* 2nd Edition, Phoenix Publishing House
D. Teaching Approach: Problem Posing
F. Materials: PowerPoint presentation, manila paper, cartolina, colored chalks and chalkboard, graphing board and duplicated materials.

III. Learning Strategies and Activities

TEACHER'S ACTIVITY *STUDENT'S ACTIVITY*

A. Preliminary Activities

a. Greetings
b. Opening Prayer
***c.* Cleanliness and Orderliness**
d. Checking of Attendance

B. Lesson Proper

Class, your goal for this lesson is to pose a problem involving theorems on proportionality.

You should develop a thorough plan to pose the problem. The approach and answers should be explicitly detailed and reasonable throughout.

a. Selection

The students will choose among the sets *of materials to come up with the concept of theorems on proportionality.*

♦Two triangles are similar if and only if the corresponding angles are congruent and the lengths of the corresponding sides are proportional.

♦The bisector of an angle separates the opposite sides into segments whose lengths are proportional to the other two sides.

♦*If three or more coplanar parallel lines are each cut by two transversals, the intercepted segments on the two transversals are proportional.*

b. Classification

Class, try to do the following:

Direction: Perform what is asked.

i. Prove that similarity between triangles has the property of transitivity.
ii. Prove that if a triangle is similar to the second triangle, and the second triangle is congruent to the third triangle, then the first triangle is similar to the third triangle.
iii. Prove that if all three pairs of corresponding sides of two triangles are proportional, then the two triangles are similar.

c. Association*(the teacher will pose a sample problem)*

(the teacher will present three more problem sets to the students)

d. ***Searching*** *(the teacher will apply the concept of ratio in proportion and let the students prepare to pose a problem)*

♦ **If a line divides two sides of a triangle proportionally, then the line is parallel to the third side**

♦ **The bisector of an angle separates the opposite sides into segments whose lengths are proportional to the other two sides.**

♦ **If three or more coplanar parallel lines are each cut by two transversals, the intercepted segments on the two transversals are proportional.**

e. ***Posing***

♦ Class you may pose now your problem.

♦ *I am so sorry class but there must be something wrong in the problem.*

f. ***Searching***

♦ *Class, we know that proportion is the equality of two ratios.*

♦ *Look back on your posed problem and try to review the given.*

g. Transformation

PROBLEM POSING:

♦ What if the measurement of one side of triangle changed?

♦ Will it be possible to obtain the same or different answer?

♦ *What if . . .?   How about . . .?   Do you think that. . .?*

h. ***Searching***

♦ Class try to do some changes in the given or you may add some questions in the problem posed. *(The students will reflect and do the necessary changes on the posed problem)*

♦ Are you now ready to pose the problem?   ♦ Yes, sir.

i. ***Posing the corrected problem***

(Students will pose the revised problem.)

♦ *Congratulations, you were able now to pose a problem.*

♦ Do you understand now the lesson?   ♦Yes, sir. We fully understand the lesson.

IV. Learning Assessment

Class prepare for the quiz. You may keep now all your notes.

Arrange your chairs and sit properly.

Do the following:

- In at least three sentences, define ratio and proportion.
- Create a problem using the concept of ratio and proportion.

Class, let us assess your work.

(The teacher will check the work of each student using the rubric)

(The teacher will record the score of students)

V. Assignment

Class, you may copy now your assignment on your notebook.

- How are ratio and proportion applied in similar triangles?
- How are the measurements of each corresponding angles of similar triangle?
- Show sample problems with solution.

PREPARED:

RAMON A. RAZA

APPENDIX B

Lesson Plan for Control Group

RATIO AND PROPORTION

I. Learning Competencies

At the end of the session, the students will be able to:

a. Define and identify ratio and proportion
b. Solve problems involving ratio and proportion
c. Participate actively in the class and group activity

II. Learning Concepts and Materials

A. Major Concept: Similarity

B. Sub Topic: Ratio and Proportion

C. References: Dildao, S. J., De Leon C. M. and Bernabe J. G. 2002. *Geometry.* Published and printed by JTW All rights reserved. 189 – 190 pp.
Fortes, E.,Garcia, T., Legaspi, M., Alcabedas, E., Bayle M., Alano, C., 2007. *Geometry*. ABIVA Publishing
Orines, F. B., and C. B. Manalo, 2004. *Next Century Mathematics, Advanced Algebra, Trigonometry and Statistics,* 2nd Edition, Phoenix Publishing House

D. Teaching Intervention: Traditional lecture-discussion intervention

F. Materials: PowerPoint presentation, manila paper, cartolina, colored chalks and chalkboard, graphing board and duplicated materials.

III. Learning Strategies / Activities

TEACHER'S ACTIVITY *STUDENT'S ACTIVITY*

A. Preliminary Activities

a. Greetings
b. Opening Prayer
3. Cleanliness and Orderliness
4. Checking of Attendance

B. Lesson Proper

Class, your goal for this lesson is to solve problems involving ratio and proportion.

You should develop in mind the importance of ratio and proportion.

a. Statement of Generality

The teacher will define *the concept of ratio.*

♦ *Ratio is a comparison of two or more quantities having the same unit.*

The teacher will present the concept of proportion to the students

♦ ***The equality of two ratios is called <u>proportion</u>.***

♦ ***In a:b = c:d, <u>b</u> and <u>c</u> are means, while <u>a</u> and <u>d</u> are extremes.***

♦ ***The law of proportion states that the product of the means is equal to the product of the extremes.***

b. Descriptive Examples for Ratio

Class, try to analyze the following:

There are 30 students in a classroom

→ *It forms Ratio (30:1)*

Five students shares two computer sets for the lesson

→ *It forms Ratio (5:1)*

A car travels 444 km in 6 hr

→ *It does not form a Ratio but a rate instead*

c. Sample Problem on Ratio *(the teacher will pose a sample problem)*

Cesar had 4 seedlings of squash and 6 seedlings of tomatoes,

while Renato had 8 seedlings of squash and 12 seedlings of tomatoes.

Guide Questions:

♦ What is the ratio of squash seedlings of to tomato seedlings each boy had?

♦ For Cesar, the ratio squash to tomato seedlings is 4:6

♦ For Renato, the ratio of squash to tomato seedlings is 8:12

♦ Can we simplify the two ratios?

♦ Yes

♦ Do they have the same ratio?

♦ They have the same ratio of 2:3

♦ Correct!

d. ***Extra Challenge***

♦ *Additional Problem sets on the text book five problems to be solved.*

♦ Are there any questions and clarification? Ask me questions?

e. Restatement of Generality

♦ Again, What is Ratio? How about Proportion?

IV. Learning Assessment

Class prepare for the quiz. You may keep now all your notes.

1. In at least three sentences, define ratio and proportion.
2. Solve the problem using the concept of ratio and proportion.
   Class, let us assess your work.

(The teacher will check the solution of each student using the rubric)

V. Assignment

Class, you may copy now your assignment on your notebook.

1. How are ratio and proportion applied in similar triangles?
2. How are the measurements of each corresponding angles of similar triangle?
3. Show sample problems with solution.

PREPARED:

RAMON A. RAZA

Mathematics Teacher

APPENDIX C

Table of Specification

TOPICS	Conceptual *Test I*			Procedural *Test II*			Item Placement
	K & C	AN & S	AP & E	K & C	AN & S	AP & E	
1. **Geometry of size and shape** (undefined terms, polygons angles, , measurements, area, circumference, perimeter, surface area and volumes)	///	~~////~~	//	//	//	/	$I_5, I_6, I_7, I_8,$ $I_{12}, I_{22}, I_{24}, I_{25},$ I_{26}, I_{31} $II_{20}, II_{23}, II_{25}$ II_{34}, II_{36}
2. **Geometry of relations** (relations involving segments and angles, angles and sides of triangle, parallel lines cut by a transversal)	//	~~////~~ - /	~~////~~ - /	///	~~////~~ - /	~~////~~ - /	$I_9, I_{10}, I_{17}, I_{18}, I_{19},$ $I_{20}, I_{21}, I_{23}, I_{27}, I_{28},$ $I_{29}, I_{30}, I_{38}, I_{39}$ $II_1, II_2, II_3, II_8, II_{11},$ $II_{14}, II_{16}, II_{17}, II_{18},$ $II_{19}, II_{26}, II_{27}, II_{30},$ II_{35}, II_{39}
3. **Triangle congruence** (conditions for triangle congruence and its applications)	//	///	/	/		/	$I_1, I_2, I_3,$ I_4, I_{11}, I_{11} II_{28}, II_{37}
4. **Quadrilaterals** (types of quadrilaterals, properties of the diagonals of special quadrilaterals, conditions that a quadrilateral is a parallelogram, properties of trapezoid)	//	///	///	/	///	//	$I_{13}, I_{14}, I_{15}, I_{32},$ $I_{33}, I_{34}, I_{35}, I_{36}$ $II_5, II_{12}, II_{21},$ $II_{24}, II_{31}, II_{33}$
5. **Similarity** (ratio and proportion, proportionality theorem, similarity between triangles, consequences of the basic proportionality theorems on areas and perimeter of similar figures)		/	/	/	///	~~////~~ - /	I_{40} $II_4, II_6, II_7,$ $II_9, II_{10}, II_{13},$ $II_{15}, II_{22}, II_{32},$ II_{38}, II_{40}
6. **Circles** (parts of circle, , angles formed by tangents and secants)		/			/	/	I_{37} II_{29}
TOTAL NUMBER OF ITEMS	**9**	**19**	**12**	**8**	**15**	**17**	**80**

K – Knowledge　　C – Comprehension　　AP – Application
AN – Analysis　　S – Synthesis　　E – Evaluation
I_N – Item number in the part of test

APPENDIX D

Initial Draft of Conceptual Understanding and Procedural Knowledge Tests

CONCEPTUAL UNDERSTANDING TEST

Name: _______________________ Date: _______________________

Name of School: _______________ Year/Section: _________________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐ ☐ ☐ ☐
A B C D
1. Given: $\Delta AOB \cong \Delta COD$, which side corresponds to $\overline{BA}$?

A. $\overline{CD}$ B. $\overline{CO}$ C. $\overline{OD}$ D. $\overline{BA}$

☐ ☐ ☐ ☐
A B C D
2. In ΔCDE, which is the included angle of sides $\overline{CE}$ and $\overline{CD}$?

A. $\angle C$ B. $\angle E$ C. $\angle D$ D. $\angle CDE$

☐ ☐ ☐ ☐
A B C D
3. Which of the following is/are property/properties of congruence?

I. Reflexive *II. Symmetric* *III. Transitive*

A. I only B. II only C. I and II D. I, II and III

☐ ☐ ☐ ☐
A B C D
4. What congruence postulate is defined, *"If two angles and included side of one triangle are congruent to the two angles and included side of another triangle, then the two triangles are congruent"*?

A. SSS B. SAS C. ASA D. SAA

☐ ☐ ☐ ☐
A B C D
5. Which of these represents the intersection of two distinct lines?

A. point B. line C. plane D. space

☐ ☐ ☐ ☐
A B C D
6. Which of the following is the union of two non collinear rays with a common endpoint?

A. line B. angle C. segment D. plane

☐ ☐ ☐ ☐ 7. Which of these is a polygon with eleven sides?
A B C D

A. nonagon B. dodecagon C. hendecagon D. decagon

☐ ☐ ☐ ☐ 8. What do you call a triangle with three congruent angles?
A B C D

A. equiangular triangle C. obtuse triangle

B. right triangle D. acute triangle

☐ ☐ ☐ ☐ 9. Which of the following refers to angles on a plane that have the same vertex and a common side but no common interior points?
A B C D

A. vertical B. corresponding C. adjacent D. exterior

☐ ☐ ☐ ☐ 10. Which segments of the given lengths **cannot be** the sides of a triangle?
A B C D

A. 4 cm, 5 cm, 6 cm C. 10 cm, 15 cm, 18 cm

B. 7 cm, 9 cm, 10 cm D. 3 cm, 4 cm, 9 cm

☐ ☐ ☐ ☐ 11. What property of equality is illustrated, if x = 7, then 7 = x?
A B C D

A. transitive B. associative C. reflexive D. symmetric

☐ ☐ ☐ ☐ 12. What is the common name for a regular quadrilateral?
A B C D

A. square B. parallelogram C. rectangle D. rhombus

☐ ☐ ☐ ☐ 13. Which of these is a quadrilateral with one pair of parallel sides?
A B C D

A. rectangle B. trapezoid C. parallelogram D. rhombus

☐ ☐ ☐ ☐ 14. Which of the following is/are true about parallelogram?
A B C D

I. The diagonals bisect each other
II. The diagonals bisects the opposite angles into two congruent parts
III. Consecutive angles are supplementary
IV. Opposite angles are congruent and supplementary

A. I and II B. II and V C. both I and IVD. both I and III

☐A ☐B ☐C ☐D 15. Which of these are the properties of a rhombus?

I. The diagonals are perpendicular bisectors with each other
II. The sides meet perpendicularly
III. The area is one-half the product of its diagonals
IV. Consecutive angles are congruent and supplementary

A. II only B. IV only C. both I and IV D. both I and III

☐A ☐B ☐C ☐D 16. Which of the following statements is NOT true about the centroid of a triangle?

A. It is point where the medians of the triangle are concurrent.

B. It may be located inside the triangle, on a side or outside of the triangle.

C. It divides the medians of the triangle into a ratio of 2:1.

D. It is the center of gravity of a triangular shape of uniform thickness and density.

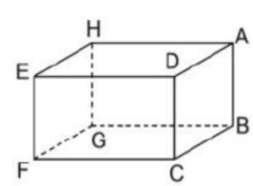
☐A ☐B ☐C ☐D 17. If the measures of three angles of a triangle are represented by $(y + 30)°$, $(4y + 30)°$, and $(10y – 30)°$, then the triangle must be:

A. obtuse B. isosceles C. scalene D. right

☐A ☐B ☐C ☐D 18. If the lengths of two sides of a triangle measure 7 and 15, which of these could be the length of the third side?

A. 7 B. 8 C. 11 D. 24

☐A ☐B ☐C ☐D 19. Which statement is logically equivalent to “If Andrea gets a job, she buys a new car”?



A. Andrea gets a job and she buys a new car.

B. If Andrea does not buy a new car, she does not get a job.

C. If Andrea does not get a job, she does not buy a new car.

D. If Andrea buys a new car, she gets a job.

□ □ □ □ 20. Which plane(s) contain *D* and is/are parallel to plane *FEH*?
A B C D

A. planes *DAB* and *HAD*

B. only plane *DAB*

C. planes *DCB* and *FCB*

D. only plane *HAD*

□ □ □ □ 21. Which statement is ***false*** about the line whose equation is
A B C D

$y = -2x - 5$?

A. Its slope is -2.

B. It is parallel to the line whose equation is $y = 2x + 5$.

C. Its y-intercept is -5.

D. It is perpendicular to the line whose equation is $y = ½ x - 5$

□ □ □ □ 22. In scalene triangle *ABC*, $m\angle B = 45$ and $m\angle C = 55$. What is the order of the sides in length, from longest to shortest?
A B C D

A. *AB*, *BC*, *AC* B. *BC*, *AC*, *AB* C. *AC*, *BC*, *AB* D. *BC*, *AB*, *AC*

□ □ □ □ 23. Point *C* (3,4) is the midpoint of *AB* . If the coordinates of *A* are (7,6), the coordinates of *B* are:
A B C D

A. (-1,2) B. (5,5) C. (2,1) D. (11,8)

□ □ □ □ 24. The sum of the measures of the interior angles of a regular pentagon is:
A B C D

A. 180º B. 360º C. 540º D. 720º

□ □ □ □ 25. A translation maps (*x*, *y*) to (*x* – 5, *y* + 3). In which quadrant does the point (–3,–2) lie under the same translation?
A B C D

A. I B. II C. III D. IV

□ □ □ □ 26. What do you call a triangle with at least two congruent sides?
A B C D

A. equilateral triangle

B. scalene triangle

C. isosceles triangle

D. none of these

☐ ☐ ☐ ☐ 27. In which triangle of which the point of intersection of the three medians
A B C D
is the same as the point of intersection of the three altitudes?

A. scalene triangle C. isosceles triangle

B. equilateral triangle D. right isosceles triangle

☐ ☐ ☐ ☐ 28. Which triangle is formed using the points (−2,3), (−2,−7), and (4,−5)as
A B C D
vertices?

A. scalene B. isosceles C. equilateral D. none of these

☐ ☐ ☐ ☐ 29. Which reason could be used to prove that a given parallelogram is a
A B C D
rhombus?

A. Diagonals are congruent C. Opposite sides are parallel

B. Diagonals are perpendicular D. Opposite angles are congruent

☐ ☐ ☐ ☐ 30. The slope of line *t*is -1/3. Which of these equations has a graph
A B C D
perpendicular to line *t*?

A. y + 2 = 13x B. −2x + 6 = 6y C. $9x - 3y = 27$ D. $3x + y = 0$

☐ ☐ ☐ ☐ 31. Which two points of concurrence could be located outside the triangle?
A B C D

A. in-center and centroid C. centroid and orthocenter

B. in-center and circumcenter D. circumcenter and orthocenter

☐ ☐ ☐ ☐ 32. The vertices of parallelogram *ABCD* are *A*(2, 0), *B*(0, −3), *C*(3, −3),
A B C D
and *D*(5, 0). If *ABCD* is reflected over the *x*-axis, how many vertices remain invariant?

A. 1 B. 2 C. 3 D. 0

☐ ☐ ☐ ☐ 33. Given three distinct quadrilaterals, a square, a rectangle, and a
A B C D
rhombus, which quadrilaterals must have perpendicular diagonals?

A. the rhombus, only C. the rectangle and the square

B. the rhombus and the square D. the rectangle, the rhombus, and the square

☐ ☐ ☐ ☐ 34. When writing a geometric proof, which angle relationship could be used alone to justify that two angles are congruent?
A B C D

A. supplementary angles
B. adjacent angles
C. linear pair of angles
D. vertical angles

☐ ☐ ☐ ☐ 35. Plane *A* is parallel to plane *B*. Plane *C* intersects plane *A* in line *m* and intersects plane *B* in line *n*. Lines *m* and *n* are
A B C D

A. intersecting
B. parallel
C. perpendicular
D. skew

For numbers 36 – 40, read the directions carefully.

Directions: Two statements labeled **A** and **B** are given. These are followed by a question which you are not expected to answer by computing but rather you have to decide whether the data given in the two statements are sufficient for answering the question. Blacken the letter that corresponds to your answer after the item number.

Blacken A – if ONE of the statements is sufficient and the other statement is NOT sufficient to answer the question asked.

B – if both statements **A** and **B** together are sufficient to answer the question asked, but neither statement alone is sufficient.

C – if each statement is sufficient by itself to answer the question asked.

D – if the statements **A** and **B** together are NOT sufficient to answer the question asked; that is, additional data specific to the problem are needed.

☐ ☐ ☐ ☐ 36. A. The length of rectangle is one meter more than its width.
A B C D

B. The diagonal is 5m.

What is the length of the rectangle?

☐ ☐ ☐ ☐ 37. A. The radius of a circle is 10 cm.
A B C D

B. The diameter of a circle is 20 cm.

What is the circumference of the circle?

☐ ☐ ☐ ☐ 38.
A B C D

A. Two candles are both 15 cm long.

B. They are lit at the same time.

How long is the third candle?

☐ ☐ ☐ ☐ 39.
A B C D

A. The formula for Pythagorean Theorem is $a^2 + b^2 = c^2$

B. Let a = 15 m and b = 20 m

How long is c?

☐ ☐ ☐ ☐ 40.
A B C D

A. Two numbers are in the ratio 4 : 3.

B. The number 20 is in the same ratio.

What is the principal root of 100?

Procedural Knowledge Test

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐ ☐ ☐ ☐ 1. What is the complement of the angle whose measure is 46º?
A B C D

A. 26 º B. 44 º C. 134 º D. 136 º

☐ ☐ ☐ ☐ 2. The measures of two supplementary angles y and x are in the ratio 2 : 3. What is the measure of angle y?
A B C D

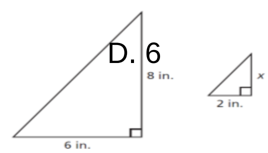
A. 72 º B. 108 º C. 18 º D. 54 º

☐ ☐ ☐ ☐ 3. Which of the following is NOT a solution to $3\,|2 - x| + 4 < 5$?
A B C D

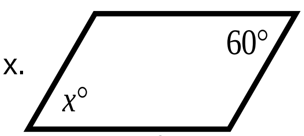
A. 2.1 B. 1.8 C. 1.6 D. 2

☐ ☐ ☐ ☐ 4. The two similar triangles shown are patterns used to create a design on a jacket.What is the value of *x*, the height of the smaller triangle, in inches?
A B C D

A. $1\frac{1}{2}$ B. $2\frac{2}{3}$ C. 4 D. 6



☐ ☐ ☐ ☐ 5. Given that the figure is a parallelogram, find x.
A B C D

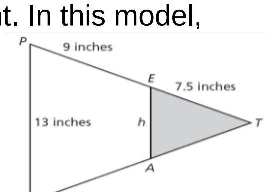


A. $130°$ B. $120°$ C. $80°$ D. $60°$

☐ ☐ ☐ ☐ 6. Two squares are similar. The area of the smaller square is 12 square inches. The area of the larger square is 768 square inches. What is the ratio of the side length of the smaller square to the side length of the larger square?
A B C D

A. $\frac{1}{64}$ B. $\frac{1}{32}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

☐ ☐ ☐ ☐ 7. The figure shows a model for a sports-team pennant. In this model, Triangle *PTL* is similar to Triangle *ETA*.Which of these is the length of $\overline{EA}$?
A B C D



A. 5.2 inches B. 5.9 inches C. 9.5 inches D. 10.8 inches

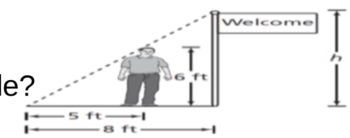
☐ ☐ ☐ ☐ A B C D 8. The measures of the acute angles of a right triangle are in the ratio 2:3.

Find their measures.

A. 40 and 50 B. 36 and 54 C. 20 and 30 D. 35 and 55

☐ ☐ ☐ ☐ A B C D 9. A flagpole casts a shadow that is 8 feet long. A 6-foot-tall man casts a shadow that is 5 feet long, as shown.

What is the value of *h*, the height of the flagpole?



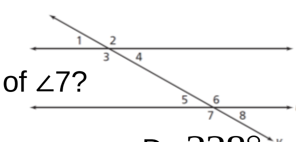
A. $9\frac{3}{5}\ ft$ B. $9\frac{3}{4}\ ft$ C. $10\frac{5}{6}\ ft$ D. $15\frac{3}{5}\ ft$

☐ ☐ ☐ ☐ A B C D 10. A rectangular painting has an area of 720 square inches. Jasmine reduced the length and width of this painting by a scale factor of 1/6 to create a miniature copy. What is the area of the miniature copy?

A. $12\ \text{in}^2$ B. $20\ \text{in}^2$ C. $60\ \text{in}^2$ D. $120\ \text{in}^2$

☐ ☐ ☐ ☐ A B C D 11. Parallel Lines *t* and *u* are cut by Transversal *v*, forming eight angles, as shown below.

The measure of $\angle 4 = 48°$. What is the measure of $\angle 7$?



A. $42°$ B. $48°$ C. $132°$ D. $228°$

☐ ☐ ☐ ☐ A B C D 12. A square has an area of 100 square meters. How long is its diagonal?

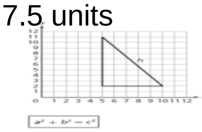
A. 10 m B. 20 m C. $10\sqrt{2}$ m D. $20\sqrt{2}$ m

☐ ☐ ☐ ☐ A B C D 13. For what value of ***n*** will make the proportion, 2 : 7 = 8 : ***n*** true?

A. 14 B. 21 C. 28 D. 35

☐ ☐ ☐ ☐ A B C D 14. The triangle on the grid represents a section of Maria's backyard. Which is closest to the length of side *h* in this triangle?

A. 14.0 units B. 10.3 units C. 9.3 units D. 7.5 units



☐ ☐ ☐ ☐ 15. Express in simplest form the ratio of $\overline{CD}:\overline{EF}$, if $\overline{CD}=30cm$ and
A B C D

$\overline{EF}=7dm$.

A. $\frac{30}{7}$ B. $\frac{7}{3}$ C. $\frac{3}{7}$ D. $\frac{7}{30}$

☐ ☐ ☐ ☐ 16. What is the measure of an angle if the measure of its supplement is 10
A B C D

less than thrice the measure of its complement?

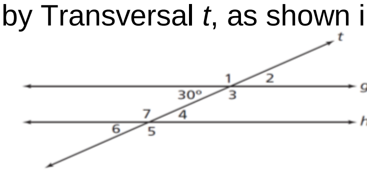
A. 40 B.50 C. 60 D. 70

☐ ☐ ☐ ☐ 17. What is the supplement of the angle with measure of 68º?
A B C D

A. 22 º B. 44 º C. 112 º D. 136 º

☐ ☐ ☐ ☐ 18. Lines *g* and *h* are parallel lines cut by Transversal *t*, as shown in the
A B C D

figure below.



What is the measure of Angle 5?

A. 30 º B. 60 º C. 150 º D. 210 º

☐ ☐ ☐ ☐ 19. The measure of two supplementary angles y and x are in the ratio 2 :
A B C D

3. What is the measure of angle x?

A. 108 º B. 72 º C. 54 º D. 18 º

☐ ☐ ☐ ☐ 20. In an isosceles triangle, the vertex angle is half the measurement of a
A B C D

base angle. What is the measure of the angle?

A. 72 º B. 60 º C. 36 º D. 30 º

☐ ☐ ☐ ☐ 21. In rhombus *ABCD*, the diagonals *AC* and *BD* intersect at *E*. If *AE* = 5
A B C D

and *BE* = 12, what is the length of *AB*?

A. 7 B. 10 C. 13 D. 17

☐ ☐ ☐ ☐ 22. In two similar triangles, the ratio of the lengths of a pair of
A B C D

corresponding sides is 7:8. If the perimeter of the larger triangle is 32, find the perimeter of the *smaller* triangle.

A. 8 B. 16 C. 21 D. 28

☐A ☐B ☐C ☐D 23. Point *F* (6,5) is the midpoint of *AB* . If the coordinates of *A* are (7,6), the coordinates of *B* are:

A. (-1,2) B. (5,4) C. (2,1) D. (11,8)

☐A ☐B ☐C ☐D 24. The lengths of the bases of an isosceles trapezoid are 6 cm and 12 cm.If the length of each leg is 5 cm, what is the area of the trapezoid?

A. 18 cm^2 B. 36 cm^2 C. 45 cm^2 D. 90 cm^2

☐A ☐B ☐C ☐D 25. Triangle *ABC* is an equilateral triangle with a perimeter of 30.

What is the length of altitude *h* of this triangle?

A. $5\sqrt{2}$ B. $5\sqrt{3}$ C. $10\sqrt{2}$ D. $10\sqrt{3}$

☐A ☐B ☐C ☐D 26. Which of the following points is a solution to the system $y = 4 - x$

and $y = -x^2 + 2x + 4$?

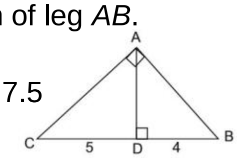
A. (1,3) B. (4,1) C. (-1,-3) D. (3,1)

☐A ☐B ☐C ☐D 27. The distance between points (4*a*,3*b*) and (3*a*,2*b*) is

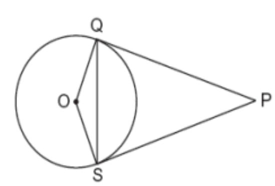
A. $a^2 + b^2$ B. $\sqrt{a^2 + b^2}$ C. $a + b$ D. $\sqrt{a + b}$

☐A ☐B ☐C ☐D 28. In the accompanying diagram of right triangle *ABC*, altitude *AD* divides hypotenuse *BC* into segments with lengths of 4 and 5. Find the length of leg *AB*.

A. 4.5 B. $2\sqrt{5}$ C. 6 D. 7.5



☐A ☐B ☐C ☐D 29. In the accompanying diagram, *PQ* and *PS* are tangents drawn to circle *O*, and chord *QS* is drawn. If $m\angle P = 40$, what is $m\angle PQS$?

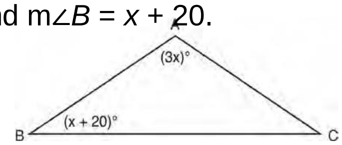


A. 140º C. 70º

B. 80º D. 60º

☐ ☐ ☐ ☐ 30. In the diagram below, $AB \cong AC$, $m\angle A = 3x$, and $m\angle B = x + 20$.
A B C D

What is the value of x?



A. 10 B. 32 C. 28 D. 40

☐ ☐ ☐ ☐ 31. Quadrilateral *MNOP* is a trapezoid with $MN \parallel OP$. If *M 'N 'O'P'* is the
A B C D
image of *MNOP* after a reflection over the *x*-axis, which two sides of quadrilateral *M 'N 'O'P'* are parallel?

A. *M 'N '* and *O'P'*

B. *P'M '* and *O'P'*

C. *M 'N '* and *N 'O'*

D. *P'M '* and *N 'O'*

☐ ☐ ☐ ☐ 32. Scalene triangle *ABC* is similar to triangle *DEF*. Which statement is *false*?
A B C D

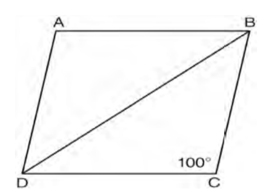
A. $AB:BC=DE:EF$

B. $\angle ACB \cong \angle DFE$

C. $AC:DF=BC:EF$

D. $\angle ABC \cong \angle EDF$

☐ ☐ ☐ ☐ 33. In the diagram below of rhombus *ABCD*, $m\angle C = 100$. What is $m\angle DBC$?
A B C D



A. 40

B. 45

C. 50

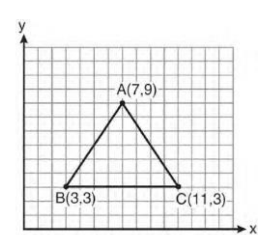
D. 80

☐ ☐ ☐ ☐ 34. In the diagram above, *ABCD*, $AC \cong BD$. Using this information,
A B C D

it could be proven that:

A. $BC = AB$ B. $AB = CD$ C. $AD - BC = CD$ D. $AB + CD = AD$

☐ ☐ ☐ ☐ 35. The vertices of the triangle in the diagram below are *A*(7, 9), *B*(3, 3),
A B C D
and *C*(11, 3). What are the coordinates of the centroid of *ABC*?



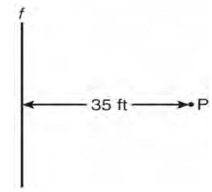
A. (5, 6)

B. (7, 5)

C. (7, 3)

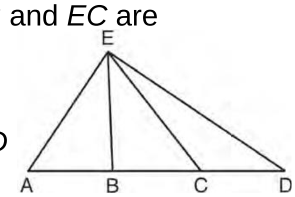
D. (9, 6)

☐A ☐B ☐C ☐D 36. A man wants to place a new bird bath in his yard so that it is 30 feet from a fence, *f*, and also 10 feet from a light pole, *P*. As shown in the diagram below, the light pole is 35 feet away from the fence. How many locations are possible for the bird bath?



A. 1 B. 3 C. 2 D. 0

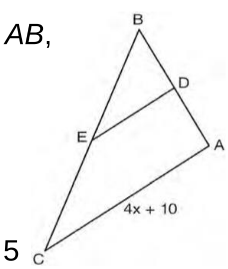
☐A ☐B ☐C ☐D 37. In *AED* with *ABCD* shown in the diagram below, *EB* and *EC* are drawn. If *AB* ≅*CD*, which statement could always be proven?



A. *AC* ≅*DB* C.*AE* ≅*ED*

B. *AB* ≅*BC* D.*EC* ≅*EA*

☐A ☐B ☐C ☐D 38. In the diagram below of *ABC*, *D* is the midpoint of *AB*, and *E* is the midpoint of *BC*.



If *AC* = 4*x* + 10, which expression represents *DE*?

A. $x + 2.5$ C. $2x + 5$

B. $2x + 10$ D. $8x + 20$

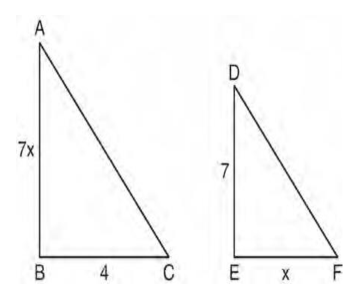
☐A ☐B ☐C ☐D 39. In *DEF*, m∠*D* = 3*x* + 5, m∠*E* = 4*x* − 15, and m∠*F* = 2*x* + 10.

Which statement is true?

A. *DF* = *FE* B. *DE* = *FE* C. m∠*E* = m∠*F*
D. m∠*D* = m∠*F*

☐A ☐B ☐C ☐D 40. As shown in the diagram below, *ABC* ~*DEF*, *AB* = 7*x*, *BC* = 4, *DE* = 7, and *EF* = *x*. What is the length of *AB*?

A. 28 B. 14 C. 2 D. 4



APPENDIX E

Evaluation Form for Conceptual Understanding

and Procedural Knowledge Tests

EVALUATION FORM FOR CONCEPTUAL UNDERSTANDING TEST

I. Directions: Kindly evaluate each conceptual understanding test item based on the set of characteristics. Please use the rating scale below.
   Characteristics for Evaluation:

   A. The test item is correct.
   B. The test item measures the conceptual understanding on the topic.
   C. Semantic relations or mathematical terms in the test item are used appropriately.
   D. The test item agrees with the principle of test construction.

   Rating Scale:

   3 – Very Good   2 – Good   1 – Poor

Test Item	CHARACTERISTICS			
	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Test Item	CHARACTERISTICS			
	A	B	C	D
21				
22				
23				
24				
25				
26				
27				
28				
29				
30				
31				
32				
33				
34				
35				
36				
37				
38				
39				
40				

II. Directions: Kindly write your comments and/or recommendations pertinent to specified test item and to entire conceptual understanding test.

____

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Evaluated by:

____

Signature Over Printed Name

Thank you very much!

EVALUATION FORM FOR PROCEDURAL KNOWLEDGE TEST

I. Directions: Kindly evaluate each procedural knowledge test item based on the set of characteristics. Please use the rating scale below.

Characteristics for Evaluation:

A. The test item is correct.
B. The test item measures the procedural knowledge on the topic.
C. Semantic relations or mathematical terms in the test item are used appropriately.
D. The test item agrees with the principle of test construction.

Rating Scale:

3 – Very Good 2 – Good 1 – Poor

Test Item	CHARACTERISTICS			
	A	B	C	D
1				
2				
3				
4				
5				
6				
7				
8				
9				

10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Test Item	CHARACTERISTICS			
	A	B	C	D
21				
22				
23				
24				
25				
26				
27				
28				
29				
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31				
32				
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40				

E.

Directions: Kindly write your comments and/or recommendations pertinent to specified test item and to entire procedural knowledge test.

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Evaluated by:

____

Signature Over Printed Name

Thank you very much!

APPENDIX F

Comments of Experts on First Draft

Conceptual Understanding Test

Name: ______________________ Date: ____________

Year/Section: _________________ Remarks: __________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐A ☐B ☐C ☐D 1. Given: $\Delta AOB \cong \Delta COD$, which side corresponds to $\overline{BA}$?

A. $\overline{CD}$ B. $\overline{CO}$ C. $\overline{OD}$ D. $\overline{BA}$

☐A ☐B ☐C ☐D 2. In ΔCDE, which is the included angle of sides $\overline{CE}$ and $\overline{CD}$?

A. $\angle C$ B. $\angle E$ C. $\angle D$ D. $\angle CDE$

☐A ☐B ☐C ☐D 3. What congruence postulate is defined, *"If two angles and included side of one triangle are congruent to the two angles and included side of another triangle, then the two triangles are congruent"*?

A. SSS B. SAS C. ASA D. SAA

☐A ☐B ☐C ☐D 4. Which of these represents the intersection of two distinct lines?

A. point B. line C. plane D. space

☐A ☐B ☐C ☐D 5. Which of the following is the union of two non collinear rays with a common endpoint?

A. line B. angle C. segment D. plane

☐A ☐B ☐C ☐D 6. Which of these is a polygon with eleven sides?

A. nonagon B. dodecagon C. hendecagon D. decagon

☐A ☐B ☐C ☐D 7. What do you call a triangle with three congruent angles?

A. equiangular triangle C. obtuse triangle
B. right triangle D. acute triangle

☐A ☐B ☐C ☐D 8. Which of the following refers to angles on a plane that have the same vertex and a common side but no common interior points?

A. vertical B. corresponding C. adjacent D. exterior

☐A ☐B ☐C ☐D 9. Which segments of the given lengths **cannot be** the sides of a triangle?

A. 4 cm, 5 cm, 6 cm C. 10 cm, 15 cm, 18 cm
B. 7 cm, 9 cm, 10 cm D. 3 cm, 4 cm, 9 cm

☐A ☐B ☐C ☐D 10. What property of equality is illustrated, if x = 7, then 7 = x?

A. transitive B. associative C. reflexive D. symmetric

☐A ☐B ☐C ☐D 11. Which of these is a quadrilateral with exactly one pair of parallel sides?

A. rectangle B. trapezoid C. parallelogram D. rhombus

□A □B □C □D 12. Which of these are the properties of a rhombus?

I. The diagonals are perpendicular bisectors with each other
II. The sides meet perpendicularly
the measures of
III. The area is one-half the product of its diagonals
IV. Consecutive angles are congruent and supplementary

A. II only B. IV only C. both I and IV D. both I and III

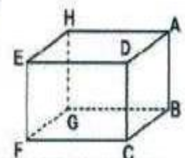
□A □B □C □D 13. If the measures of three angles of a triangle are represented by $(y + 30)°$, $(4y + 30)°$, and $(10y - 30)°$, then the triangle must be:

A. obtuse B. isosceles C. scalene D. right

□A □B □C □D 14. If the lengths of two sides of a triangle measure 7 and 15, which of these could be the length of the third side?

A. 7 B. 8 C. 11 D. 24

□A □B □C □D 15. Which plane(s) contain *D* and is/are parallel to plane *FEH*?



A. planes *DAB* and *HAD*
B. only plane *DAB*
C. planes *DCB* and *FCB*
D. only plane *HAD*

□A □B □C □D 16. In scalene triangle *ABC*, $m\angle B = 45$ and $m\angle C = 55$. What is the order of the sides in length, from longest to shortest?

A. *AB, BC, AC*
B. *BC, AC, AB*
C. *AC, BC, AB*
D. *BC, AB, AC*

□A □B □C □D 17. Point *C* (3,4) is the midpoint of *AB* . If the coordinates of *A* are (7,6), the coordinates of *B* are:

A. (-1,2) B. (5,5) C. (2,1) D. (11,8)

□A □B □C □D 18. The sum of the measures of the interior angles of a regular pentagon is:

A. 180º B. 360º C. 540º D. 720º

□A □B □C □D 19. A translation maps (x, y) to $(x - 5, y + 3)$. In which quadrant does the point $(-3,-2)$ lie under the same translation?

A. I B. II C. III D. IV

□A □B □C □D 20. Which triangle is formed using the points $(-2,3)$, $(-2,-7)$, and $(4,-5)$ as vertices?

A. scalene B. isosceles C. equilateral D. none of these

□A □B □C □D 21. Which reason could be used to prove that a given parallelogram is a rhombus?

A. Diagonals are congruent
B. Diagonals are perpendicular
C. Opposite sides are parallel
D. Opposite angles are congruent

□A □B □C □D 22. The slope of line *t* is -1/3. Which of these linear equations has a graph perpendicular to line *t* ?

A. $y + 2 = 13x$ B. $-2x + 6 = 6y$ C. $9x - 3y = 27$ D. $3x + y = 0$

□ □ □ □ 23. Which two points of concurrence could be located outside the triangle?
A B C D

A. in-center and centroid
B. in-center and circumcenter
C. centroid and orthocenter
D. circumcenter and orthocenter

□ □ □ □ 24. The vertices of parallelogram *ABCD* are *A*(2, 0), *B*(0, −3), *C*(3, −3), and *D*(5, 0).
A B C D

If *ABCD* is reflected over the *x*-axis, how many vertices remain invariant?

A. 1 B. 2 C. 3 D. 0

□ □ □ □ 25. When writing a geometric proof, which angle relationship could be used alone
A B C D

to justify that two angles are congruent?

A. supplementary angles
B. adjacent angles
C. linear pair of angles
D. vertical angles

space

For numbers 26 – 30, read the directions carefully.

Directions: Two statements labeled **A** and **B** are given. These are followed by a question which you are not expected to answer by computing but rather you have to decide whether the data given in the two statements are sufficient for answering the question. Blacken the letter that corresponds to your answer after the item number.

Blacken A – if ONE of the statements is sufficient and the other statement is NOT sufficient to answer the question asked.

B – if both statements **A** and **B** together are sufficient to answer the question asked, but neither statement alone is sufficient.

C – if each statement is sufficient by itself to answer the question asked.

D – if the statements **A** and **B** together are NOT sufficient to answer the question asked; that is, additional data specific to the problem are needed.

□ □ □ □ 26. A. The length of rectangle is one meter more than its width.
A B C D

B. The diagonal is 5m.
What is the length of the rectangle?

□ □ □ □ 27. A. The radius of a circle is 10 cm.
A B C D

B. The diameter of a circle is 20 cm.
What is the circumference of the circle?

□ □ □ □ 28. A. Two candles are both 15 cm long.
A B C D

B. They are lit at the same time.
How long is the third candle?

□ □ □ □ 29. A. The formula for Pythagorean Theorem is $a^2 + b^2 = c^2$
A B C D

B. Let a = 15 m and b = 20 m
How long is c?

□ □ □ □ 30. A. Two numbers are in the ratio 4 : 3.
A B C D

B. The number 20 is in the same ratio.
What is the principal root of 100?

Procedural Knowledge Test

Name: ______________________ Date: _____________

Year/Section: _________________ Remarks: ___________

Directions: Blacken the box that corresponds to the letter of the correct answer.

□ A □ B □ C □ D 1. What is the measure of the complement of the angle whose measure is 46º?

A. 26 º B. 44 º C. 134 º D. 136 º

#19

□ A □ B □ C □ D 2. The measures of two supplementary angles y and x are in the ratio 2 : 3, respectively. What is the measure of angle y?

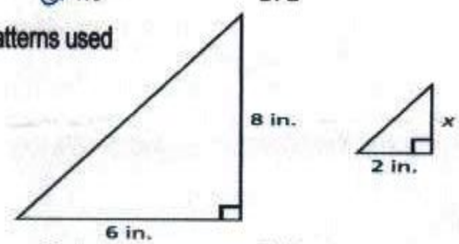
A. 72 º B. 108 º C. 18 º D. 54 º arrange options

□ A □ B □ C □ D 3. Which of the following is NOT a solution to $3\left|2 - x\right| + 4 < 5$?

A. 2.1 B. 1.8 C. 1.6 D. 2

□ A □ B □ C □ D 4. The two similar triangles shown are patterns used to create a design on a jacket.

What is the value of *x*, the height of the smaller triangle, in inches?

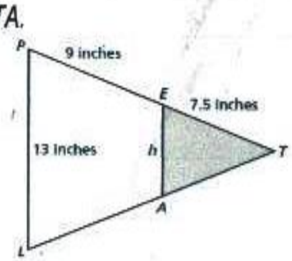


A. $1\frac{1}{2}$ B. $2\frac{2}{3}$ C. 4 D. 6

□ A □ B □ C □ D 6. Two squares are similar. The area of the smaller square is 12 square inches. The area of the larger square is 768 square inches. What is the ratio of the side length of the smaller square to the side length of the larger square?

A. $\frac{1}{64}$ B. $\frac{1}{32}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

□ A □ B □ C □ D 7. The figure shows a model for a sports-team pennant. In this model, Triangle *PTL* is similar to Triangle *ETA*.



Which of these is the length of $\overline{EA}$?

A. 5.2 inches B. 5.9 inches C. 9.5 inches D. 10.8 inches

☐ ☐ ☐ ☐ A B C D

8. The measures of the acute angles of a right triangle are in the ratio 2:3.

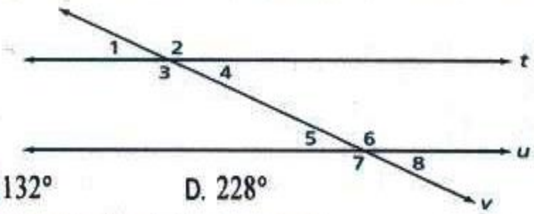
Find their measures.

A. 40 and 50 B. 36 and 54 C. 20 and 30 D. 35 and 55

☐ ☐ ☐ ☐ A B C D

11. Parallel Lines *t* and *u* are cut by Transversal *v*, forming eight angles, as shown below.

The measure of $\angle 4 = 48°$.



What is the measure of $\angle 7$?

A. 42° B. 48° C. 132° D. 228°

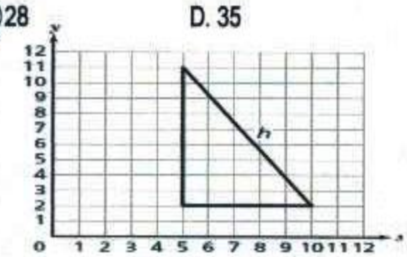
☐ ☐ ☐ ☐ A B C D

13. For what value of ***n*** will make the proportion, 2 : 7 = 8 : ***n*** true?

A. 14 B. 21 C. 28 D. 35

☐ ☐ ☐ ☐ A B C D

14. The triangle on the grid represents a section of Maria's backyard.



Which is closest to the length of side *h* in this triangle?

$a^2 + b^2 = c^2$

A. 14.0 units B. 10.3 units C. 9.3 units D. 7.5 units

☐ ☐ ☐ ☐ A B C D

16. What is the measure of an angle if the measure of its supplement is 10 less than thrice the measure of its complement?

A. 40 B. 50 C. 60 D. 70

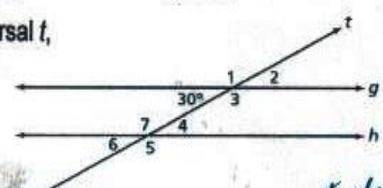
☐ ☐ ☐ ☐ A B C D

17. What is the supplement of the angle with measure of 68°?

A. 22 ° B. 44 ° C. 112 ° D. 136 °

☐ ☐ ☐ ☐ A B C D

18. Lines *g* and *h* are parallel lines cut by Transversal *t*,



as shown in the figure below.

What is the measure of Angle 5?

~~A. 30 °~~ ~~B. 60 °~~ C. 150 ° ~~D. 210 °~~

#2

☐ ☐ ☐ ☐ A B C D

19. The measure of two supplementary angles y and x are in the ratio 2 : 3, respectively What is the measure of angle *x*?

A. 108 ° B. 72 ° ~~C. 54 °~~ D. 18 °

☐ ☐ ☐ ☐ A B C D

20. In an isosceles triangle, the vertex angle is half the measurement of a base angle. What is the measure of the vertex angle?

A. 72 ° B. 60 ° C. 36 ° D. 30 °

☐ ☐ ☐ ☐ A B C D

21. In rhombus *ABCD*, the diagonals *AC* and *BD* intersect at *E*. If $|AE| = 5$ and $|BE| = 12$, what is the length of *AB*?

A. 7 B. 10 C. 13 D. 17

□ □ □ □ 22. In two similar triangles, the ratio of the lengths of a pair of corresponding sides
A B C D

is 7:8. If the perimeter of the larger triangle is 32, find the perimeter of the *smaller* triangle.

A. 8 B. 16 C. 21 D. 28

□ □ □ □ 23. Point *F* (6,5) is the midpoint of *AB* . If the coordinates of *A* are (7,6),
A B C D

the coordinates of *B* are:

A. (-1,2) B. (5,4) C. (2,1) D. (11,8)

□ □ □ □ 24. The lengths of the bases of an isosceles trapezoid are 6 cm and 12 cm.
A B C D

If the length of each leg is 5 cm, what is the area of the trapezoid?

A. 18 cm^2 B. 36 cm^2 C. 45 cm^2 D. 90 cm^2

□ □ □ □ 26. Which of the following points is a solution to the system $y = 4 - x$
A B C D

and $y = -x^2 + 2x + 4$?

A. (1,3) B. (4,1) C. (-1,-3) D. (3,1)

□ □ □ □ 27. The distance between points $(4a,3b)$ and $(3a,2b)$ is
A B C D

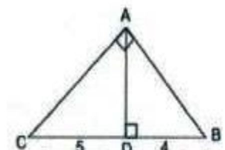
A. $a^2 + b^2$ B. $\sqrt{a^2 + b^2}$ C. $a + b$ D. $\sqrt{a + b}$

□ □ □ □ 28. In the accompanying diagram of right triangle *ABC*, altitude *AD* divides hypotenuse *BC*
A B C D

into segments with lengths of 4 and 5. Find the length of leg *AB*.

A. 4.5 C. 6

B. $2\sqrt{5}$ D. 7.5

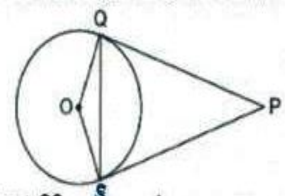


□ □ □ □ 29. In the accompanying diagram, *PQ* and *PS* are tangents drawn to circle *O*, and chord
A B C D

QS is drawn. If $m\angle P = 40$, what is $m\angle PQS$?

A. 140° C. 70°

B. 80° D. 60°

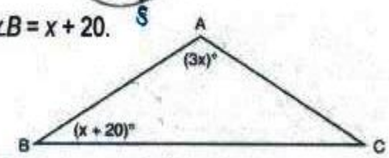


□ □ □ □ 30. In the diagram below, $AB \cong AC$, $m\angle A = 3x$, and $m\angle B = x + 20$.
A B C D

What is the value of *x*?

A. 10° C. 28°

B. 32° D. 40°



Appendix G

Item Analysis of the First Draft

Conceptual Understanding Test

Item No.	H	B	H+ B	Facility Index	INTER-PRETATION	H – B	Discrimi-nation Index	INTERPRETATION	DECISION
1	8	5	13	72.22%	Easy	3	0.33	Good Discrimination	Retain
2	7	5	12	66.67%	Easy	2	0.22	Moderately Discriminating	Retain
3	0	2	2	11.11%	Very Difficult	-2	-0.22	Not at all Discriminating	Reject
4	4	0	4	22.22%	Difficult	7	0.78	Very High Discriminator	Retain
5	7	4	11	61.11%	Easy	3	0.33	Good Discrimination	Retain
6	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
7	4	3	7	38.89%	Difficult	1	0.11	Very Low Discriminator	Retain
8	5	3	8	44.44%	Average	2	0.22	Moderately Discriminating	Retain
9	5	4	9	50.00%	Average	1	0.11	Very Low Discriminator	Retain
10	6	2	8	44.44%	Average	4	0.44	Good Discrimination	Retain
11	2	0	2	11.11%	Very Difficult	2	0.22	Moderately Discriminating	Retain
12	4	10	14	77.78%	Easy	-6	-0.67	Not at all Discriminating	Reject
13	3	2	5	27.78%	Difficult	1	0.11	Very Low Discriminator	Revise
14	2	2	4	22.22%	Difficult	0	0.00	Not at all Discriminating	Reject
15	3	2	5	27.78%	Difficult	1	0.11	Very Low Discriminator	Revise
16	1	2	3	16.67%	Very Difficult	-1	-0.11	Not at all Discriminating	Reject
17	5	3	8	44.44%	Average	2	0.22	Moderately Discriminating	Reject
18	5	1	6	33.33%	Difficult	4	0.44	Good Discrimination	Retain
19	0	4	4	22.22%	Difficult	-4	-0.44	Not at all Discriminating	Reject
20	5	1	6	33.33%	Difficult	4	0.44	Good Discrimination	Retain
21	0	2	2	11.11%	Very Difficult	-2	-0.22	Not at all Discriminating	Reject

22	6	1	7	38.89%	Difficult	5	0.56	Highly Discriminating	Retain
23	3	0	3	16.67%	Very Difficult	3	0.33	Good Discrimination	Retain
24	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
25	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
26	1	2	3	16.67%	Very Difficult	-1	-0.11	Not at all Discriminating	Reject
27	4	2	6	33.33%	Difficult	2	0.22	Moderately Discriminating	Retain
28	8	4	12	66.67%	Easy	4	0.44	Good Discrimination	Retain
29	4	3	7	38.89%	Difficult	1	0.11	Very Low Discriminator	Revise
30	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
31	5	1	6	33.33%	Difficult	4	0.44	Good Discrimination	Retain
32	6	1	7	38.89%	Difficult	5	0.56	Highly Discriminating	Retain
33	1	2	3	16.67%	Very Difficult	-1	-0.11	Not at all Discriminating	Reject
34	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain
35	2	2	4	22.22%	Difficult	0	0.00	Not at all Discriminating	Reject
36	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
37	6	2	8	44.44%	Average	4	0.44	Good Discrimination	Retain
38	3	2	5	27.78%	Difficult	1	0.11	Very Low Discriminator	Revise
39	4	1	5	27.78%	Difficult	3	0.33	Good Discrimination	Retain
40	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain

Item Analysis of the First Draft

Procedural Knowledge Test

Item No.	H	B	H+ B	Facility Index	INTER-PRETATION	H – B	Discrimi-nation Index	INTERPRETATION	DECISION
1	7	2	9	50.00%	Average	5	0.56	Highly Discriminating	Revise
2	5	3	8	44.44%	Average	2	0.22	Moderately Discriminating	Retain
3	6	3	9	50.00%	Average	3	0.33	Good Discrimination	Retain
4	7	3	10	55.56%	Average	3	0.33	Good Discrimination	Retain
5	6	9	15	83.33%	Very Easy	-3	-0.33	Not at all Discriminating	Reject
6	4	2	6	33.33%	Difficult	2	0.22	Moderately Discriminating	Retain
7	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain
8	4	1	5	27.78%	Difficult	3	0.33	Good Discrimination	Retain
9	5	1	6	33.33%	Difficult	4	0.44	Good Discrimination	Retain
10	0	2	2	11.11%	Very Difficult	-2	-0.22	Not at all Discriminating	Reject
11	4	1	5	27.78%	Difficult	3	0.33	Good Discrimination	Retain
12	0	3	3	16.67%	Very Difficult	-3	-0.33	Not at all Discriminating	Reject
13	7	2	9	50.00%	Average	5	0.56	Highly Discriminating	Reject
14	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
15	0	2	2	11.11%	Very Difficult	-2	-0.22	Not at all Discriminating	Reject
16	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain
17	0	1	1	5.56%	Very Difficult	-1	-0.11	Not at all Discriminating	Reject
18	3	2	5	27.78%	Difficult	1	0.11	Very Low Discriminator	Retain
19	1	5	6	33.33%	Difficult	-4	-0.44	Not at all Discriminating	Reject
20	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
21	2	0	2	11.11%	Very Difficult	2	0.22	Moderately Discriminating	Retain
22	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
23	4	2	6	33.33%	Difficult	2	0.22	Moderately Discriminating	Retain
24	5	1	6	33.33%	Difficult	4	0.44	Good Discrimination	Retain

25	0	2	2	11.11%	Very Difficult	-2	-0.22	Not at all Discriminating	Reject
26	4	0	4	22.22%	Difficult	4	0.44	Good Discrimination	Retain
27	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain
28	6	4	10	55.56%	Average	2	0.22	Moderately Discriminating	Retain
29	0	3	3	16.67%	Very Difficult	-3	-0.33	Not at all Discriminating	Reject
30	4	2	6	33.33%	Difficult	2	0.22	Moderately Discriminating	Retain
31	2	1	3	16.67%	Very Difficult	1	0.11	Very Low Discriminator	Reject
32	3	0	3	16.67%	Very Difficult	3	0.33	Good Discrimination	Retain
33	3	0	3	16.67%	Very Difficult	3	0.33	Good Discrimination	Retain
34	4	3	7	38.89%	Difficult	1	0.11	Very Low Discriminator	Retain
35	3	0	3	16.67%	Very Difficult	3	0.33	Good Discrimination	Retain
36	4	1	5	27.78%	Difficult	3	0.33	Good Discrimination	Retain
37	3	1	4	22.22%	Difficult	2	0.22	Moderately Discriminating	Retain
38	5	2	7	38.89%	Difficult	3	0.33	Good Discrimination	Retain
39	4	2	6	33.33%	Difficult	2	0.22	Moderately Discriminating	Retain
40	6	2	8	44.44%	Average	4	0.44	Good Discrimination	Retain

APPENDIX H

Second Draft of Conceptual and Procedural Knowledge Tests

CONCEPTUAL UNDERSTANDING TEST

Name: ___________________________ Date: _______________

Year/Section: ____________________ Remarks: ____________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐ ☐ ☐ ☐ 1. Given: $\Delta AOB \cong \Delta COD$, which side corresponds to $\overline{BA}$?
A B C D

A. $\overline{CD}$ B. $\overline{CO}$ C. $\overline{OD}$ D. $\overline{BA}$

☐ ☐ ☐ ☐ 2. In ΔCDE, which is the included angle of sides $\overline{CE}$ and $\overline{CD}$?
A B C D

A. $\angle C$ B. $\angle E$ C. $\angle D$ D. $\angle CDE$

☐ ☐ ☐ ☐ 3. What congruence postulate is defined, *"If two angles and included side of one triangle are congruent to the two angles and included side of another triangle, then the two triangles are congruent"*?
A B C D

A. SSS B. SAS C. ASA D. SAA

☐ ☐ ☐ ☐ 4. Which of these represents the intersection of two distinct lines?
A B C D

A. point B. line C. plane D. space

☐ ☐ ☐ ☐ 5. Which of the following is the union of two non collinear rays with a common endpoint?
A B C D

A. line B. angle C. segment D. plane

☐ ☐ ☐ ☐ 6. Which of these is a polygon with eleven sides?
A B C D

A. nonagon B. dodecagon C. hendecagon D. decagon

☐ ☐ ☐ ☐ 7. What do you call a triangle with three congruent angles?
A B C D

A. equiangular triangle C. obtuse triangle
B. right triangle D. acute triangle

☐ ☐ ☐ ☐ 8. Which of the following refers to angles on a plane that have the same vertex and a common side but no common interior points?
A B C D

A. vertical B. corresponding C. adjacent D. exterior

☐ ☐ ☐ ☐ 9. Which segments of the given lengths **cannot be** the sides of a triangle?
A B C D

A. 4 cm, 5 cm, 6 cm C. 10 cm, 15 cm, 18 cm

B. 7 cm, 9 cm, 10 cm D. 3 cm, 4 cm, 9 cm

☐ ☐ ☐ ☐ 10. What property of equality is illustrated, if x = 7, then 7 = x?
A B C D

A. transitive B. associative C. reflexive D. symmetric

☐ ☐ ☐ ☐ 11. Which of these is a quadrilateral with exactly one pair of parallel sides?
A B C D

A. rectangle B. trapezoid C. parallelogram D. rhombus

☐ ☐ ☐ ☐ 12. Which of these are the properties of a rhombus?
A B C D

I. The diagonals are perpendicular bisectors with each other
II. The sides meet perpendicularly
III. The area is one-half the product of the measures of its diagonals
IV. Consecutive angles are congruent and supplementary

A. II only B. IV only C. both I and IV D. both I and III

☐ ☐ ☐ ☐ 13. If the measures of three angles of a triangle are represented by $(y + 30)°$, $(4y + 30)°$, and $(10y – 30)°$, then the triangle must be:
A B C D

A. obtuse B. isosceles C. scalene D. right

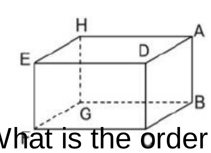
☐ ☐ ☐ ☐ 14. If the lengths of two sides of a triangle measure 7 and 15, which of these could be the length of the third side?
A B C D

A. 7 B. 8 C. 11 D. 24

☐ ☐ ☐ ☐ 15. Which plane(s) contain *D* and is/are parallel to plane *FEH*?
A B C D

A. planes *DAB* and *HAD* C. planes *DCB* and *FCB*

B. only plane *DAB* D. only plane *HAD*



☐ ☐ ☐ ☐ 16. In scalene triangle *ABC*, m∠*B* = 45 and m∠*C* = 55. What is the order of the sides in length, from longest to shortest?
A B C D

A. *AB*, *BC*, *AC* C. *AC*, *BC*, *AB*

B. *BC*, *AC*, *AB* D. *BC*, *AB*, *AC*

□A □B □C □D 17. Point *C* (3,4) is the midpoint of $\overline{AB}$. If the coordinates of *A* are (7,6), the coordinates of *B* are:

A. (-1,2) B. (5,5) C. (2,1) D. (11,8)

□A □B □C □D 18. The sum of the measures of the interior angles of a regular pentagon is:

A. 180º B. 360º C. 540º D. 720º

□A □B □C □D 19. A translation maps (*x*, *y*) to (*x* – 5, *y* + 3). In which quadrant does the point (–3,–2) lie under the same translation?

A. I B. II C. III D. IV

□A □B □C □D 20. Which triangle is formed using the points (−2,3), (−2,−7), and (4,−5) as vertices?

A. scalene B. isosceles C. equilateral D. none of these

□A □B □C □D 21. Which reason could be used to prove that a given parallelogram is a rhombus?

A. Diagonals are congruent

B. Diagonals are perpendicular

C. Opposite sides are parallel

D. Opposite angles are congruent

□A □B □C □D 22. The slope of line *t* is -1/3. Which of these linear equations has a graph perpendicular to line *t* ?

A. $y + 2 = 13x$ B. $-2x + 6 = 6y$ C. $9x - 3y = 27$ D. $3x + y = 0$

□A □B □C □D 23. Which two points of concurrence could be located outside the triangle?

A. in-center and centroid

B. in-center and circumcenter

C. centroid and orthocenter

D. circumcenter and orthocenter

□A □B □C □D 24. The vertices of parallelogram *ABCD* are *A*(2, 0), *B*(0, −3), *C*(3, −3), and *D*(5, 0). If *ABCD* is reflected over the *x*-axis, how many vertices remain invariant?

A. 1 B. 2 C. 3 D. 0

☐ ☐ ☐ ☐ 25. When writing a geometric proof, which angle relationship could be
A B C D

used alone to justify that two angles are congruent?

A. supplementary angles | C. linear pair of angles

B. adjacent angles | D. vertical angles

For numbers 26 – 30, read the directions carefully.

Directions: Two statements labeled **A** and **B** are given. These are followed by a question which you are not expected to answer by computing but rather you have to decide whether the data given in the two statements are sufficient for answering the question. Blacken the letter that corresponds to your answer after the item number.

Blacken A – if ONE of the statements is sufficient and the other statement is NOT sufficient to answer the question asked.

B – if both statements **A** and **B** together are sufficient to answer the question asked, but neither statement alone is sufficient.

C – if each statement is sufficient by itself to answer the question asked.

D – if the statements **A** and **B** together are NOT sufficient to answer the question asked; that is, additional data specific to the problem are needed.

☐ ☐ ☐ ☐ 26. A. The length of rectangle is one meter more than its width.
A B C D

B. The diagonal is 5m.

What is the length of the rectangle?

☐ ☐ ☐ ☐ 27. A. The radius of a circle is 10 cm.
A B C D

B. The diameter of a circle is 20 cm.

What is the circumference of the circle?

☐ ☐ ☐ ☐ 28. A. Two candles are both 15 cm long.
A B C D

B. They are lit at the same time.

How long is the third candle?

☐ ☐ ☐ ☐ 29. A. The formula for Pythagorean Theorem is $a^2 + b^2 = c^2$
A B C D

B. Let a = 15 m and b = 20 m

How long is c?

☐ ☐ ☐ ☐ 30. A. Two numbers are in the ratio 4 : 3.
A B C D

B. The number 20 is in the same ratio.

What is the principal root of 100?

PROCEDURAL KNOWLEDGE TEST

Name: ___________________________ Date: ________________

Year/Section: _____________________ Remarks: _____________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐A ☐B ☐C ☐D 1. What is the measure of the complement of the angle whose measure is 46^o?

A. 26^o B. 44^o C. 134^o D. 136^o

☐A ☐B ☐C ☐D 2. The measures of two supplementary angles y and x are in the ratio 2 : 3, respectively. What is the measure of angle y?

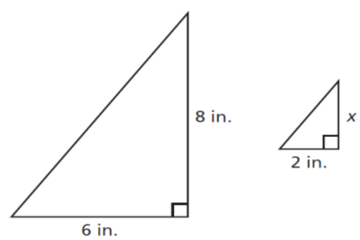
A. 18^o B. 54^o C. 72^o D. 108^o

☐A ☐B ☐C ☐D 3. Which of the following is NOT a solution to $3\left|2 - x\right| + 4 < 5$?

A. 2.1 B. 1.8 C. 1.6 D. 2

☐A ☐B ☐C ☐D 4. The two similar triangles shown are patterns used to create a design on a jacket.

What is the value of *x*, the height of the smaller triangle, in inches?



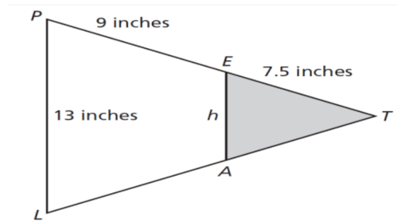
A. $1\frac{1}{2}$ B. $2\frac{2}{3}$ C. 4 D. 6

☐A ☐B ☐C ☐D 5. Two squares are similar. The area of the smaller square is 12 square inches. The area of the larger square is 768 square inches. What is the ratio of the side length of the smaller square to the side length of the larger square?

A. $\frac{1}{64}$ B. $\frac{1}{32}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$

☐ ☐ ☐ ☐
A B C D 6. The figure shows a model for a sports-team pennant. In this model,

Triangle *PTL* is similar to ΔETA.



Which of these is the length of $\overline{EA}$?

A. 5.2 inches B. 5.9 inches C. 9.5 inches D. 10.8 inches

☐ ☐ ☐ ☐
A B C D 7. The measures of the acute angles of a right triangle are in the ratio 2:3.

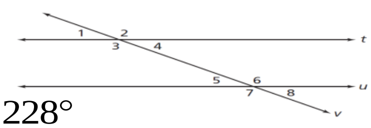
Find their measures.

A. 40 and 50 B. 36 and 54 C. 20 and 30 D. 35 and 55

☐ ☐ ☐ ☐
A B C D 8. Parallel Lines *t* and *u* are cut by Transversal *v*, forming eight angles, as

shown below. The measure of ∠4 = 48°.

What is the measure of ∠7?



A. $42°$ B. $48°$ C. $132°$ D. $228°$

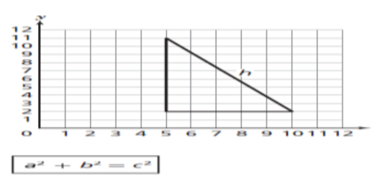
☐ ☐ ☐ ☐
A B C D 9. For what value of ***n*** will make the proportion, 2 : 7 = 8 : ***n*** true?

A. 14 B. 21 C. 28 D. 35

☐ ☐ ☐ ☐
A B C D 10. The triangle on the grid represents

a section of Maria's backyard. Which is closest

to the length of side *h* in this triangle?



A. 14.0 units B. 10.3 units C. 9.3 units D. 7.5 units

☐ ☐ ☐ ☐
A B C D 11. What is the measure of an angle if the measure of its supplement is 10

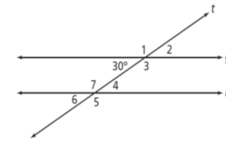
less than thrice the measure of its complement?

A. 40 B. 50 C. 60 D. 70

☐ ☐ ☐ ☐
A B C D 12. What is the supplement of the angle with measure of 68º?

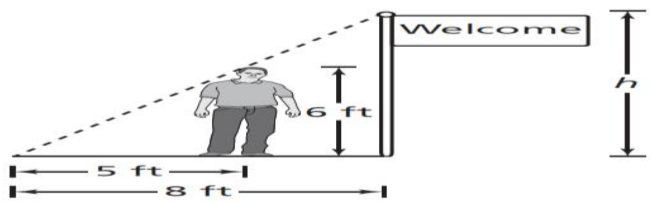
A. 22 º B. 44 º C. 112 º D. 136 º

☐A ☐B ☐C ☐D 13. Lines *g* and *h* are parallel lines cut by Transversal *t*, as shown in the figure below. What is the measure of ∠5?



A. 30 º   B. 60 º   C. 150 º   D. 210 º

☐A ☐B ☐C ☐D 14. A flagpole casts a shadow that is 8 feet long. A 6-foot-tall man casts a shadow that is 5 feet long, as shown.What is the value of *h*, the height of the flagpole?



A. $9\frac{3}{5}\ ft$ B. $9\frac{3}{4}\ ft$ C. $10\frac{5}{6}\ ft$ D. $15\frac{3}{5}\ ft$

☐A ☐B ☐C ☐D 15. In an isosceles triangle, the vertex angle is half the measurement of a base angle. What is the measure of the vertex angle?

A. 72 º   B. 60 º   C. 36 º   D. 30 º

☐A ☐B ☐C ☐D 16. In rhombus *ABCD*, the diagonals *AC* and *BD* intersect at *E*. If $|\mathrm{AE}|$ I = 5 and *BE* = 12, what is the length of *AB*?

A. 7   B. 10   C. 13   D. 17

☐A ☐B ☐C ☐D 17. In two similar triangles, the ratio of the lengths of a pair of corresponding sides is 7:8. If the perimeter of the larger triangle is 32, find the perimeter of the *smaller* triangle.

A. 8   B. 16   C. 21   D. 28

☐A ☐B ☐C ☐D 18. Point *F* (6,5) is the midpoint of *AB* . If the coordinates of *A* are (7,6), the coordinates of *B* are:

A. (-1,2)   B. (5,4)   C. (2,1)   D. (11,8)

☐A ☐B ☐C ☐D 19. The lengths of the bases of an isosceles trapezoid are 6 cm and 12 cm. If the length of each leg is 5 cm, what is the area of the trapezoid?

A. 18 cm^2 B. 36 cm^2 C. 45 cm^2 D. 90 cm^2

☐A ☐B ☐C ☐D 20. Which of the following points is a solution to the system $y = 4 - x$ and $y = -x^2 + 2x + 4$?

A. (1,3) B. (4,1) C. (-1,-3) D. (3,1)

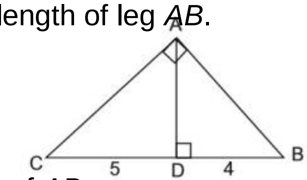
☐A ☐B ☐C ☐D 21. The distance between points A($4a$,$3b$) and B($3a$,$2b$) is

A. $a^2 + b^2$ B. $\sqrt{a^2 + b^2}$ C. $a + b$ D. $\sqrt{a + b}$

☐A ☐B ☐C ☐D 22. In the accompanying diagram of right triangle *ABC*, altitude *AD* divides hypotenuse *BC* into segments with lengths of 4 and 5. Find the length of leg *AB*.

A. 4.5 C. 6

B. $2\sqrt{5}$ D. 7.5

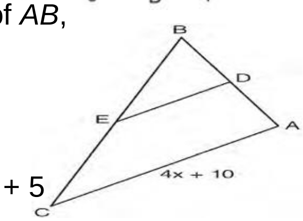


☐A ☐B ☐C ☐D 23 In the diagram below of *ABC*, *D* is the midpoint of *AB*, and *E* is the midpoint of *BC*.

If *AC* = 4*x* + 10, which expression represents *DE*?

A. *x* + 2.5 C. 2*x* + 5

B. 2*x* + 10 D. 8*x* + 20

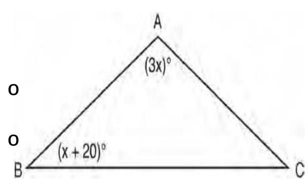


☐A ☐B ☐C ☐D 24. In the diagram below, $AB \cong AC$, $m\angle A = 3x$, and $m\angle B = x + 20$.

What is the value of *x*?

A. 10 º C. 28 º

B. 32 º D. 40 º



☐A ☐B ☐C ☐D 25. Scalene triangle *ABC* is similar to triangle *DEF*. Which statement is *false*?

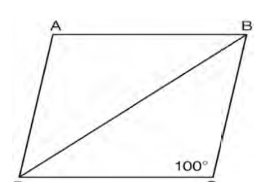
A. *AB*:*BC*=*DE*:*EF* *C. AC*:*DF*=*BC*:*EF*

B. $\angle ACB \cong \angle DFE$ *D.* $\angle ABC \cong \angle EDF$

☐A ☐B ☐C ☐D 26. In the diagram below of rhombus *ABCD*, $m\angle C = 100$. What is $m\angle DBC$?

A. 40 C. 50

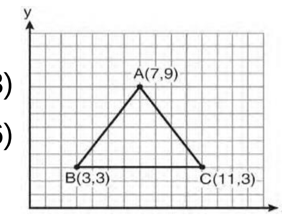
B. 45 D. 80



☐A ☐B ☐C ☐D 27. In the diagram above, *ABCD*, $AC \cong BD$. Using this information, it could be proven that:

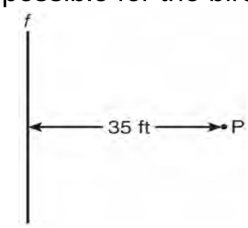
A. $BC = AB$ B. $AB = CD$ C. $AD - BC = CD$ D. $AB + CD = AD$

☐A ☐B ☐C ☐D 28. The vertices of the triangle in the diagram below are *A*(7, 9), *B*(3, 3), and *C*(11, 3). What are the coordinates of the centroid of *ABC*?



A. (5, 6)
C. (7, 3)
B. (7, 5)
D. (9, 6)

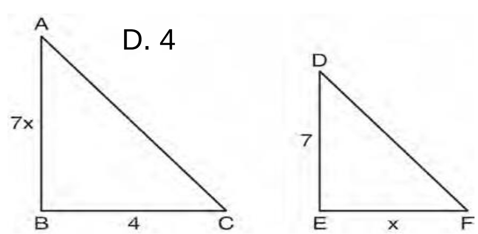
☐A ☐B ☐C ☐D 29. A man wants to place a new bird bath in his yard so that it is 30 feet from a fence, *f*, and also 10 feet from a light pole, *P*. As shown in the diagram below, the light pole is 35 feet away from the fence. How many locations are possible for the bird bath?



A. 1
C. 2
B. 3
D. 0

☐A ☐B ☐C ☐D 30. As shown in the diagram below, $ABC \sim DEF$, $AB = 7x$, $BC = 4$, $DE = 7$, and $EF = x$. What is the length of *AB*?

A. 28
C. 2
B. 14
D. 4



APPENDIX I

Item Analysis of the Second Draft

Conceptual Understanding Test

Item No.	H	B	H+B	Facility Index	INTER-PRETATION	H – B	Discrimi-nation Index	INTERPRETATION	DECISION
1	15	8	23	76.67%	Easy	7	0.47	Highly Discriminating	Retain
2	11	8	19	63.33%	Easy	3	0.20	Moderately Discriminating	Retain
3	9	6	15	50.00%	Average	3	0.20	Moderately Discriminating	Retain
4	12	7	19	63.33%	Easy	4	0.27	Moderately Discriminating	Retain
5	11	5	16	53.33%	Average	6	0.40	Good Discrimination	Retain
6	2	6	8	26.67%	Difficult	-4	-0.27	Not at all Discriminating	Reject
7	12	8	20	66.67%	Easy	4	0.27	Moderately Discriminating	Retain
8	7	2	9	30.00%	Difficult	5	0.33	Good Discrimination	Retain
9	9	4	13	43.33%	Average	5	0.33	Good Discrimination	Retain
10	2	5	7	23.33%	Difficult	-3	-0.20	Not at all Discriminating	Reject
11	8	2	10	33.33%	Difficult	6	0.40	Good Discrimination	Retain
12	7	2	9	30.00%	Difficult	5	0.33	Good Discrimination	Retain
13	8	13	21	70.00%	Easy	-5	-0.33	Not at all Discriminating	Reject
14	2	4	6	20.00%	Very Difficult	-2	-0.13	Not at all Discriminating	Reject
15	11	8	19	63.33%	Easy	3	0.20	Moderately Discriminating	Retain
16	9	0	9	30.00%	Difficult	9	0.60	Highly Discriminating	Retain
17	6	0	6	20.00%	Very Difficult	6	0.40	Good Discrimination	Retain
18	8	5	13	43.33%	Average	3	0.20	Moderately Discriminating	Retain
19	6	2	8	26.67%	Difficult	4	0.27	Moderately Discriminating	Retain
20	7	4	11	36.67%	Difficult	3	0.20	Moderately Discriminating	Retain
21	6	2	8	26.67%	Difficult	4	0.27	Moderately Discriminating	Retain
22	8	2	10	33.33%	Difficult	6	0.40	Good Discrimination	Retain
23	9	4	13	43.33%	Average	5	0.33	Good Discrimination	Retain
24	10	4	14	46.67%	Average	6	0.40	Good Discrimination	Retain
25	13	4	17	56.67%	Average	9	0.60	Highly Discriminating	Retain

Item Analysis of the Second Draft

Procedural Knowledge Test

Item No.	H	B	H+B	Facility Index	INTER-PRETATION	H – B	Discrimi-nation Index	INTERPRETATION	DECISION
1	13	6	19	63.33%	Easy	7	0.47	Highly Discriminating	Retain
2	9	3	12	40.00%	Difficult	6	0.40	Good Discrimination	Retain
3	11	7	18	60.00%	Average	4	0.27	Moderately Discriminating	Retain
4	11	6	17	56.67%	Average	5	0.33	Good Discrimination	Retain
5	6	2	8	26.67%	Difficult	4	0.27	Moderately Discriminating	Retain
6	7	0	7	23.33%	Difficult	7	0.47	Highly Discriminating	Retain
7	7	4	11	36.67%	Difficult	3	0.20	Moderately Discriminating	Retain
8	14	8	22	73.33%	Easy	6	0.40	Good Discrimination	Retain
9	11	15	26	86.67%	Very Easy	-4	-0.27	Not at all Discriminating	Reject
10	6	10	16	53.33%	Average	-4	-0.27	Not at all Discriminating	Reject
11	8	4	12	40.00%	Difficult	4	0.27	Moderately Discriminating	Retain
12	15	9	24	80.00%	Easy	6	0.40	Good Discrimination	Retain
13	15	7	22	73.33%	Easy	8	0.53	Highly Discriminating	Retain
14	8	2	10	33.33%	Difficult	6	0.40	Good Discrimination	Retain
15	9	2	11	36.67%	Difficult	7	0.47	Highly Discriminating	Retain
16	10	5	15	50.00%	Average	5	0.33	Good Discrimination	Retain
17	1	6	7	23.33%	Difficult	-5	-0.33	Not at all Discriminating	Reject
18	3	9	12	40.00%	Difficult	-6	-0.40	Not at all Discriminating	Reject
19	1	3	4	13.33%	Very Difficult	-2	-0.13	Not at all Discriminating	Reject
20	8	5	13	43.33%	Average	3	0.20	Moderately Discriminating	Retain
21	12	8	20	66.67%	Easy	4	0.27	Moderately Discriminating	Retain
22	8	3	11	36.67%	Difficult	5	0.33	Good Discrimination	Retain
23	13	6	19	63.33%	Easy	7	0.47	Highly Discriminating	Retain
24	7	2	9	30.00%	Difficult	5	0.33	Good Discrimination	Retain
25	9	4	13	43.33%	Average	5	0.33	Good Discrimination	Retain

APPENDIX J

Reliability Test Results

Descriptive Statistics

	Mean	Std. Deviation	N
Odd Number Conceptual Understanding	4.9583	1.68798	48
Even Number Conceptual Understanding	4.7500	1.68220	48
Odd Number Procedural Knowledge	4.5417	2.11337	48
Even Number Procedural Knowledge	3.6042	1.80707	48

Correlation Between the Odd and Even Numbered Items
for Conceptual Understanding Test

		Odd Number Conceptual Understanding	Even Number Conceptual Understanding
Odd Number Conceptual Understanding	Correlation	1.000	.318
	Significance (2-	.	.027
	Df	0	46
Even Number Conceptual Understanding	Correlation	.318	1.000
	Significance (2-tailed)	.027	.
	Df	46	0

Correlation Between the Odd and Even Numbered Items
for Procedural Knowledge Test

		Odd Number Procedural Understanding	Even Number Procedural Understanding
Odd Number Procedural Knowledge	Correlation	1.000	.314
	Significance (2-tailed)	.	.030
	df	0	46
Even Number Procedural Knowledge	Correlation	.314	1.000
	Significance (2-tailed)	.030	.
	df	46	0

APPENDIX K

Final Draft of Conceptual and Procedural Knowledge Tests

CONCEPTUAL UNDERSTANDING TEST

Name: ______________________ Date: _______________________

Name of School: _______________ Year/Section: ________________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐ ☐ ☐ ☐ 1. Given: $\Delta AOB \cong \Delta COD$, which side corresponds to $\overline{BA}$?
A B C D

A. $\overline{CD}$ B. $\overline{CO}$ C. $\overline{OD}$ D. $\overline{BA}$

☐ ☐ ☐ ☐ 2. In ΔCDE, which is the included angle of sides $\overline{CE}$ and $\overline{CD}$?
A B C D

A. $\angle C$ B. $\angle E$ C. $\angle D$ D. $\angle CDE$

☐ ☐ ☐ ☐ 3. What congruence postulate is defined, *"If two angles and included side of one triangle are congruent to the two angles and included side of another triangle, then the two triangles are congruent"*?
A B C D

A. SSS B. SAS C. ASA D. SAA

☐ ☐ ☐ ☐ 4. Which of these represents the intersection of two distinct lines?
A B C D

A. point B. line C. plane D. space

☐ ☐ ☐ ☐ 5. Which of the following is the union of two non collinear rays with a common endpoint?
A B C D

A. line B. angle C. segment D. plane

☐ ☐ ☐ ☐ 6. What do you call a triangle with three congruent angles?
A B C D

A. equiangular triangle C. obtuse triangle
B. right triangle D. acute triangle

☐ ☐ ☐ ☐ 7. Which of the following refers to angles on a plane that have the same vertex and a common side but no common interior points?
A B C D

A. vertical B. corresponding C. adjacent D. exterior

☐ ☐ ☐ ☐ 8. Which segments of the given lengths **cannot be** the sides of a triangle?
A B C D

A. 4 cm, 5 cm, 6 cm | C. 10 cm, 15 cm, 18 cm

B. 7 cm, 9 cm, 10 cm | D. 3 cm, 4 cm, 9 cm

□ □ □ □ 9. Which of these is a quadrilateral with exactly one pair of parallel sides?
A B C D

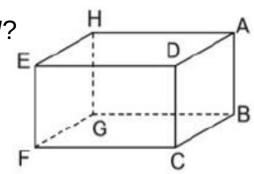
A. rectangle B. trapezoid C. parallelogram D. rhombus

□ □ □ □ 10. Which of these are the properties of a rhombus?
A B C D

I. The diagonals are perpendicular bisectors with each other
II. The sides meet perpendicularly
III. The area is one-half the product of the measures of its diagonals
IV. Consecutive angles are congruent and supplementary

A. II only B. IV only C. both I and IV D. both I and III

□ □ □ □ 11. Which plane(s) contain *D* and is/are parallel to plane *FEH*?
A B C D



A. planes *DAB* and *HAD* | C. planes *DCB* and *FCB*

B. only plane *DAB* | D. only plane *HAD*

□ □ □ □ 12. In scalene triangle *ABC*, m∠*B* = 45 and m∠*C* = 55. What is the order of the sides in length, from longest to shortest?
A B C D

A. *AB*, *BC*, *AC* B. *BC*, *AC*, *AB* C. *AC*, *BC*, *AB* D. *BC*, *AB*, *AC*

□ □ □ □ 13. Point *C* (3,4) is the midpoint of *AB* . If the coordinates of *A* are (7,6), the coordinates of *B* are:
A B C D

A. (-1,2) B. (5,5) C. (2,1) D. (11,8)

□ □ □ □ 14. The sum of the measures of the interior angles of a regular pentagon is:
A B C D

A. 180º B. 360º C. 540º D. 720º

□ □ □ □ 15. A translation maps (*x*, *y*) to (*x* – 5, *y* + 3). In which quadrant does the point (–3,–2) lie under the same translation?
A B C D

A. I B. II C. III D. IV

□ □ □ □ 16. Which triangle is formed using the points (−2,3), (−2,−7), and (4,−5) as vertices?
A B C D

A. scalene B. isosceles C. equilateral D. none of these

□ □ □ □ 17. Which reason could be used to prove that a given parallelogram is a rhombus?
A B C D

A. Diagonals are congruent | C. Opposite sides are parallel

B. Diagonals are perpendicular

D. Opposite angles are congruent

☐ ☐ ☐ ☐ 18. The slope of line *t* is -1/3. Which of these linear equations has a graph
A B C D

perpendicular to line *t* ?

A. y + 2 = 13x

B. −2x + 6 = 6y

C. $9x - 3y = 27$

D. $3x + y = 0$

☐ ☐ ☐ ☐ 19. Which two points of concurrence could be located outside the triangle?
A B C D

A. in-center and centroid

B. in-center and circumcenter

C. centroid and orthocenter

D. circumcenter and orthocenter

☐ ☐ ☐ ☐ 20. The vertices of parallelogram *ABCD* are *A*(2, 0), *B*(0, −3), *C*(3, −3), and *D*(5, 0). If *ABCD* is reflected over the *x*-axis, how many vertices remain invariant?
A B C D

A. 1

B. 2

C. 3

D. 0

☐ ☐ ☐ ☐ 21. When writing a geometric proof, which angle relationship could be used alone to justify that two angles are congruent?
A B C D

A. supplementary angles

B. adjacent angles

C. linear pair of angles

D. vertical angles

For numbers 22 – 25, read the directions carefully.

Directions: Two statements labeled **A** and **B** are given. These are followed by a question which you are not expected to answer by computing but rather you have to decide whether the data given in the two statements are sufficient for answering the question. Blacken the letter that corresponds to your answer.

Blacken A – if ONE of the statements is sufficient and the other statement is NOT sufficient to answer the question asked.

B – if both statements **A** and **B** together are sufficient to answer the question asked, but neither statement alone is sufficient.

C – if each statement is sufficient by itself to answer the question asked.

D – if the statements **A** and **B** together are NOT sufficient to answer the question asked; that is, additional data specific to the problem are needed.

☐ ☐ ☐ ☐ 22. A. The length of rectangle is one meter more than its width.
A B C D

B. The diagonal is 5m.

What is the length of the rectangle?

☐ ☐ ☐ ☐ 23. A. The radius of a circle is 10 cm.
A B C D

B. The diameter of a circle is 20 cm.

What is the circumference of the circle?

☐ ☐ ☐ ☐ 24.
A B C D

A. The formula for Pythagorean Theorem is $a^2 + b^2 = c^2$

B. Let a = 15 m and b = 20 m

How long is c?

☐ ☐ ☐ ☐ 25.
A B C D

A. Two numbers are in the ratio 4 : 3.

B. The number 20 is in the same ratio.

What is the principal root of 100?

PROCEDURAL KNOWLEDGE TEST

Name: ________________________ Date: _________________________

Name of School: ________________ Year/Section: _________________

Directions: Blacken the box that corresponds to the letter of the correct answer.

☐ ☐ ☐ ☐ 1. What is the measure of the complement of the angle whose measure is 46º?
A B C D

A. 26 º B. 44 º C. 134 º D. 136 º

☐ ☐ ☐ ☐ 2. The measures of two supplementary angles y and x are in the ratio 2 : 3,
A B C D

respectively. What is the measure of angle y?

A. 18 º B. 54 º C. 72 º D. 108 º

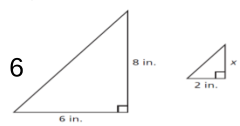
☐ ☐ ☐ ☐ 3. Which of the following is NOT a solution to $3\,|2 - x| + 4 < 5$?
A B C D

A. 2.1 B. 1.8 C. 1.6 D. 2

☐ ☐ ☐ ☐ 4. The two similar triangles shown are patterns used to create a design on a
A B C D

jacket. What is the value of *x*, the height of the smaller triangle, in inches?

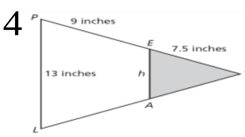
A. $1\frac{1}{2}$ B. $2\frac{2}{3}$ C. 4 D. 6



☐ ☐ ☐ ☐ 5. Two squares are similar. The area of the smaller square is 12 square inches.
A B C D

The area of the larger square is 768 square inches. What is the ratio of the side length of the smaller square to the side length of the larger square?

A. $\frac{1}{64}$ B. $\frac{1}{32}$ C. $\frac{1}{8}$ D. $\frac{1}{4}$



☐ ☐ ☐ ☐ 6. The figure shows a model for a sports-team pennant.
A B C D

In this model, Triangle *PTL* is similar to ΔETA .Which of these is the length of $\overline{EA}$?

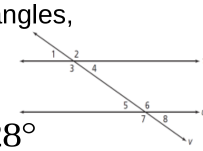
A. 5.2 inches B. 5.9 inches C. 9.5 inches D. 10.8 inches

☐ ☐ ☐ ☐ 7. The measures of the acute angles of a right triangle are in the ratio 2:3.
A B C D

Find their measures.

A. 40 and 50 B. 36 and 54 C. 20 and 30 D. 35 and 55

☐ ☐ ☐ ☐ 8. Parallel Lines *t* and *u* are cut by Transversal *v*, forming eight angles,
A B C D



as shown. The measure of ∠4 = 48°. What is the measure of ∠7?

A. $42°$ B. $48°$ C. $132°$ D. $228°$

☐ ☐ ☐ ☐ 9. What is the measure of an angle if the measure of its supplement is 10 less
A B C D

than thrice the measure of its complement?

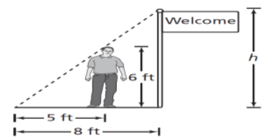
A. 40 B. 50 C. 60 D. 70

☐ ☐ ☐ ☐ 10. What is the supplement of the angle with measure of 68º?
A B C D

A. 22 º B. 44 º C. 112 º D. 136 º

☐ ☐ ☐ ☐ 11. A flagpole casts a shadow that is 8 feet long. A 6-foot-tall man casts a shadow
A B C D

that is 5 feet long, as shown. What is the height of the flagpole?



A. $9\frac{3}{5}\ ft$ B. $9\frac{3}{4}\ ft$ C. $10\frac{5}{6}\ ft$ D. $15\frac{3}{5}\ ft$

☐ ☐ ☐ ☐ 12. In an isosceles triangle, the vertex angle is half the measurement of a base
A B C D

angle. What is the measure of the vertex angle?

A. 72 º B. 60 º C. 36 º D. 30 º

☐ ☐ ☐ ☐ 13. In rhombus *ABCD*, the diagonals *AC* and *BD* intersect at *E*. If $|\mathrm{AE}|$ = 5 and
A B C D

$|\mathrm{BE}|$ = 12, what is the length of *AB*?

A. 7 B. 10 C. 13 D. 17

☐ ☐ ☐ ☐ 14. In two similar triangles, the ratio of the lengths of a pair of corresponding sides
A B C D

is 7:8. If the perimeter of the larger triangle is 32, find the perimeter of the *smaller* triangle.

A. 8 B. 16 C. 21 D. 28

☐ ☐ ☐ ☐ 15. Which of the following points is a solution to the system
A B C D

$y = 4 - x$ and $y = -x^2 + 2x + 4$?

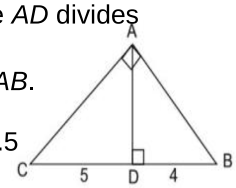
A. (1,3) B. (4,1) C. (-1,-3) D. (3,1)

☐ ☐ ☐ ☐ 16. The distance between points A(4*a*,3*b*) and B(3*a*,2*b*) is
A B C D

A. $a^2 + b^2$ B. $\sqrt{a^2 + b^2}$ C. $a + b$ D. $\sqrt{a + b}$

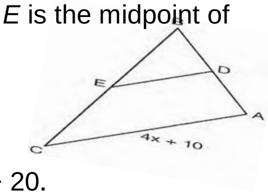
☐ ☐ ☐ ☐ 17. In the accompanying diagram of right triangle *ABC*, altitude *AD* divides
A B C D

hypotenuse *BC* into segments with lengths of 4 and 5. Find the length of leg *AB*.



A. 4.5 B. $2\sqrt{5}$ C. 6 D. 7.5

☐ ☐ ☐ ☐ 18 In the diagram below of *ABC*, *D* is the midpoint of *AB*, and *E* is the midpoint of
A B C D

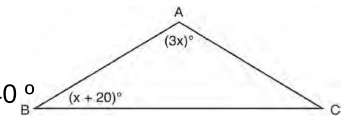


BC. If *AC* = 4*x* + 10, which expression represents *DE*?

A. *x* + 2.5 B. 2*x* + 10 C. 2*x* + 5 D. 8*x* + 20

☐ ☐ ☐ ☐ 19. In the diagram below, *AB* ≅ *AC*, m∠*A* = 3*x*, and m∠*B* = *x* + 20.
A B C D

What is the value of *x*?



A. 10 º B. 32 º C. 28 º D. 40 º

☐ ☐ ☐ ☐ 20. Scalene triangle *ABC* is similar to triangle *DEF*. Which statement is *false*?
A B C D

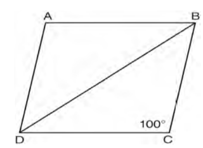
A. *AB*:*BC*=*DE*:*EF* C. *AC*:*DF*=*BC*:*EF*

B. ∠*ACB* ≅ ∠*DFE* D. ∠*ABC* ≅ ∠*EDF*

☐ ☐ ☐ ☐ 21. In the diagram below of rhombus *ABCD*, m∠*C* = 100. What is m∠*DBC*?
A B C D

A. 40 C. 50

B. 45 D. 80



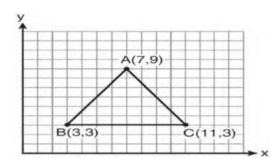
☐ ☐ ☐ ☐ 22. In the diagram above, *ABCD*, *AC* ≅ *BD*. Using this information,
A B C D

it could be proven that:

A. *BC* = *AB* B. *AB* = *CD* C. *AD* − *BC* = *CD* D. *AB* + *CD* = *AD*

☐ ☐ ☐ ☐ 23. The vertices of the triangle in the diagram below are *A*(7, 9), *B*(3, 3), and *C*(11,
A B C D

3). What are the coordinates of the centroid of *ABC*?



A. (5, 6) C. (7, 3)

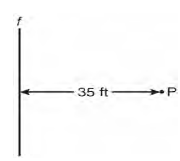
B. (7, 5) D. (9, 6)

☐ ☐ ☐ ☐ 24. A man wants to place a new bird bath in his yard so that it is 30 feet from a
A B C D

fence, *f*, and also 10 feet from a light pole, *P*. As shown in the diagram below, the light pole is 35 feet away from the fence. How many locations are possible for the bird bath?

A. 1 C. 2

B. 3 D. 0

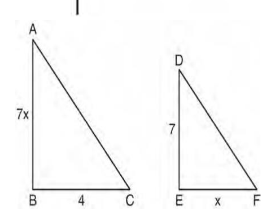


☐ ☐ ☐ ☐ 25. As shown in the diagram below, *ABC* ~ *DEF*, *AB* = 7*x*,
A B C D

BC = 4, *DE* = 7, and *EF* = *x*. What is the length of *AB*?

A. 28 C. 2

B. 14 D. 4



APPENDIX L

Answer Key

Conceptual Understanding

1.	A	16.	B
2.	A	17.	B
3.	C	18.	C
4.	A	19.	D
5.	B	20.	B
6.	A	21.	D
7.	C	22.	B
8.	D	23.	C
9.	B	24.	B
10.	D	25.	D
11.	B		
12.	D		
13.	A		
14.	C		
15.	B		

Procedural Knowledge

1.	B	16.	B
2.	C	17.	C
3.	C	18.	C
4.	B	19.	C
5.	C	20.	D
6.	B	21.	A
7.	B	22.	B
8.	C	23.	B
9.	A	24.	C
10.	C	25.	B
11.	A		
12.	C		
13.	C		
14.	D		
15.	D		

APPENDIX M

Students' Evaluation on Problem Posing

Name: ___________________________________ Date: _________________

A. Directions: The following statements refer to the use of problem posing approach on the topic, Triangle Similarity. Rate each statement from 1 to 5, with 1 being the lowest and 5 being the highest. Encircle the number that corresponds to your rating on the use of problem posing.

STATEMENT	RATING
1. Problem posing is helpful in understanding the concept in the lesson.	1 2 3 4 5
2. The procedural knowledge on the lesson is easily attained through problem posing.	1 2 3 4 5
3. The use of systematic procedures in lesson was attained using problem posing.	1 2 3 4 5
4. The length of time used in problem posing was effective and efficient.	1 2 3 4 5
5. The teacher applied the problem posing clearly.	1 2 3 4 5

B. Direction: Write your personal reflections on the following questions:

1. What do you like the most about problem posing?
____
____

2. What do you like the least about problem posing?
____
____

3. Will you continue using the problem posing in other lessons? Why?
____
____

4. Will you recommend the problem posing to others? Why?
____
____

Thank you and God Bless!

APPENDIX N

Letter of Permission to the Principals and School President

August 25, 2015

SISTER CLOTILDE L. MADRIDANO, FHMA
Elementary and High School Principal
NAMEI Polytechnic Institute
Mandaluyong City, Metro Manila

Dear Sister Clotilde:

Greetings!

The undersigned is an M.A. in Mathematics Education student of Philippine Normal University – Manila, and currently working on his thesis entitled, **"Effect of Problem Posing on Student's Conceptual Understanding and Procedural Knowledge in Geometry"**. Two of the instruments that will be employed in the study for data gathering are the conceptual understanding test and procedural knowledge test. These instruments must be validated through a review and evaluation of the items in the instruments to determine the reliability, suitability and acceptability.

In line with this, the researcher would like to request your approval to conduct the pilot test of research instruments to a group of Grade 10 students.

Your favorable response to this request will be highly valued.

Thank you very much and let the blessing flow.

Respectfully,

RAMON A. RAZA
Researcher

NOTED:

DR. GLADYS C. NIVERA
Adviser

DR. RENE R. BELECINA
Adviser

Recd. 08/28/15
Rec. 08/28/15

PHILIPPINE NORMAL UNIVERSITY
College of Graduate Studies and Teacher Education Research
Taft Avenue, Manila

August 25, 2015

MA. CELEDONIA P. PATAG, M.D.
Director, Basic Education Department
NAMEI Polytechnic Institute
Mandaluyong City, Metro Manila

Dear Dr. Celine:

Greetings!

The undersigned is an M.A. in Mathematics Education student of Philippine Normal University – Manila, and currently working on his thesis entitled, **"Effect of Problem Posing on Student's Conceptual Understanding and Procedural Knowledge in Geometry"**. Two of the instruments that will be employed in the study for data gathering are the conceptual understanding test and procedural knowledge test. These instruments must be validated through a review and evaluation of the items in the instruments to determine the reliability, suitability and acceptability.

In line with this, the researcher would like to request your approval to conduct the pilot test of research instruments to a group of Grade 10 students.

Your favorable response to this request will be highly valued.

Thank you very much and let the blessing flow.

Respectfully,

RAMON A. RAZA
Researcher

NOTED:

DR. GLADYS C. NIVERA
Adviser

DR. RENE R. BELECINA
Adviser

8/25/15

PHILIPPINE NORMAL UNIVERSITY
College of Graduate Studies and Teacher Education Research
Taft Avenue, Manila

August 25, 2015

SISTER CLOTILDE L. MADRIDANO, FHMA
High School Principal, NAMEI Polytechnic Institute
Mandaluyong City, Metro Manila

Dear Sister Clotilde:

Greetings!

The undersigned is an M.A. in Mathematics Education student of Philippine Normal University – Manila, and currently working on his thesis entitled, **"Effect of Problem Posing on Student's Conceptual Understanding and Procedural Knowledge in Geometry"**. The study aims to find out the effect of problem posing on students' conceptual understanding and procedural knowledge in Geometry.

Seventy (70) Grade 9 students will be the target respondents of the study. These students will undergo four-week formal instruction. Thirty-five (35) of the students will undertake problem posing instruction while the other thirty-five (35) will not undertake the problem posing. Each session will be conducted for sixty (60) minutes.

In this line, the researcher would like to request your approval to conduct the study in the institution. He intends to administer the problem posing lessons on November 2-27, 2015.

The researcher is confident that the findings of the study will be very significant in improving the mathematics instruction and mathematical skills (conceptual understanding and procedural knowledge) of the students.

Your favorable response to this request will be highly valued. Thank you very much and let the blessing flow.

Respectfully,

RAMON A. RAZA
Researcher

NOTED:

DR. GLADYS C. NIVERA
Adviser

DR. RENE R. BELECINA
Adviser

Recd. 08/28/15
Rc. 08/28/15

PHILIPPINE NORMAL UNIVERSITY
College of Graduate Studies and Teacher Education Research
Taft Avenue, Manila

November 3, 2015

MR. ERNESTO JOSE ALBEA
Principal, Ilaya Barangka Integrated School
Mandaluyong City, Metro Manila

Sir:

Greetings!

The undersigned is an M.A. in Mathematics Education student of Philippine Normal University – Manila, and currently working on his thesis entitled, **"Effect of Problem Posing on Student's Conceptual Understanding and Procedural Knowledge in Geometry"**. Two of the instruments that will be employed in the study for data gathering are the conceptual understanding test and procedural knowledge test. These instruments must be validated through a review and evaluation of the items in the instruments to determine the reliability, suitability and acceptability.

In line with this, the researcher would like to request your approval to conduct the pilot test of research instruments to a group of Grade 10 students.

Your favorable response to this request will be highly valued.

Thank you very much and let the blessings flow.

Respectfully,

RAMON A. RAZA
Researcher

NOTED:

DR. GLADYS C. NIVERA
Adviser

for

DR. RENE R. BELECINA
Adviser

APPROVED

ILAYA BARANGKA INTEGRATED SCHOOL
RECEIVED
By:
Date: 11 - 9 - 15
Time: 12:30 pm

APPENDIX O

Request Letter to the Experts

August 25, 2015

To Whom It May Concern:

Greetings!

The undersigned is an M.A. in Mathematics Education student of Philippine Normal University – Manila, and currently working on his thesis entitled, **"Effect of Problem Posing on Student's Conceptual Understanding and Procedural Knowledge in Geometry"**. The study aims to find out the effect of problem posing on students' conceptual understanding and procedural knowledge in Geometry through an experiment and test.

Two of the instruments that will be employed in the study for data gathering are the conceptual understanding test and procedural knowledge tests. These instruments must be validated through a review and evaluation of the items in the instruments to determine the reliability, suitability and acceptability.

In line with this, the researcher would like to request your expertise to evaluate the abovementioned instruments based on the determined criteria. Attached are the Conceptual Understanding and Procedural Knowledge Tests and Evaluation Form.

Your favorable response to this request will be highly valued.

Thank you very much and let the blessing flow.

Respectfully,

RAMON A. RAZA
Researcher

NOTED:

DR. GLADYS C. NIVERA
Adviser

DR. RENE R. BELECINA
Adviser

APPENDIX P
Calendar of Activities

DATE	ACTIVITIES
November 06, 2015	• Orientation to the Respondents
November 09, 2015	• Administration of Conceptual Understanding and Procedural Knowledge Pretests
November 10, 2015	• Presentation of Problem Posing
November 11, 2015	• Presentation of Lesson 1: Ratio and Proportion • Selection, Classification, Association, Transformation of objects/idea/problem
November 12, 2015	• Searching, transformation, classification, association of problem for posing • Conduct of Problem Posing
November 13, 2015	• Quiz # 1 • Assignment
November 16, 2015	• Checking of Assignment • Feed backing on students' works
November 23, 2015	• Group Activity on Ratio and Proportion • Conduct of Adversarial Problem Posing
November 24, 2015	• Presentation of Lesson 2: Theorems on Proportionality • Selection, Classification, Association, Transformation of objects/idea/problem
November 25, 2015	• Searching, transformation, classification, association of problem for posing • Conduct of Problem Posing

November 26, 2015	• Quiz # 2 • Assignment
November 27, 2015	• Checking of Assignment • Feed backing on students' works
December 01, 2015	• Group Activity on Theorems of Proportionality • Conduct of Problem Posing
December 02, 2015	• Presentation of Lesson 3: Triangles Similarity
December 03, 2015	• Searching, transformation, classification, association of problem for posing • Conduct of Problem Posing
December 04, 2015	• Quiz # 3 • Assignment
December 07, 2015	• Checking of Assignment • Feed backing on students' works
December 08, 2015	• Group Activity on Triangle Similarity • Conduct of Problem Posing
December 09, 2015	• Administration of Conceptual Understanding and Procedural Knowledge Posttests
December 10, 2015	• Conduct of Students' Evaluation on Problem Posing

APPENDIX Q
Attendance Record

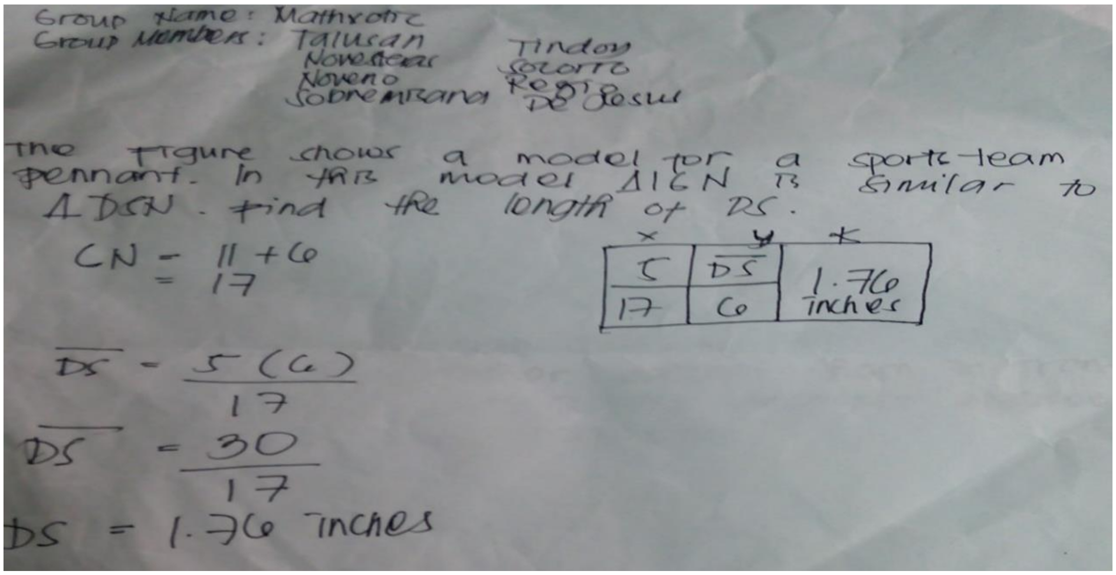
Appearance of the Students in Experimental Group

Student	November, 2015												December, 2015							
	06	09	10	11	12	13	16	23	24	25	26	27	01	02	03	04	07	08	09	10
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
5	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
12	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
17	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Note: 1 – Present ; 0 – absent

APPENDIX R

Sample Problems Posed by the Students



Group Name: Mathxotic
Group Members: Talusan, Novesteras, Noveno, Sobremisana, Tindoy, Socorro, Regio, De Jesus

The figure shows a model for a sports-team pennant. In this model $\triangle IGN$ is similar to $\triangle DSN$. Find the length of DS.

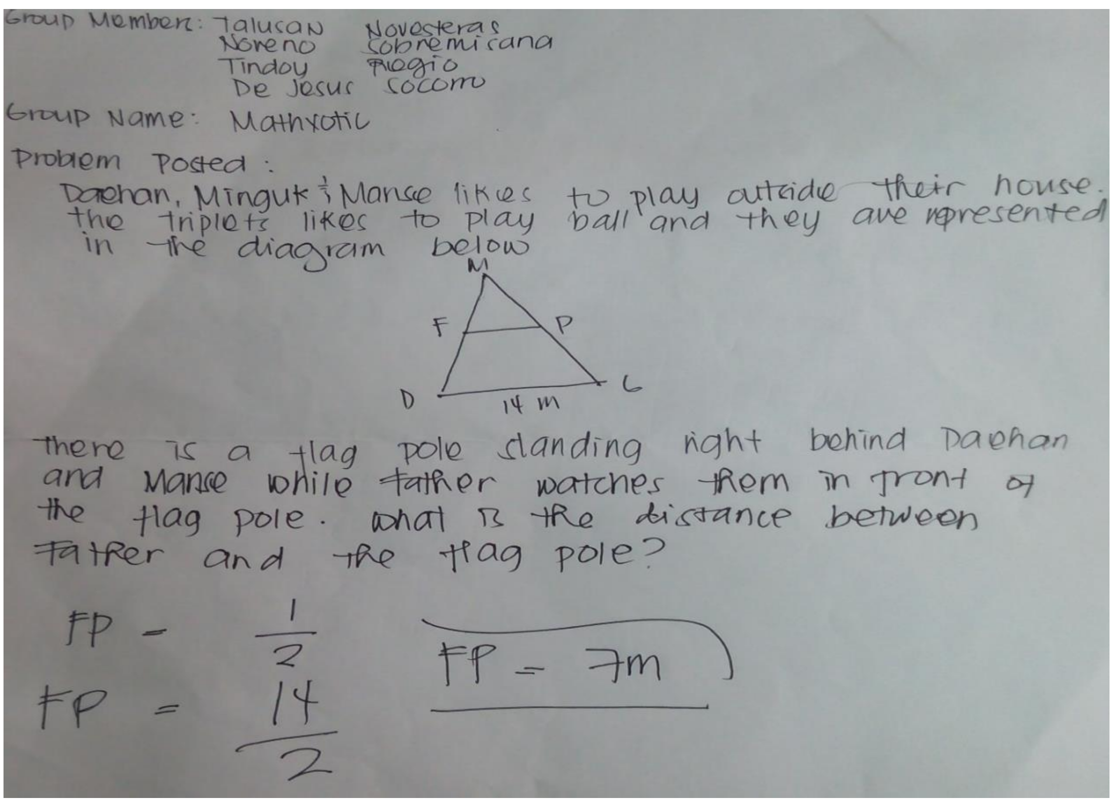
$CN = 11 + 6$
$= 17$

x	y	k
5	$\overline{DS}$	1.76 inches
17	6	

$\overline{DS} = \frac{5(6)}{17}$

$\overline{DS} = \frac{30}{17}$

$DS = 1.76$ inches



Group Member: Talusan, Novesteras, Noveno, Sobremisana, Tindoy, Regio, De Jesus, Socorro

Group Name: Mathxotic

Problem Posed:

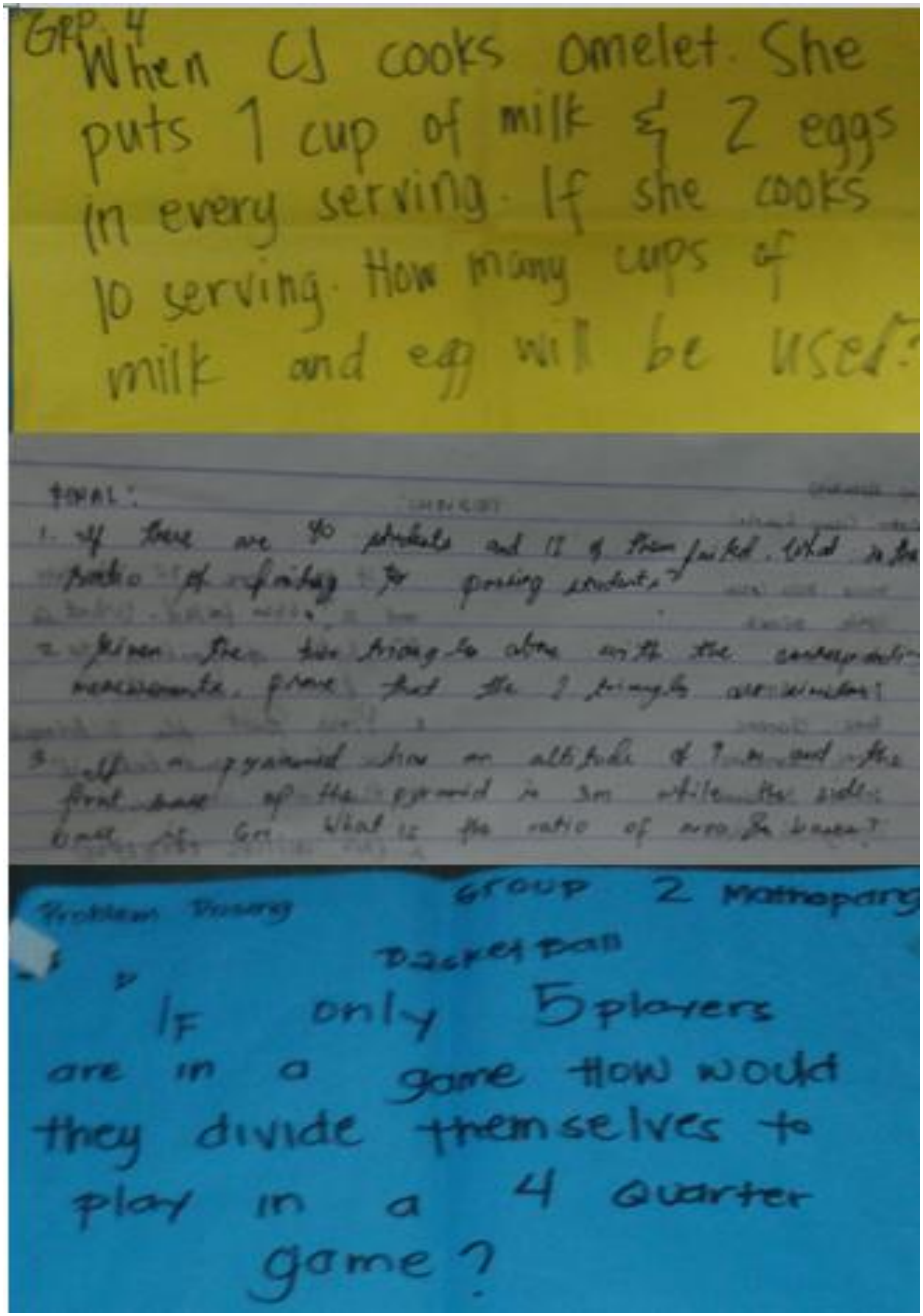
Daehan, Minguk & Manse likes to play outside their house. The triplets likes to play ball and they are represented in the diagram below

There is a flag pole standing right behind Daehan and Manse while father watches them in front of the flag pole. What is the distance between father and the flag pole?

$FP = \frac{1}{2}$

$FP = \frac{14}{2}$

$FP = 7m$


GRP. 4
When CJ cooks omelet. She puts 1 cup of milk & 2 eggs in every serving. If she cooks 10 serving. How many cups of milk and egg will be used?
Problem Posing
Group 2
Basket ball
If only 5 players are in a game how would they divide themselves to play in a 4 quarter game?

APPENDIX S
Students' Test Result in Conceptual Understanding Test

Student	Pretest				Posttest			
	Experimental Mean = 6.714 SD = 2.432		Control Group Mean = 6.629 SD = 2.474		Experimental Mean = 11.171 SD = 2.051		Control Mean = 10.657 SD = 2.890	
	Score	%	Score	%	Score	%	Score	%
1	8	32%	9	36%	12	48%	6	24%
2	6	24%	7	28%	11	44%	11	44%
3	6	24%	12	48%	10	40%	5	20%
4	7	28%	4	16%	11	44%	6	24%
5	4	16%	4	16%	12	48%	11	44%
6	8	32%	7	28%	18	72%	9	36%
7	7	28%	9	36%	11	44%	12	48%
8	8	32%	9	36%	11	44%	13	52%
9	1	4%	7	28%	11	44%	11	44%
10	3	12%	11	44%	7	28%	12	48%
11	6	24%	8	32%	9	36%	10	40%
12	9	36%	9	36%	12	48%	9	36%
13	8	32%	9	36%	11	44%	7	28%
14	7	28%	6	24%	13	52%	7	28%
15	10	40%	11	44%	13	52%	12	48%
16	3	12%	5	20%	10	40%	11	44%
17	5	20%	6	24%	12	48%	9	36%
18	7	28%	8	32%	10	40%	14	56%
19	6	24%	4	16%	8	32%	9	36%
20	3	12%	6	24%	11	44%	15	60%
21	10	40%	4	16%	12	48%	8	32%
22	13	52%	5	20%	10	40%	12	48%
23	10	40%	9	36%	10	40%	10	40%
24	5	20%	6	24%	11	44%	11	44%
25	5	20%	3	12%	9	36%	15	60%
26	7	28%	4	16%	10	40%	7	28%
27	10	40%	6	24%	13	52%	10	40%
28	7	28%	5	20%	8	32%	10	40%
29	9	36%	3	12%	12	48%	14	56%
30	8	32%	8	32%	13	52%	11	44%
31	6	24%	6	24%	8	32%	12	48%
32	5	20%	9	36%	13	52%	10	40%
33	5	20%	5	20%	13	52%	11	44%
34	7	28%	6	24%	13	52%	18	72%
35	6	24%	2	8%	13	52%	15	60%

APPENDIX T
Students' Test Result in Procedural Knowledge Test

Student	Pretest				Posttest			
	Experimental Mean = 6.229 SD = 1.664		Control Group Mean = 6.286 SD = 2.371		Experimental Mean = 10.14 SD = 1.665		Control Mean = 9.943 SD = 3.404	
	Score	%	Score	%	Score	%	Score	%
1	5	20%	2	8%	11	44%	9	36%
2	9	36%	2	8%	12	48%	7	28%
3	6	24%	7	28%	8	32%	6	24%
4	4	16%	7	28%	8	32%	6	24%
5	5	20%	3	12%	10	40%	3	12%
6	7	28%	9	36%	14	56%	6	24%
7	8	32%	9	36%	12	48%	13	52%
8	6	24%	8	32%	13	52%	18	72%
9	7	28%	6	24%	10	40%	9	36%
10	7	28%	9	36%	8	32%	12	48%
11	7	28%	7	28%	10	40%	10	40%
12	5	20%	7	28%	11	44%	8	32%
13	3	12%	5	20%	10	40%	12	48%
14	8	32%	6	24%	11	44%	7	28%
15	8	32%	8	32%	13	52%	12	48%
16	3	12%	8	32%	11	44%	13	52%
17	4	16%	7	28%	12	48%	6	24%
18	5	20%	6	24%	9	36%	13	52%
19	9	36%	8	32%	8	32%	9	36%
20	5	20%	2	8%	9	36%	13	52%
21	6	24%	3	12%	8	32%	5	20%
22	9	36%	3	12%	12	48%	8	32%
23	8	32%	2	8%	9	36%	8	32%
24	5	20%	8	32%	9	36%	12	48%
25	5	20%	9	36%	8	32%	15	60%
26	6	24%	2	8%	9	36%	5	20%
27	7	28%	7	28%	10	40%	10	40%
28	8	32%	7	28%	10	40%	13	52%
29	8	32%	7	28%	9	36%	11	44%
30	5	20%	7	28%	8	32%	7	28%
31	6	24%	8	32%	10	40%	11	44%
32	5	20%	8	32%	11	44%	14	56%
33	8	32%	9	36%	11	44%	11	44%
34	6	24%	8	32%	9	36%	12	48%
35	5	20%	6	24%	12	48%	14	56%

APPENDIX U

SPSS Outputs

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
ExpCUpretest	35	100.0%	0	0.0%	35	100.0%
ContCUpretest	35	100.0%	0	0.0%	35	100.0%
ExpPKpretest	35	100.0%	0	0.0%	35	100.0%
ContPKpretest	35	100.0%	0	0.0%	35	100.0%

Descriptive Statistics

			Statistic	Std. Error
ExpCUpretest	Mean		6.7143	.41113
	95% Confidence Interval for Mean	Lower Bound	5.8788	
		Upper Bound	7.5498	
	5% Trimmed Mean		6.7063	
	Median		7.0000	
	Variance		5.916	
	Std. Deviation		2.43228	
	Minimum		1.00	
	Maximum		13.00	
	Range		12.00	
	Interquartile Range		3.00	
	Skewness		.081	.398
	Kurtosis		.558	.778
ContCUpretest	Mean		6.6286	.41825
	95% Confidence Interval for Mean	Lower Bound	5.7786	
		Upper Bound	7.4786	
	5% Trimmed Mean		6.5873	
	Median		6.0000	
	Variance		6.123	
	Std. Deviation		2.47441	
	Minimum		2.00	
	Maximum		12.00	
	Range		10.00	
	Interquartile Range		4.00	
	Skewness		.245	.398
	Kurtosis		-.599	.778
ExpPKpretest	Mean		6.2286	.28131
	95% Confidence Interval for Mean	Lower Bound	5.6569	

		Upper Bound	6.8003	
	5% Trimmed Mean		6.2540	
	Median		6.0000	
	Variance		2.770	
	Std. Deviation		1.66426	
	Minimum		3.00	
	Maximum		9.00	
	Range		6.00	
	Interquartile Range		3.00	
	Skewness		-.020	.398
	Kurtosis		-.827	.778
ContPKpretest	Mean		6.2857	.40078
	95% Confidence Interval for Mean	Lower Bound	5.4712	
		Upper Bound	7.1002	
	5% Trimmed Mean		6.3730	
	Median		7.0000	
	Variance		5.622	
	Std. Deviation		2.37104	
	Minimum		2.00	
	Maximum		9.00	
	Range		7.00	
	Interquartile Range		3.00	
	Skewness		-.839	.398
	Kurtosis		-.639	.778

Paired Samples Statistics

Compared pretest and posttest on conceptual understanding and procedural knowledge of control and experimental groups

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	ContCUpretest	6.6286	35	2.47441	.41825
	ContCUposttest	10.6571	35	2.88956	.48843
Pair 2	ContPLpretest	6.2857	35	2.37104	.40078
	ContPKposttest	9.9429	35	3.40366	.57532
Pair 3	ExpCUpretest	6.7143	35	2.43228	.41113
	ExpCUposttest	11.1714	35	2.05062	.34662
Pair 4	ExpPKpretest	6.2286	35	1.66426	.28131
	ExpPKposttest	10.1429	35	1.66527	.28148

Paired Samples Statistics

Compared experimental and control groups on conceptual understanding and procedural knowledge tests (pretest and posttest)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	ExpCUpretest	6.7143	35	2.43228	.41113
	ContCUpretest	6.6286	35	2.47441	.41825
Pair 2	ExpCUposttest	11.1714	35	2.05062	.34662
	ContCUposttest	10.6571	35	2.88956	.48843
Pair 3	ExpPKpretest	6.2286	35	1.66426	.28131
	ContPKpretest	6.2857	35	2.37104	.40078
Pair 4	ExpPKposttest	10.1429	35	1.66527	.28148
	ContPKposttest	9.9429	35	3.40366	.57532

Paired Samples Correlations

Compared experimental and control groups on conceptual understanding and procedural knowledge tests (pretest and posttest)

		N	Correlation	Significance
Pair 1	ExpCUpretest & ContCUpretest	35	.065	.711
Pair 2	ExpCUposttest & ContCUposttest	35	-.029	.866
Pair 3	ExpPKpretest & ContPKpretest	35	-.017	.923
Pair 4	ExpPKposttest & ContPKposttest	35	.147	.400

Paired Samples Correlations

Compared pretest and posttest on conceptual understanding and procedural knowledge of control and experimental groups

		N	Correlation	Significance
Pair 1	ContCUpretest & ContCUposttest	35	-.228	.187
Pair 2	ContPKpretest & ContPKposttest	35	.447	.007
Pair 3	ExpCUpretest & ExpCUposttest	35	.234	.176
Pair 4	ExpPKpretest & ExpPKposttest	35	.168	.334

Paired Samples Test

Compared experimental and control groups on conceptual understanding and procedural knowledge tests (pretest and posttest)

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	ExpCUpretest - ContCUpretest	.08571	3.35517	.56713	-1.06683	1.23825	.151	34	.881
Pair 2	ExpCUposttest - ContCUposttest	.51429	3.59224	.60720	-.71969	1.74826	.847	34	.403
Pair 3	ExpPKpretest - ContPKpretest	-.05714	2.91994	.49356	-1.06018	.94589	-.116	34	.909
Pair 4	ExpPKposttest - ContPKposttest	.20000	3.56288	.60224	-1.02389	1.42389	.332	34	.742

Paired Samples Test

Compared pretest and posttest on conceptual understanding and procedural knowledge of control and experimental groups

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	ContCUpretest - ontCUposttest	-4.0285	4.21123	.71183	-5.47518	-2.58196	-5.659	34	.000
Pair 2	ContPKpretest - ContPKposttest	-3.6571	3.16175	.53443	-4.74324	-2.57105	-6.843	34	.000
Pair 3	ExpCUpretest - ExpCUposttest	-4.4571	2.79014	.47162	-5.41559	-3.49870	-9.451	34	.000
Pair 4	ExpPKpretest - ExpPKposttest	-3.91429	2.14711	.36293	-4.65184	-3.17673	-10.785	34	.000

Paired Samples Statistics

Compared pretests and posttest mean scores of experimental and control groups on conceptual understanding and procedural knowledge tests

T – Test for Independent Sample

Group Statistics

	Group	N	Mean	Std. Deviation	Std. Error Mean
CU Pretest	E	35	6.7143	2.43228	.41113
	C	35	6.6286	2.47441	.41825
PK Pretest	E	35	6.2286	1.66426	.28131
	C	35	6.2857	2.37104	.40078
CU Posttest	E	35	11.1714	2.05062	.34662
	C	35	10.6571	2.88956	.48843
PK Posttest	E	35	10.1429	1.66527	.28148
	C	35	9.9429	3.40366	.57532

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Differ-ence	Std. Error Differ-ence	95% Confidence Interval of the Difference Lower	Upper
CU Pretest	Equal variances assumed	.339	.562	.146	68	.884	.08571	.58648	-1.08459	1.25602
	Equal variances not assumed			.146	67.980	.884	.08571	.58648	-1.08460	1.25603
PK Pretest	Equal variances assumed	3.759	.057	-.117	68	.907	-.05714	.48965	-1.03423	.91994
	Equal variances not assumed			-.117	60.958	.907	-.05714	.48965	-1.03628	.92199
CU Posttest	Equal variances assumed	3.197	.078	.859	68	.394	.51429	.59892	-.68084	1.70941
	Equal variances not assumed			.859	61.318	.394	.51429	.59892	-.68320	1.71177
PK Posttest	Equal variances assumed	19.49	.000	.312	68	.756	.20000	.64049	-1.07808	1.47808
	Equal variances not assumed			.312	49.395	.756	.20000	.64049	-1.08685	1.48685

APPENDIX V
Characterization Scale for Conceptual Knowledge, Procedural Knowledge, and Coupled Conceptual and Procedural Knowledge

Knowledge Type	Characterization Criteria
Procedural Knowledge (P)	P1.Solving procedures step by step P2.Using mathematical knowledge which was learnt before in the level of knowledge (the level of Bloom's taxonomy) P3.Being able to use algebraic relations and to conduct basic procedures
Conceptual Knowledge (C)	C1.Knowing basic concepts and meaning of these concepts C2.Finding solution way by grasping the meaning of question and correlating given information with intended result C3.Using mathematics knowledge which was learnt before in the level of comprehension and application C4.By perceiving question as whole, appreciating hints properly and relevantly C5.Dividing problem into easy sub-steps C6.Making generalization and drawing shape and figures to back up a complex and hard problem C7.Matching the problem with given figure and graph C8.Matching problem with the features of this problem after determining these features
Coupled Conceptual and Procedural Knowledge (C-P)	C-P1.Understanding, using, writing, retrenchment, and simplifying the symbols and statements which constitute language of mathematic C-P2.Solving equation after converting problem into equation and checking rationality of solutions C-P3.Converting given relation into another relation by associating these relations among themselves

APPENDIX W

Certification of Language Editing

Philippine Normal University

The National Center for Teacher Education

COLLEGE OF GRADUATE STUDIES AND TEACHER EDUCATION RESEARCH

CERTIFICATION OF EDITING

12 February 2016

This certifies that I have edited the thesis titled **"The Effect of Problem Posing on Students' Conceptual Understanding and Procedural Knowledge in Geometry"** by MR. RAMON A. RAZA, a candidate in Master of Arts in Education with specialization in Mathematics Education.

This is issued for whatever legal purpose it may serve Mr. Raza.

MERRY RUTH M. GUTIERREZ

Faculty, CGSTER

APPENDIX X

Originality Report

Philippine Normal University
College of Graduate Studies and Teacher Education Research
PUBLICATION OFFICE

AUTHENTICITY RESULT

Date of Release: February 12, 2016
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Author(s): Ramon A. Raza

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APPENDIX Y

Curriculum Vitae

RAMON A. RAZA
Tawiran Sta. Cruz, Marinduque
Mobile Phone: 0936-9509127
Email: ramon_raza@yahoo.com.ph

EDUCATIONAL QUALIFICATION

November 2010 – March 2016	**Philippine Normal University** Taft, Manila – Master of Arts in Education with specialization in Mathematics
June 2006 – April 2010	**Marinduque State College** Tanza, Boac, Marinduque – Bachelor of Secondary Education major in Mathematics – Outstanding Education Student
June 2002 – March 2006	**Makapuyat National High School** Napo, Sta. Cruz, Marinduque
June 1996 – March 2002	**Tawiran Elementary School** Tawiran, Sta. Cruz, Marinduque

TEACHING EXPERIENCES

May 2011 – Present	High School Math Teacher Instructor and Research Head NAMEI Polytechnic Institute A. Mabini St., Mandaluyong City
October 2010 – May 2011	Math Teacher Golden Link College Foundation Inc. Waling-waling St., Caloocan City

PERSONAL DATA

Date of Birth:	April 8, 1990	Civil Status:	Single
Place of Birth:	San Miguel, Ilo-Ilo	Religion:	Catholic
Nationality:	Filipino	Known Language:	English, Filipino

AWARDS RECEIVED

2013 Great Teacher Award
Japan – East Asia Network of Exchange Students and Youth (student ambassador)