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Heuristics in Solving Nonroutine Problems of Selected Students in San Pablo City Science High School

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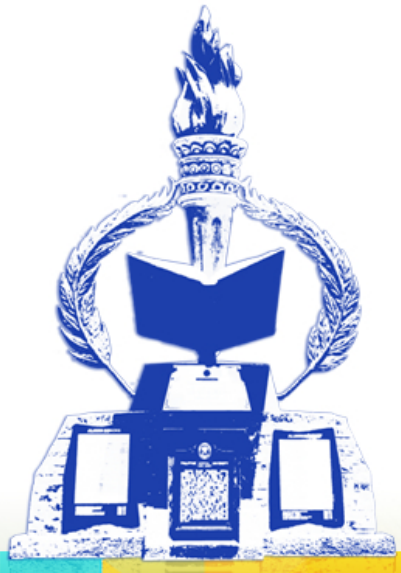

PHILIPPINE NORMAL UNIVERSITY
The National Center for Teacher Education

HEURISTICS IN SOLVING NONROUTINE PROBLEMS OF SELECTED STUDENTS IN SAN PABLO CITY SCIENCE HIGH SCHOOL

A Special Project Paper
Presented to the
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In Partial Fulfillment
of the Requirements for the Degree
Master of Arts in Teaching
with Specialization in Mathematics

by
IAN A. VILLAREAL
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APPROVAL SHEET

In partial fulfillment of the requirements for the degree Master of Arts in Teaching with specialization in Mathematics, this special project entitled **"HEURISTICS IN SOLVING NONROUTINE PROBLEMS OF SELECTED STUDENTS IN SAN PABLO CITY SCIENCE HIGH SCHOOL",** prepared and submitted by **IAN A. VILLAREAL,** has been examined and is hereby recommended for acceptance.

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DEDICATION

This special project is dedicated to…

my family especially to my mother Anita and father Inigo

my bothers Arvin, Al John and Mark Angelo

my only sister Anne Princess Joy and to my fiancée, Madeline.

May this work serve its purpose of

helping every teacher in making Math

an enjoyable, relevant and meaningful subject to everyone.

I.A.V.

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Researcher:	Ian A. Villareal
Title:	Heuristics in Solving Nonroutine Problems of Selected Students in San Pablo City Science High School
Key Concepts:	Problem Solving Heuristics
Degree:	Master of Arts in Teaching (MAT)
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Adviser:	Myla B. Esperanza

The main purpose of this special project was to identify the heuristics and performance of selected students in a public city science high school (ESEP) in solving nonroutine problems (NRPs). The study also aimed to compare and analyze the consistency of Krulik and Rudnick's 5-Stage Heuristic Model (1996) against the solutions and answers of the students in the NRPs. These NRPs are problems that require higher order thinking skills among students and the use of their conceptual understanding and procedural knowledge of Mathematics concepts. The results in this study showed that 30 out of the 60 solutions to the 5 NRPs or 50% shows consistency with Krulik and Rudnick's 5-Stage Heuristic Model (1996) except that the *extending part* of the 5th stage was not clearly seen

in their solutions. Seven out of the eight possible strategies were used to solve the 5 NRPs with an average of four strategies per student. “Compute and Simplify” was the most frequently used strategy. After using the Oregon Mathematics Problem Solving Rubric in evaluating the student’s problem solving skills, the result showed that out of the twelve students, two were considered as proficient problem solver, nine were apprentice problem solver and one was a novice problem solver. Among the four year levels, the third year level performed the best indicating that the level of expertise of solving nonroutine problems in this study is not directly proportional to their year levels. Factors such as attitude towards Math, trainings, exposures to various Math competitions and even hobbies and parents’ support should be considered which, in one way or another may have an effect on the problem solving performance of the students.

Chapter I
The Problem and Its Background

Introduction

From the everyday conversations with my colleagues, an issue regarding problem solving among students is a perennial topic. Most often, many of them will exclaim: “Why is it that our students find difficulty whenever we gave them problems that are not similar to that of the given in their mathematics book?” Truly, problem solving is really a quandary to many students.

One of the primary skills that students must take with them when they leave the classrooms and enter the real world is problem solving (Krulik and Rudnick 1996). It requires skills in analysis and good reasoning that eventually leads to critical thinking. Hence, emphasis on problem solving is given importance not only in math but also in various areas in sciences and psychology that will help prepare our students in their future careers to be productive citizens of our country.

At present, our country has adopted the K to 12 Basic Education Program. Its basic feature is that it makes the curriculum relevant to the learners. In the K to 12 Curriculum Guide (2012), they placed critical thinking and problem solving as the twin goals of Mathematics in the basic education (elementary and

secondary). According to Scriven and Paul (1987), critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. On the other hand, Polya (1945 & 1962) stated that problem solving has something to do with finding a way around a difficulty, around an obstacle and finding a solution to a problem that is unknown.

The Department of Education (DepEd) is very much aware of the importance of honing the critical thinking skills of our students that is why in the two of the national examinations we have here in our country like the National Career Assessment Examination (NCAE) and National Achievement Test (NAT), there include the *critical thinking skills test* which helps our students in their decision making which course to be taken in college that best suits their abilities and skills. Hence, critical thinking also helps us to make good and sound decisions in the future.

In mathematics, there are two classifications of problems: routine and nonroutine problems. Routine problems are generally found in textbooks both used in public and privates schools. These are problems that require a direct formula or an algorithm learned during class discussion. Hence, such problems

produce only a limited number of strategies in problem solving. However, problems that entail the application of various concepts in math and are not usually found in textbooks are called nonroutine problems. Moreover, nonroutine problems are given to students to help them think critically and creatively since such problems represents real life situations that are more appropriate in training our students to face life after school.

To successfully solve problems, Polya (1957) suggested four steps which are as follows: 1.understanding the problem; 2. selecting a strategy; 3. solving the problem; and 4. looking back. However, according to Krulik and Rudnick in their book The New Sourcebook for Teaching Reasoning and Problem Solving in Junior and Senior High School (1996), there are five stages that constitute the heuristic process. A heuristic is simply a problem solving process as defined by Candelaria and Limjap (2002). The five stages are as follows: 1. read and think; 2. explore and plan; 3. select a strategy; 4. find an answer; and 5. reflect and extend. So, the question now is: What problem solving heuristics do students often use especially in dealing with unfamiliar problems like nonroutine ones? Are their heuristics same as those stated by Krulik and Rudnick's 5-stages heuristics? Also, can they perform better even if these problems are not usually taught in classroom setting? This special project aims to provide answers to these questions.

To put the problem into a study, San Pablo City Science High School (SPCSHS) was chosen. The San Pablo City Science High School (SPCSHS) is a science-oriented school for San Pablo City's gifted students and was formally inaugurated on August 25, 2005. The K to 12 program is offered for Grades 7 and 8 students while the ESEP program is offered by the school for the third year up to the fourth year students.

The Science Technology Engineering Mathematics Education Program (ESEP) is a science and mathematics-oriented curriculum devised for high schools in the Philippines. Whether in public or private, the ESEP program is offered by specialized high schools, supervised by the Department of Education. At present, there are 110 high schools offering the ESEP program, the majority are public schools.

There are three types of science high schools in the Philippines, namely: ESEP high schools, high schools in the Regional Science High School Union and the Philippine Science High School System. All three types of science high schools in the Philippines offer a curriculum placing importance in mathematics and the sciences, as well as research. With this description, SPCSHS is very much suited as the subject of the study.

Graduating elementary pupils are eligible to take the qualifying exam provided that they belong to the upper 20% of the graduating class with no grade lower than 83% in English, Science and Math as of the third quarter and that they are recommended by their respective principals. Admission to the San Pablo City Science High School requires passing the two screening tests. The first screening test consists of elementary Mathematics, Science and English. The student applicant should have a grade equivalent to the cut-off score identified by the admission committee to advance to the second screening test. The second test consists of elementary Science, advanced Mathematics and an interview to be conducted by the language teachers. With this selection process, SPCSHS students are among the best students in the country. Moreover, the top 3 SPCSHS students who are participants in various Math competitions like the MTAP Math Challenge from each of the four year levels (a total of 12) were chosen as subjects of this study to ensure that the selected students have above average skills in Mathematics and are well-equipped to solve mathematics problems.

Below are the mathematics courses included in the ESEP (before the implementation of the K to 12 program) taken by SPCSHS students that really help them be well equipped in different Math concepts.

First Year	Second Year	Third Year	Fourth Year
Elementary Algebra	Intermediate Algebra; Geometry; Basic Statistics	Trigonometry Advanced Algebra	Analytic Geometry; Calculus

Conceptual Framework

This study was based on Krulik and Rudnick's 5-Stage Heuristic Model (1996) in nonroutine problem solving. The assessment of the performance of the students in problem solving was based on the Oregon Department of Education (1991) Nonroutine Problem Solving Levels of Understanding Rubric and Piaget's theory of the chronological stages of intellectual development was utilized regarding the respondents.

Krulik and Rudnick (1996) defines problem and problem solving in their book entitled The New Sourcebook for Teaching Reasoning and Problem Solving in Junior and Senior High School as follows:

A problem is a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtaining a solution.

Problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation.

Hence, problem solving is a process. They added that this process represented by a series of steps is referred to as a *heuristic plan,* or simply, *heuristics*. Furthermore, heuristics provides a “compass”; it represents a plan that directs one’s course toward a solution and resolution of a problem situation.

According to Polya (1945), there are four steps in problem solving: first is understanding the problem; second is devising a plan; next is carrying out the plan; and lastly, looking back. Also, according to the Singapore Ministry of Education (1997), there are four steps in problem solving:

a. Understanding the problem - In this step, the solver must know the conditions surrounding the problem and the things being asked. They added that the solver must know how to go through the pertinent information from the sets of data that are being put forward by the problem statement.

b. Devising a plan - In this 2nd pace, the formulation of strategies or the means of going about the problem must be seen. The solver must be familiar

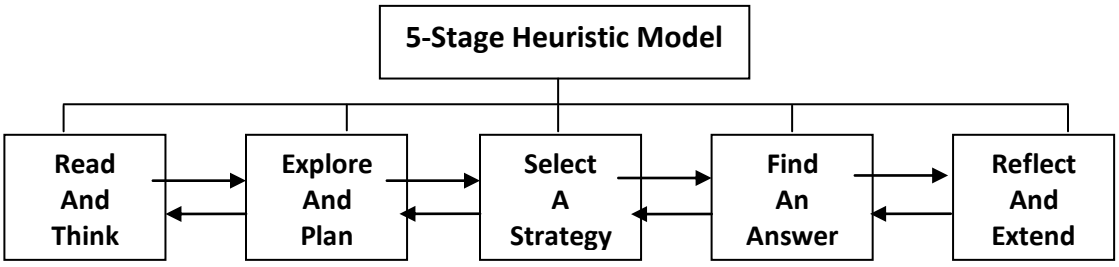
with the specific strategy that applies to the present problem. It is very vital that the solver knows different strategies of problem solving for different problem situations.

c. Carrying out the plan - This step is where one applies the appropriate strategies to the present problem and must be able to solve the problem by calculating.

d. Looking Back – It is the act of checking the correctness of the answer and finding ways to solve the same problem by another means.

However, Krulik and Rudnick (1996) identified not only four but five stages of heuristics in problem solving compared to that of Polya (1945) and Singapore Ministry of Education (1997) and this is the model used in this study: The 5 stages are as follows:

1. Read and Think
2. Explore and Plan
3. Select a Strategy
4. Find an Answer
5. Reflect and Extend



In each stage, Krulik and Rudnick stated that there are corresponding mental and physical activities a person engages in when solving a problem and has sub-skills under it. The stages are neither independent nor consecutive.

In the 1st heuristic "Read and Think", the problem is analyzed, facts are determined and evaluated where critical thinking begins. The physical setting is visualized and understood. The problem is translated into the language of the problem solver.

In the "Explore and Plan" stage" which is the 2nd stage, here the problem solver analyzes the data and determines whether there is sufficient information. The data are then organized and a plan for finding the answer is developed.

In stage three, "Select a Strategy", appropriate strategy is chosen among many according to the skill and experience of the problem solver. According to Krulik and Rudnick (1996), there are nine most frequently used strategies in all ages. These are: 1. pattern recognition; 2. working backwards; 3. guess and test;

4. experimentation or simulation; 5. reduction expansion; 6. organized listing; 7. logical deduction; 8. divide and conquer; 9. write an equation.

In the 4th stage “Find an Answer”, the two pointed out that to find an answer, one must employ appropriate mathematical skills. Estimation and the use calculators and other technologies are also encouraged.

Lastly, in the 5th and final stage which is the “Reflect and Extended”, ones’ answer must be checked against the statement of the problem. Creative thinking can be taken full advantage here. An alternate solution should be found and discussed and by changing some of the initial conditions or interpretations, the problem can be altered. In addition, they also said that if possible, the process should be extended to discover a generalization or the mathematical concepts that underlie the situation. Interesting variations of the original problem should be produced and discussed.

In view of the theoretical bases and history, this study was conceptualized. The capsulated presentation of the study is shown Figure 1.

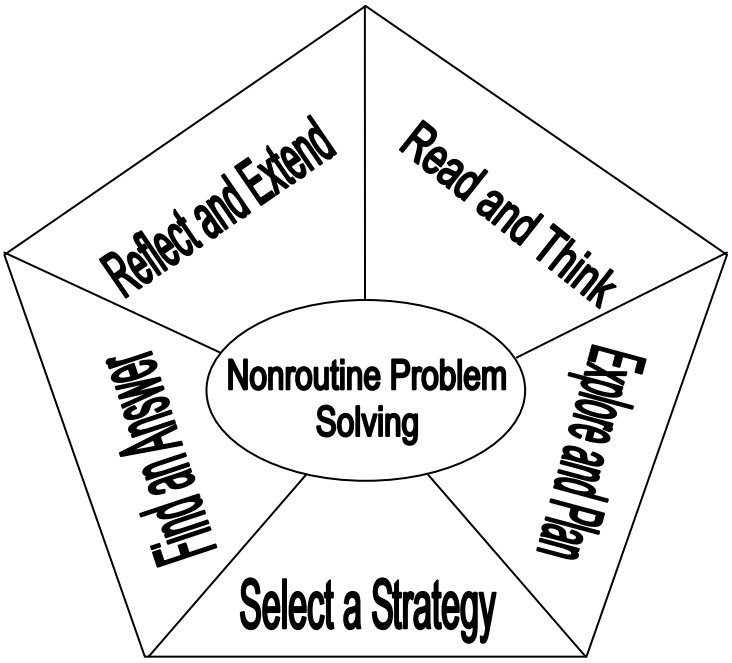


Figure 1. Conceptual Paradigm

Nonroutine problem solving is organized into 5 stages: read and think, explore and plan, select a strategy, find an answer, and reflect and extend.

As used in the study of Lee (2011), the students' problem solving strategies are classified as follows:

a. Compute or Simplify
b. Use a Formula
c. Make a Model or Diagram
d. Make a Table, Chart, or List
e. Guess, Check, and Revise
f. Consider a Simpler Case

g. Eliminate

h. Look or patterns

"Compute or Simplify" includes straightforward application o Arithmetic rules, order of operations, etc. "Use a Formula" involves substituting values into a formula or selecting the proper formula to use. "Make a Model" includes use of objects, drawings or sketches, acting out, or writing an equation. "Make a Table, Chart or List" is organizing the data by making a table, chart, graph, or list. "Guess, Check and Revise" is making a reasonable guess, checking the guess, and revising the guess if necessary. "Consider a Simpler Case" includes rewording the problem, using smaller numbers, using a more familiar problem setting, dividing the problem into simpler problems, or working backwards. "Eliminate" is elimination of incorrect answers. "Look for Patterns" is recognizing a pattern to find a solution.

As stated in the study of Mabilangan (2011), the main concept in Piaget's theory is the chronological stages of intellectual development. Since this study is concerned on how the students think and solve mathematical problems specifically nonroutine problems, the focus would be in the formal operational stage. In addition, the age range of the respondents (12-16) would fall under the said stage.

Basically, Piaget's theory will be used to describe the intended respondents.

a. Sensory-motor Stage (ages 0-2 years). Recognizes self as agent of action and begins to act intentionally: e.g. pulls a string to set mobile in motion or shakes a rattle to make a noise. Achieves object permanence: realizes that things continue to exist even when no longer present to the sense.

.b. Pre-Operational Stage (ages 2-7 years). Learns to use language and to represent objects by images and words. Thinking is still egocentric: has difficulty taking the viewpoint of others. Classifies objects by a single feature: e.g. groups together all the red blocks regardless of shape or all the square blocks regardless of color.

c. Concrete Operational Stage (ages 7-11). Can think logically about objects and events. Achieves conservation of number (age 6), mass (age 7), and weight (age 9). Classifies objects according to several features and can order them in series along a single dimension such as size.

d. Formal Operational Stage (ages 11-16). Can think logically about abstract propositions and test hypotheses systematically. Becomes concerned with the hypothetical, the future, and ideological problems.

According to the NCTM (1977), the principal reason for studying mathematics is learning to solve problems. In simple words, problem solving is the core of mathematics learning. In learning to solve problems, one must understand the basic concepts of heuristics. Heuristics that, in one way or another, may help the solver reach a possible answer.

In solving nonroutine problems, a student‘s understanding on this type of problems was described by the Oregon Department of Education (1991) which was used in this study as stated in the as stated in work of Mabilangan, Limjap and Belecina (2011) in their case study: Problem Solving Strategies of High School Students on Nonroutine Problems. It consists of the following:

a. Conceptual Understanding – includes the ability to interpret the problem and select appropriate concepts and information to apply a strategy for solution. Evidence is communicated through making connections between the problem situation, relevant information, appropriate mathematical concepts and logical/reasonable responses.

b. Procedural Knowledge – deals with the student's ability to demonstrate appropriate use of algorithms. Evidence includes verifying and justifying of a procedure using concrete models, or the modifying of procedures to deal with factors inherent to the problem.

c. Problem Solving Skills and Strategies – includes clearly focused good reasoning and insightful thinking that leads to a successful resolution of the problem.

Statement of the Purpose

Nonroutine problems (NRPs) are problems that make use of heuristics and often requires little to no use of algorithms. Heuristics are procedures or strategies important in problem solving. These do not guarantee a direct solution or answer to a problem but offer a jumping board for discovering the solution/answer to a problem. It is for that reason important to find out which heuristics are usually used by students and what strategies do they employ in solving non-routine problems correctly.

This study is intended to identify the nonroutine problem solving heuristics of selected students in San Pablo City Science High School (SPCSHS).

Specifically, this study sought to:

1. compare and analyze the consistency of Krulik and Rudnick's 5-Stage Heuristic Model (1996) against the solutions and answers of the students in the NRPs;

2. identify the students' problem solving strategies in solving nonroutine problems;

3. classify the students into three levels of expertise; and

4. compare the students' problem solving performance across year levels.

Research Hypothesis

Below is the research hypothesis:

The students' performance in solving non-routine problems is not directly related to students' related factors.

Significance of the Study

According to an article "Problem-Solving Strategies: Algorithms and Heuristics" written by J.E. Ormrod (2010), some problems can be successfully solved by following specific, step-by-step instructions—that is, by using an algorithm. To illustrate, he further added that we can correctly assemble the pieces of a new bookcase by following the directions for assembly that comes

with the package and that we can calculate the length of a slanted roof by using the Pythagorean Theorem. When we follow an algorithm faithfully, we invariably arrive at a correct solution.

He also added that the world presents many problems for which no algorithms exist. There are no rules we can follow to identify a substitute metal ship, no list of instructions to help us address the destruction of rain forests. In the absence of an algorithm, learners must instead use a heuristic, general problem-solving strategy that may or may not yield a successful outcome. For example, one heuristic that we might use in solving the deforestation problem is this: Identify a new behavior that adequately replaces the problem behavior (i.e., identify another way that peasant farmers can meet their survival needs). To illustrate, we may consider the addition problem in the exercise that follows. At school, students typically get far more practice solving well-defined problems than ill-defined ones, and they are taught many more algorithms than heuristics.

Nonroutine problems represent real life problems or situations. Students should be trained using this type of problems rather than less realistic routine ones. These non-routine problems promote logical and critical thinking skills necessary for daily life. Hence, students would be better problem solvers when they leave the school and face the real world. This study reveals the importance of using nonroutine problems in the classroom. With this, educators may bring

about changes in the Mathematics Curriculum regarding the type of problems being used in our schools and in SPCSHS in particular. Moreover, teachers may include nonroutine problems in their own subjects or lessons whenever applicable.

When solving non-routine problems, students may demonstrate various heuristics to solve each problem. We may observe how students think about certain problem, come up with strategy, and finally reach an answer to solve a problem. This study identified different strategies in solving nonroutine problems and how students used each of them.

From the views above we can say that solving problems is truly the very nature and essence of mathematics learning and as a discipline. Through this discipline, one can develop critical thinking skills that will lead to analytical and sound reasoning for us to make clear and right decisions in our life. This is the very reason why the researcher wants to know the *heuristics* that are usually used by students and the specific strategies do they make use of in solving unusual word problems called nonroutine problems.

This study will be of significance to the following:

DepEd, Division of San Pablo City

This study will give the institution some recommendations and suggestions on how to improve the weak points and areas of the present curriculum in the field of Mathematics.

Mathematics Teachers

The study will be beneficial to the Mathematics teachers since the result of the study will give them idea about the real scenario of the heuristics being used by the students. Also, they may find out ways to increase students' level of skills in problem solving and make this a basis for adjustment in lesson planning and syllabus content as well as their teaching methods and strengths and made them conceptualize action research to strive for improved learning.

School Administrators

This study may suggest to the school administrators some ways and means on how to implement the alternative approaches that can redirect the learning activities in Mathematics that will help the students in developing logical and critical thinking through problem solving in math courses.

Parents

This study will be beneficial to the parents of the students. The study will show the parents how their children think about certain problem, come up with strategy, and finally reach an answer that may solve the problem.

Students

This study is beneficial to the students for the study will instill in the students the awareness of the importance and application of math in real life situations and their problem solving skills as science curriculum students. Once they had addressed the possible problems, they would be much more ready to study harder so that they may be more prepared for college education.

Scope and Delimitation of the Study

The focus of this study is the heuristics in solving nonroutine problems of selected students of San Pablo City Science High School, San Pablo City, Laguna.

The researcher observed and gathered data from 12 selected students of San Pablo City Science High School. The top students who have represented the school in various mathematics contests were chosen. Three were chosen from each grade/year level. These students are well-trained participants in the Metro

Bank-DepEd Math Challenge (MTAP). Five nonroutine problems were solved by each student even though the SPCSHS does not require this type of problems to be used. Inferences were made from this set of data only and were not representative of any other set of data. The data gathering was held February 13, 2014 from 1:15 pm to 4 pm at the Computer laboratory of SPCSHS.

Definition of Terms

The following terms are operationally defined for thorough understanding of the study.

Algorithm. It is a step-by-step procedure for calculations and is a set of rules for solving a math problem which, if done properly, will give a correct answer each time. These are usually formulas in math.

Availability of IT materials at home. These include computers, calculators and other electronic gadgets that students may use in their studies especially in math.

Conceptual Understanding. Includes the ability to interpret the problem and select appropriate information and concepts to apply a strategy for solution.

Favorite Subject. Refers to the enthusiasm and the degree of likeness of a student to a particular subject in school.

Heuristics. A strategy or strategies to solve a problem that provides a starting point which can be followed either to successfully solve a problem or switch to another plan or approach.

Hobbies. Refers to the activities the students enjoy doing in their free time.

Learning Experience. Refers to the strategies and methods used by students' math teacher in delivering lessons and the exposure of the students in dealing with math like math contest, math camps and other math related activities

Math Ability. Refers to the level of the students understanding and skill in Math based on their assumption and self-evaluation.

Nonroutine Problem. Mathematics problems that are usually not found in textbooks and is mostly concerned with developing students' mathematical reasoning power and fostering the understanding that mathematics is a creative endeavor.

Parents' Support. Includes moral, spiritual and financial support coming from the parent's with regards to school activities and other related things to motivate the students.

Performance of the Students in Solving Non-Routine Problems. Refers to the score of the students in 5 item non-routine problems given in this study. *Problem*

Solving. The process of working through details of a problem to reach a solution using mathematical or systematic operations that can be a gauge of an individual's critical thinking skills.

Problem Solving Skills and Strategies. Includes clear focus, good reasoning and insightful thinking that leads to a successful resolution of the problem

Procedural Knowledge. Deals with the student's ability to demonstrate appropriate use of algorithm

School Math Courses. Refers to the math subjects offered by the school to be taken by the students

School Math Curriculum. Refers to the program being adopted by the school to give the students mastery of the competencies in mathematics.

School Math Program. This refers to the plan of activities that help students in improving their skills in math like enhancement program through peer tutoring.

Study Habit. Refers to the routines and ways the students do to study or review on a particular subject especially in math.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter contains a survey of the previous researches and literatures which provided the researcher with confirmation related to and relevant to the present study.

Description & Importance of Problem Solving

Since this study is all about mathematical problem solving, the review of related literature starts with the definition of Polya (the *Father of Problem Solving*) about problem solving. According to him, problem solving has something to do with finding a way around a difficulty, around an obstacle and finding a solution to a problem (Polya 1945 & 1962).

A common view among the researchers, mathematics teachers, students and parents is that, doing mathematics is considered as the heart of mathematics (Cockcroft 1982, Schoenfeld 1985, Kaur 1997, NCTM 2000, Cankoy & Ozder 2011). An important component of mathematics training is solving word problem. The National Council of Supervisors of Mathematics (1977) added that learning to solve problems is the principal reason for studying mathematics (as stated by M. Seifi 2012).

Cockcroft (1982) also attempted to characterize problem solving: The ability to solve problems is at the heart of mathematics. Mathematics is only useful to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name problem solving.

From the views above it seems that some writers believe that solving problem is the fundamental nature of mathematics learning, while some consider mathematics as a body of knowledge which provides the tools for the process of solving mathematical problems. But what then is the problem in mathematical setting? Krulik and Rudnick (1988) defined a problem as "a situation that requires resolution and for which the individual sees no apparent or obvious means or path to obtaining the solution. Schoenfeld (1989) further stated that for any student, a mathematical problem is a task in which the student is interested and engaged in, for which he/she wishes to obtain a resolution, and for which the student does not have a readily accessible mathematical means by which to achieve that resolution.

According to Arslan (2007), many research studies and projects have pointed out the importance of learning problem solving in school mathematics

courses (Ford, 1994; Higgins, 1997; National Council of Teachers in Mathematics, 1989; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaert, Ratinckx, 1999). In fact, for the past decades, problem solving has been the main focus of the Singapore mathematics curriculum. Revisions are made to the syllabus for the mathematics curriculum in schools by Singapore's Ministry of Education which aims to enable students develop their abilities in mathematical problem solving (Ministry of Education, 2006). Today, in our country, the DepEd in line with the implementation of the K to 12 program is determined to strengthen the basic education of the Filipinos. Critical thinking and problem solving will be the main focus of the mathematics learning in both elementary and secondary level.

Routine and Nonroutine Problems

According to Polya (1962), routine problem solving does not contribute to the mental development of the student. He supposed that nonroutine problems should be employed to provide an opportunity for students to develop higher-order thinking in the process of understanding, analysis, exploration and application of mathematical concepts.

Much of our classroom mathematics is composed of routine problems. Routine problems require only the application of a formula or a prescribed

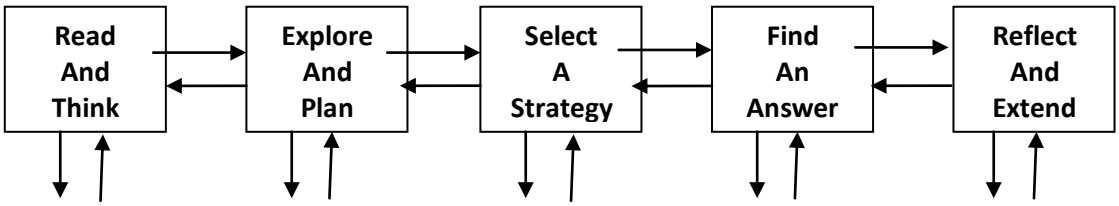
procedure (algorithm). These are the usual problems which we see in our textbooks that hold back our student's capability in solving problems. On the other hand, nonroutine problems are problems (NRPs) that make use of heuristics and often requires little to no use of algorithms.

According to MATHCOUNTS Foundation (1999), there are many problems that in first glance may seem difficult if attacked by setting up a series of equations, but quite simple when one use different problem solving strategies such as looking for a pattern, drawing a diagram, making an organized list, and others which are called as *heuristics*.

Problem Solving Heuristics

In solving any mathematical problem, heuristics are needed to serve as a guide and lead the solver to arrive at a certain answer. But what then are heuristics? According to Polya (1957), heuristics are possible solutions for certain problems but are not definite rules in getting the correct answer. It simply means that heuristics are the lines of attack in which if we follow may lead us to successfully solve a certain problem.

Krulik and Rudnick's 5-Stage Heuristic Model.



Identify the facts Identify the question Visualize the situation Describe the setting Restate the action	Organize the information Is there sufficient information? Is there too much information? Draw a diagram or construct a model Make a chart, a table, a graph or a drawing	Pattern Recognition Working backwards Guess and test Simulation or experimentation Reduction/ Expansion Organized listing/ exhaustive listing Logical deduction Write an equation	Estimate Use computational skills Use algebraic skills Use geometric skills Use a calculator when appropriate	Check your answer 1. Is the computation correct? 2. Is the question answered? 3. Is the answer reasonable? 4. How does your answer compare with your estimate? Find alternate solutions What if? Extend to either a generalization or a mathematical concept Discuss the solutions Create interesting variations on the original problem

Krulik and Rudnick (1996) identified 5 stages of solving a problem namely: 1. read and think; 2. explore and plan; 3. select a strategy; 4. find an answer; and 5. reflect and extend. The 5-Stage Heuristic Model of Krulik and Rudnick is the one used in this study. Below is the diagram showing Krulik and Rudnick's (1996) 5-Stage Heuristic Model and the description in each stage.

Krulik and Rudnick's Eight Problem Solving Strategies

Furthermore, Krulik and Rudnick (1996) recognized eight strategies that can be applied to solve mathematical problems at the secondary level. Below are the eight strategies and their definitions as stated in the work of Mabilangan, Limjap and Belecina (2011) in their case study: Problem Solving Strategies of High School Students on Nonroutine Problems.

a. Computing or Simplifying (CS) – it includes straightforward application of arithmetic rules, order of operations, and other procedures.

b. Using a Formula (F) – this strategy involves substituting values into a formula or selecting the proper formula to use.

c. Making a Model or Diagram (MD) – it includes the use of objects, drawings, acting out or writing an equation.

d. Making a Table, Chart or List (TCL) – this strategy involves organizing the data by making a table, chart, graph or list.

e. Guessing, Checking and Revising (GCR) – it involves making a reasonable guess, checking the guess and revising the guess, if necessary.

f. Considering a Simpler Case (SC) – includes rewording the problem, using smaller numbers, using a more familiar problem setting, dividing the problem into simpler problems or working backwards.

e. Eliminating (E) – this strategy involves eliminating possible solutions based on information presented in the problem or elimination of incorrect answers.

f. Looking for Patterns (LP) – it involves determining certain common characteristics that can be generalized and used to solve the problem.

ODE (1991) Problem Solving Levels of Understanding

In solving nonroutine problems, a student‘s understanding on this type of problems was described by the Oregon Department of Education (1991) which was used in this study as stated in the as stated in work of Mabilangan, Limjap and Belecina (2011) in their case study: Problem Solving Strategies of High School Students on Nonroutine Problems. It consists of the following:

a. Conceptual Understanding – includes the ability to interpret the problem and select appropriate concepts and information to apply a strategy for solution. Evidence is communicated through making connections between the problem

situation, relevant information, appropriate mathematical concepts and logical/reasonable responses.

b. Procedural Knowledge – deals with the student's ability to demonstrate appropriate use of algorithms. Evidence includes verifying and justifying of a procedure using concrete models, or the modifying of procedures to deal with factors inherent to the problem.

c. Problem Solving Skills and Strategies – includes clearly focused good reasoning and insightful thinking that leads to a successful resolution of the problem.

Related Studies on Problem Solving Heuristics

In 2005, Candelaria and Limjap conducted a case analysis on nine students entitled "Problem Solving Heuristics of College Freshmen". They stated that the problem solving heuristics of the students were varied and that they had difficulty in understanding the problem supported by the result that no one was found to be an expert problem solver.

In a study of Infante (2005) on the "Idiosyncratic Heuristics in Solving Nonroutine Problems of Schooled and Partially-Schooled Students," she stated that mathematics teachers, as well as students, should be more determinate

towards nonroutine problem solving as it intends to improve mathematical instructions

.

In 2006, Samonte studied the problem solving abilities in nonroutine and open-ended problems of 6 sophomore students. It was found out that in the results that the problem solving abilities of the students vary depending on the type of problem the students are solving. Likewise, he added that the students' abilities vary in terms of conceptual understanding, strategies and reasoning, computation and execution.

In a study conducted by Laset and Limjap (2005) on problem solving heuristics of six high-performing UP Rural High School Senior Students, the results showed that the problem solving process of the students was consistent with the following model: Read and Think, Explore and Plan, Select a Strategy, Find an answer and Reflect and extend. The problem solving heuristics of students were varied.

Lastly, in 2011, Lee conducted a similar study with those of Laset and Limjap (2005) entitled "Nonroutine Problem Solving Heuristics of Selected High-Performing Students in UP Rural High School." He found out that the students were able to use different strategies in solving the nonroutine problems (NRPs)

and that the NRPs enabled the students to discover and develop their reasoning and problem solving skills even more.

Synthesis

The studies and literatures presented in this chapter are all pertinent to genuinely understand the essence of the use of heuristics or strategies in solving nonroutine problems in the field of mathematics and how they can facilitate in developing students problem solving and crictical thinking skills. Most of the literature presented the distinctiveness of nonroutine problems to elicit different heuristics in solving it rather than focusing on a particular procedure or algorithm. Promoting the use of heuristics can help in making problem solving a stimulating endeavor rather than a dull and boring part of learning mathematics.

Most researchers and studies recommend the use of heuristics in solving problems especially if that problem is unfamiliar to the solver. In the exploratory study of Mabilangan (2011) entitled “Problem solving strategies of high school students on nonroutine problems”, she found out that a single problem (nonroutine) could be solved in more than one way and that there are many ways in solving a given problem. She further added that the problem solving strategy could be employed by students in different ways depending on the nature of the

problem. That is the very reason why nonroutine problems will be utilized in this study.

Another study entitled “Effects of using varied problem solving heuristics on the mathematics achievement of students at Imus National High School by Martin (2003) suggests the idea of using different heuristics in teaching mathematics. She found out that using heuristics is an effective tool in teaching mathematics.

The different studies and literature presented therein are essential in the conduct of this special project because they provide bases on the use and importance of heuristics in solving problems in mathematics specifically nonroutine type of problems.

CHAPTER III

METHODOLOGY

Research Design

The researcher observed and gathered data from 12 selected students of San Pablo City Science High School. Top students who have represented the school in various mathematics contests were chosen. Three were chosen from each grade/year level. These students are well-trained participants in the Metro Bank-DepEd Math Challenge (MTAP). Five nonroutine problems were solved by each student even though SPCSHS does not openly require this type of problems to be used. At the data gathering on February 13, 2014 held at the Computer laboratory of SPCSHS from 1:15 to 4:00 pm, all twelve (12) students participated in the conduct of the study. Inferences were made from this set of data only and were not representative of any other set of data.

Respondents and Sampling

In the selection of the participants for this study, the researcher used a non-random sampling method particularly the purposive sampling method. In this study, the participants were chosen based on researcher's personal judgment. This judgment was based on the idea about problem solving and which participants would most likely provide the needed data.

As regards the use of purposive sampling, Wallen (2003) writs that researchers assume that they can use their knowledge of the population to judge whether or not a particular sample will be a representative.

Instruments Used

The following were used to gather the necessary data:

1. Preliminary Questions (Profile of the Students)

Students were asked regarding their:

a. background in mathematics specifically their experiences in doing mathematics; their attitude towards Math; trainings; and exposures to various Math competitions;

b. personal data (students profile) which include year level, math ability (personal rating) and even hobbies and parents' support in their studies.

2. Nonroutine Problem (NRP) Worksheets and Journal (Appendix B)

A worksheet with a journal page was given to each student for each of the five non-routine problems to reflect on his or her solutions to each NRP. Scratch papers were also provided. Also guide questions will be provided to give the students the idea on what to reflect on.

3. Nonroutine Problems (NRPs)

Five nonroutine problems were used in this study. These problems were variations of problems for two books namely: The New Sourcebook for Teaching Reasoning and Problem Solving in Junior and Senior High School (Krulik and Rudnick), and Brain-Teasers for Mathematics Students (Fostanes 2005).The first NRP is about counting, the second is about logic, the third is about looking for patterns, the fourth is about the number properties and combinations and the fifth is about logic and computation. Four teachers were asked to face validate the NRPs. 10 problems were made and 5 were selected from it. A pre-testing was done to 12 MTAP contestants/students of Col. Lauro D. Dizon Memorial National High School for the face validity of the problems. Some of the problems were found readable and understandable and some were revised to better suit the students' level and understanding as per recommendation of the professors and teachers who validated the NRPs.

Methodology

The descriptive design was utilized since the study requires an analysis of heuristics in solving nonroutine problems. Preliminary questions and nonroutine problem (NRP) worksheets with scratch papers and journal page were given to the students to collect data. The five NRPs have undergone face validation from educators (two university professors from the Philippine Normal University and 2

public teachers from Col. Lauro D. Dizon Memorial national High School) and a pre-testing of the 5 NRPs was conducted among students.

The data gathered were described and analyzed quantitatively. The data were subjected to statistical treatments and by a close examination of the students' answers in the NRPs.

Data Gathering Procedure

The 12 students first filled out the preliminary questionnaire (see Appendix A). Upon its completion, the first worksheet was given to the students with scratch papers provided. After solving each non-routine problem, each student was asked to make a journal of his/her reflection on how they solved the problem. The journal includes guide questions to help the students focus on their reflection. A script was read during the administration of the 5 NRPs.

Data Analysis

The consistency of Krulik and Rudnick's 5-Stage Heuristic Model with each student's problem solving process in each NRP was analyzed. The strategy or strategies used by each student in each of the five nonroutine problems was identified. The students' problem solving strategies were classified as follows:

a. Compute or Simplify CS

b. Use a Formula	F
c. Make a Model or Diagram	MD
d. Make a Table, Chart or List	TCL
e. Guess, Check, and Revise	GCR
f. Consider a Simpler Case	SC
g. Eliminate	E
h. Look for Patterns	LP

Statistical Analysis

To evaluate students' conceptual understanding, procedural knowledge and problem solving strategies and skills, the Oregon Mathematics Problem Solving Rubrics, (Oregon Department of Education 1991) were used.

Moreover, the problem solving expertise of the students was classified as Proficient (P), Apprentice (Ap) and Novice (N). The Proficient problem solver is the highest level of expertise, followed by the Proficient problem solver and the Apprentice Problem Solver.

The students' level of understanding of the nonroutine problem has three components namely: Conceptual Understanding, Procedural Knowledge and Problem Solving Skills and Strategies (ODE 1991). Each of these components has three categories. A point system corresponding to the three categories of

each component was utilized to evaluate the over-all problem solving performance of the students for the five nonroutine problems.

The Conceptual Understanding is categorized into:

a. Full Conceptual Understanding – 5pts

b. Partial Conceptual Understanding – 3pts

c. Lack Conceptual Understanding – 1pt

The Procedural Understanding is categorized into:

a. Full Use of Appropriate Procedures – 5pts

b. Partial Use of Appropriate Procedures – 3pts

c. Lack Use of Appropriate Procedures – 1pt

The Problem Solving Skills and Strategies are categorized into:

a. Full Use of Problem Solving Skills and Strategies – 5pts

b. Partial Use of Problem Solving Skills and Strategies – 3pts

c. Lack Use of Problem Solving Skills and Strategies – 1pt

Scale points 2 and 4 are not explicitly defined. A score of 2 would be assigned to work that exceeded criteria for a score of 1, but did not meet criteria for a score of 2. Similarly, a score of 4 would be assigned to work that exceeded criteria for a score of 3, but did not meet criteria for a score of 5.

The average total score of each year level was also determined to compare the problem solving abilities of students across year levels. Factors affecting problem solving performance such as trainings in Mathematics, exposures to various Math competitions and the like were also considered.

CHAPTER IV

PRESENTATION, ANALYSIS AND INTERPRETATION OF DATA

This chapter contains the outcome of the data gathering and discussions that were based on the results of the solutions of the 12 students on the 5 NRPs given.

Consistency of the 5-Stage Heuristic Model

Thirty (30) out of the 60 solutions to the 5 NRPs of 12 students or 50% showed consistency Krulik and Rudnick's 5-Stage Heuristic Model (1996) except that the *extending part* of the 5th stage was not fully or clearly seen in their solutions but shows evidence of the *checking part*. On the other hand, 30 solutions followed the first four stages only and did not show the checking of answer(s) which is part of the 5th stage Reflect and Extend.

Out of 12 students, stages 1 to 4 of the 5-Stage Model were evident as seen and observed in their solutions and journals. The first stage (Read and Think) was seen clearly during the administration of the study and in every solution made by the students. The second stage (Explore and Plan) was clearly seen in the attempt of each student in solving the problem and using different strategies. The third stage (Select a Strategy) was seen due to the fact there

was a computation and solution in every NRP and each student ticked a strategy from the choices given in each journal page. The fourth stage (Find an Answer) was definitely seen in each of the solutions of the students as everyone executed his strategy to arrive at an answer. The fifth and final stage (Reflect and Extend) which includes the checking of answer(s) and extending the problem or process to discover a generalization was not fully observed and seen. The checking of solution was seen only in 30 out of 60 solutions to the 5 NRPs solved while the extending part of the problem was not observed from any of the 60 solutions to the 5 NRPs.

Sample Solutions and Journal Entries

Let us look at 5 examples from the 30 NRPs which showed consistency with the 5-Stage Model which include the *checking part* of the 5th stage.

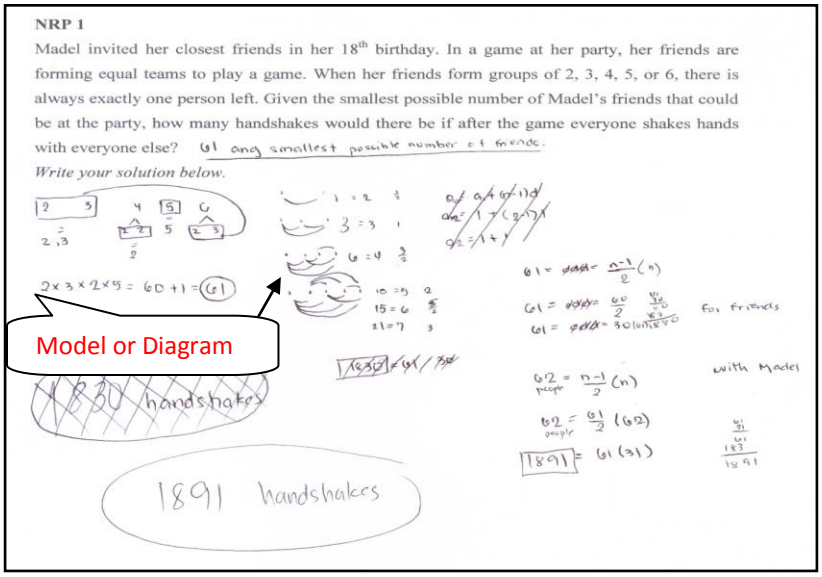
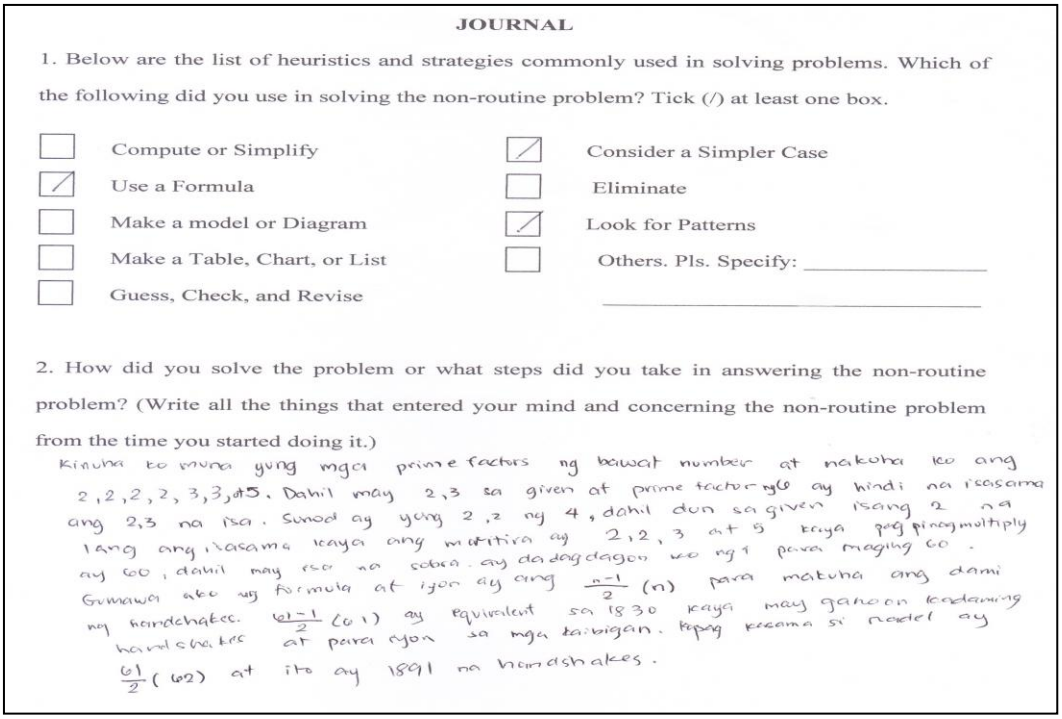


Figure 2. Student G8C's solution to NRP 1.



JOURNAL

1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

- ☐ Compute or Simplify
- ☑ Use a Formula
- ☐ Make a model or Diagram
- ☐ Make a Table, Chart, or List
- ☐ Guess, Check, and Revise
- ☑ Consider a Simpler Case
- ☐ Eliminate
- ☑ Look for Patterns
- ☐ Others. Pls. Specify: _____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.)

Kinuha ko muna yung mga prime factors ng bawat number at nakuha ko ang 2,2,2,2,3,3,at 5. Dahil may 2,3 sa given at prime factor nga ay hindi na isasama ang 2,3 na isa. Sunod ay yung 2,2 ng 4, dahil dun sa given isang 2 na lang ang isasama kaya ang matitira ay 2,2,3 at 5 kaya pag pinagmultiply ay 60, dahil may isa na sobra. ay dadagdagan ko ng 1 para maging 60. Gumawa ako ng formula at iyon ay ang $\frac{n-1}{2}(n)$ para makuha ang dami ng handshakes. $\frac{61-1}{2}(61)$ ay equivalent sa 1830 kaya may ganoon kadaming handshakes at para iyon sa mga taibigan. Kaya kasama si noel ay $\frac{61}{2}(62)$ at ito ay 1891 na handshakes.

Figure 3. Student G8C's journal page to NRP 1.

Figure 2 shows the solution of student C of Grade 8 level to NRP 1 which shows consistency in the 5-Stage heuristic model. The first thing he did was to get the prime factors. Then, multiplied them and got 60, then added 1 because there is "always one person left" having 61 friends indicating the 1st stage (Read and Think) and 2nd stage (Explore and Plan). Student G8C didn't actually know the formula in finding the handshakes. Looking closely at his solution, he actually drew points then looked for patterns (3rd stage – Select a Strategy). He then thought of the formula of arithmetic series then erased it and ended up

using $\frac{n-1}{2}(n)$. The 4th stage (Find an Answer) and the 5th stage (Reflect and Extend) of his problem solving can be seen in his journal. He stated there that if Madel is not included in the handshakes, there were 1830 handshakes while if Madel was included then there were 1891 handshakes.

Figure 2 also shows that aside from the 3 strategies that student G8C ticked in his journal page in Figure 3, he also used unconsciously the strategy MD (Make a Model or Diagram) to get the pattern and made use of a formula and arrived at the correct answer. He also used LP, SC and F.

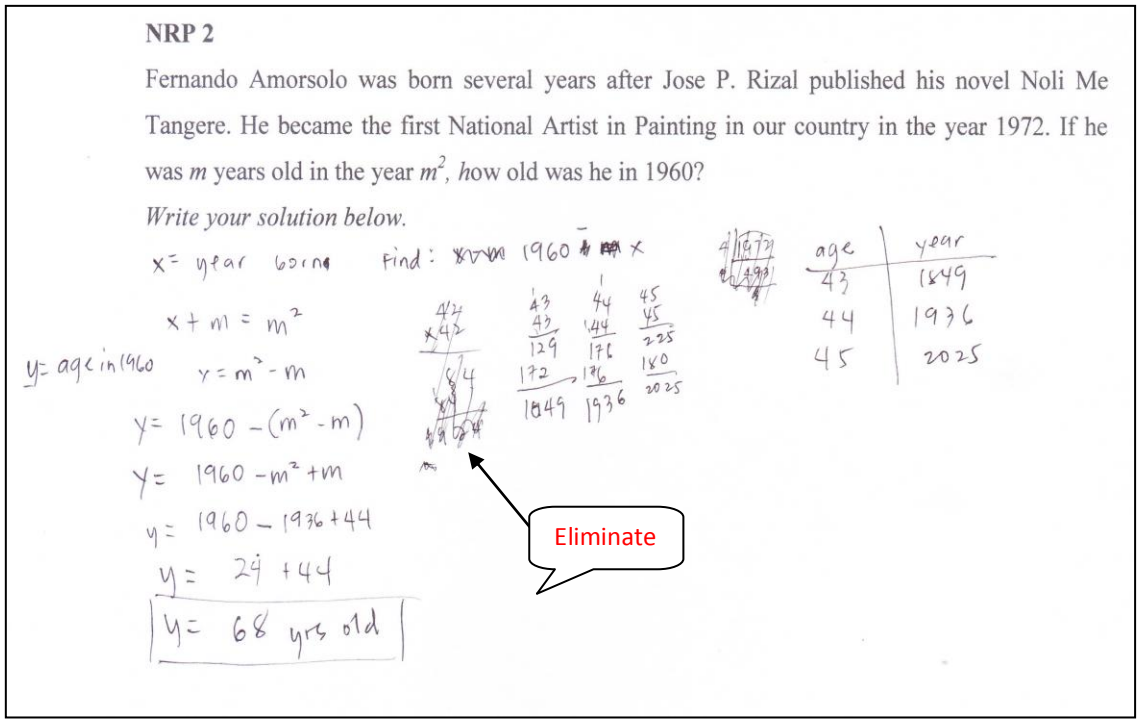
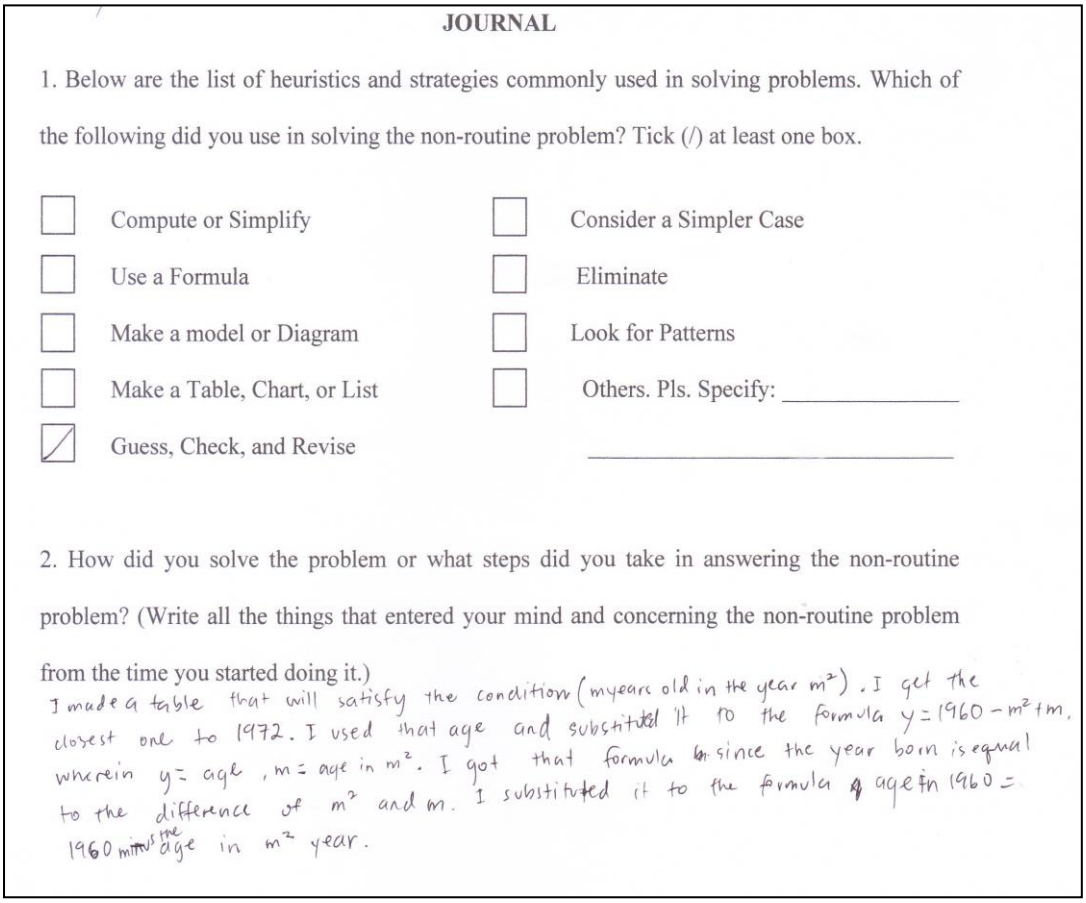


Figure 4. Student Y3C's solution to NRP 2.



JOURNAL

1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

- ☐ Compute or Simplify
- ☐ Use a Formula
- ☐ Make a model or Diagram
- ☐ Make a Table, Chart, or List
- ☑ Guess, Check, and Revise
- ☐ Consider a Simpler Case
- ☐ Eliminate
- ☐ Look for Patterns
- ☐ Others. Pls. Specify: _____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.)

I made a table that will satisfy the condition (m years old in the year m^2). I get the closest one to 1972. I used that age and substituted it to the formula $y = 1960 - m^2 + m$, wherein y = age, m = age in m^2. I got that formula since the year born is equal to the difference of m^2 and m. I substituted it to the formula of age in 1960 = 1960 minus the age in m^2 year.

Figure 5. Student Y3C's solution to NRP 2.

Figure 4 shows the solution of student C of the third year level to NRP 2 which shows consistency in the 5-Stage heuristic model. What he did was he wanted to make a table that will satisfy the condition *m years old in the year* m^2 indicating the 1st stage (Read and Think) and 2nd stage (Explore and Plan). Student Y3C selected GCR as the strategy together with CS since he came up with an equation $y = 1960 - m^2 + m$ (3rd stage – Select a Strategy). During the

GCR and CS, we can notice that in his solution he eliminated 42 as the value of *m* and erased his computation. He completed his table getting the ages from 43 to 45. The 4th stage (Find an Answer) and the 5th stage (Reflect and Extend) of his problem solving can be verified in his journal. He substituted the assumed values of *m* in the equation he made and chose the age that if you squared it will be closest to 1972 and that is with *m =44* in the year m^2 1936. Then he finally arrived at the correct answer 68 years old.

Aside from that, Figure 4 also shows that aside from one (1) strategy that student Y3C ticked in his journal page (Figure 5) which is the GCR, he also used unconsciously the strategy MD (Making a Model or Diagram) as seen when he wrote the equation. He also used TCL (Make a Table, Chart or List) to get the possible values of *m* and did the CS (Compute or Simplify) for the useful equation and arrived at the correct answer through E (Elimination) and substitution.

Figure 6 shows the solution of student A of the Grade 7 level to NRP 3 which shows consistency in the 5-Stage heuristic model. He started solving the problem by assigning variables for the unknown in the problem indicating the 1st stage (Read and Think) and translated the problem into a mathematical sentence signifying the 2nd stage (Explore and Plan).

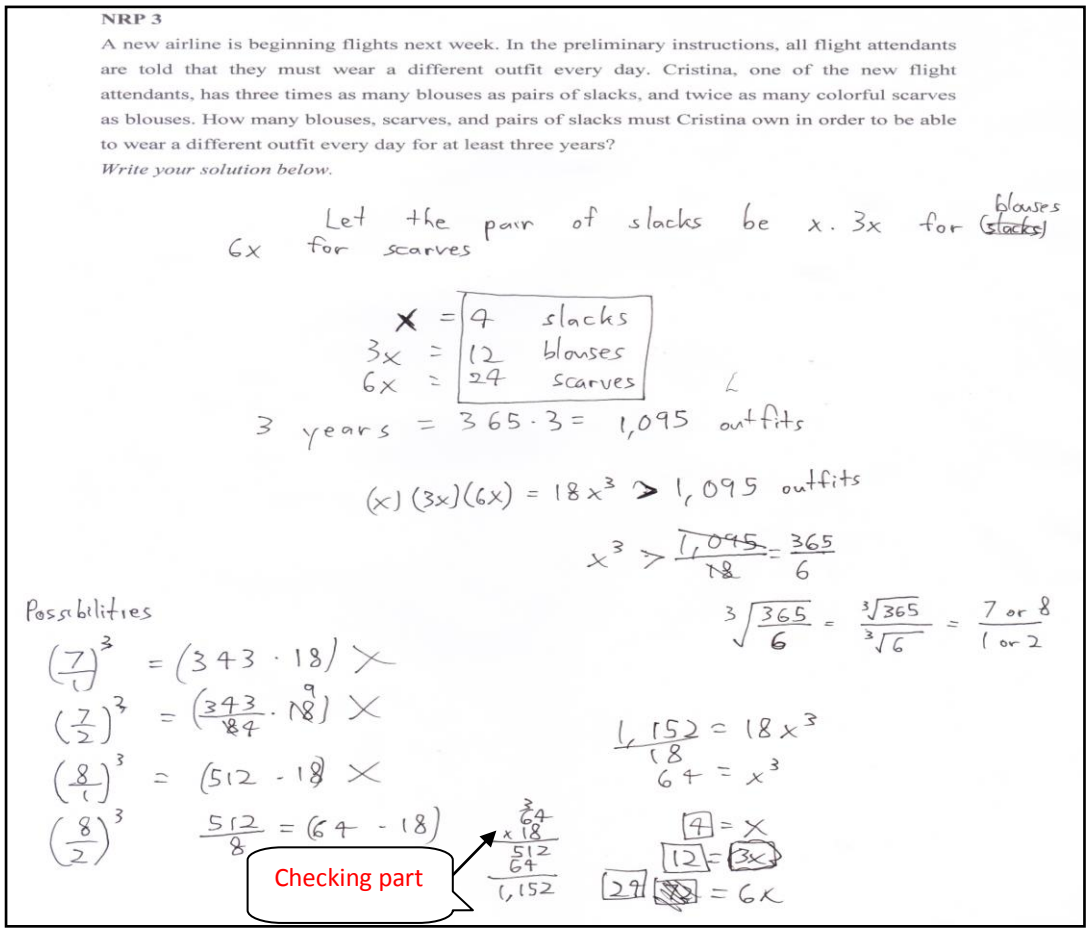
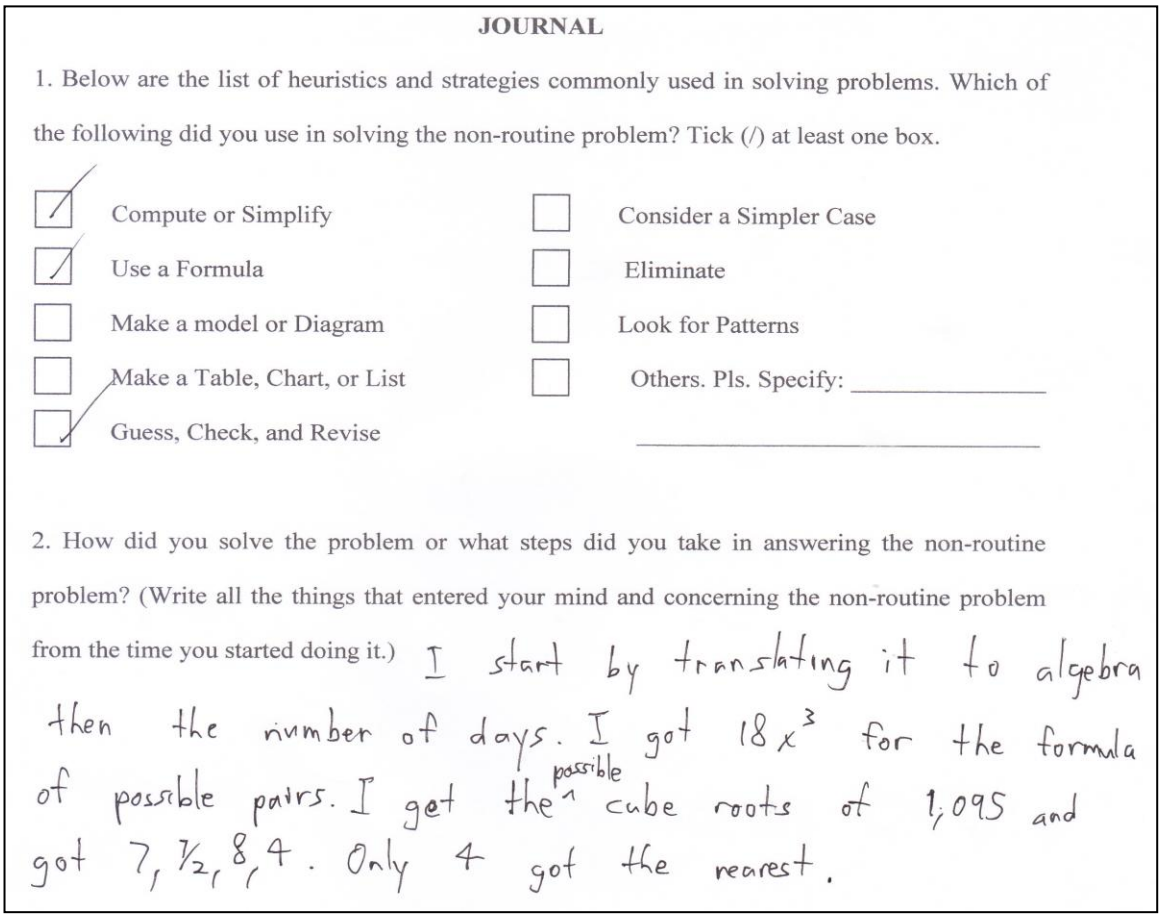


Figure 6. Student G7A's solution to NRP 3.

Student G7A selected CS to identify the number of outfits in 3 years as seen in his solution in figure 5 (3rd stage – Select a Strategy). He then used another strategy which is F since he came up with an inequality $(x)(3x)(6x) > 1095$ for the possible pairs of outfits. The 4th stage (Find an Answer) of his problem solving can be verified in his journal. He computed the possible cube roots of 1095 and got many possible value of *x.* Using GCR, he

arrived at the conclusion that among the possible values of *x* that he got, 4 was the closest. The 5th stage (Reflect and Extend) was seen in the solution of student G7A where he did the checking of his answer by multiplying 4, 12 and 24 and getting a product of 1152 which is greater than 1095, getting the correct number of outfits.



JOURNAL

1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

- [x] Compute or Simplify
- [x] Use a Formula
- [ ] Make a model or Diagram
- [ ] Make a Table, Chart, or List
- [x] Guess, Check, and Revise
- [ ] Consider a Simpler Case
- [ ] Eliminate
- [ ] Look for Patterns
- [ ] Others. Pls. Specify: _____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.) I start by translating it to algebra then the number of days. I got $18x^3$ for the formula of possible pairs. I get the possible cube roots of 1,095 and got 7, 7½, 8, 4. Only 4 got the nearest.

Figure 7. Student G7A's journal page to NRP 3.

In figure 6, aside from the strategies that student G7A ticked (Figure 7), it shows that he also used some sort of elimination (E) where one can see that he wrote some possibilities of the values of *x.* Then after some computations (CS) and GCR, he eliminated 3 possibilities and ended up with the equation (MD) $x^3 = 64$ and ended up having $x = 4$. After that, he did the substitution and checked his answer.

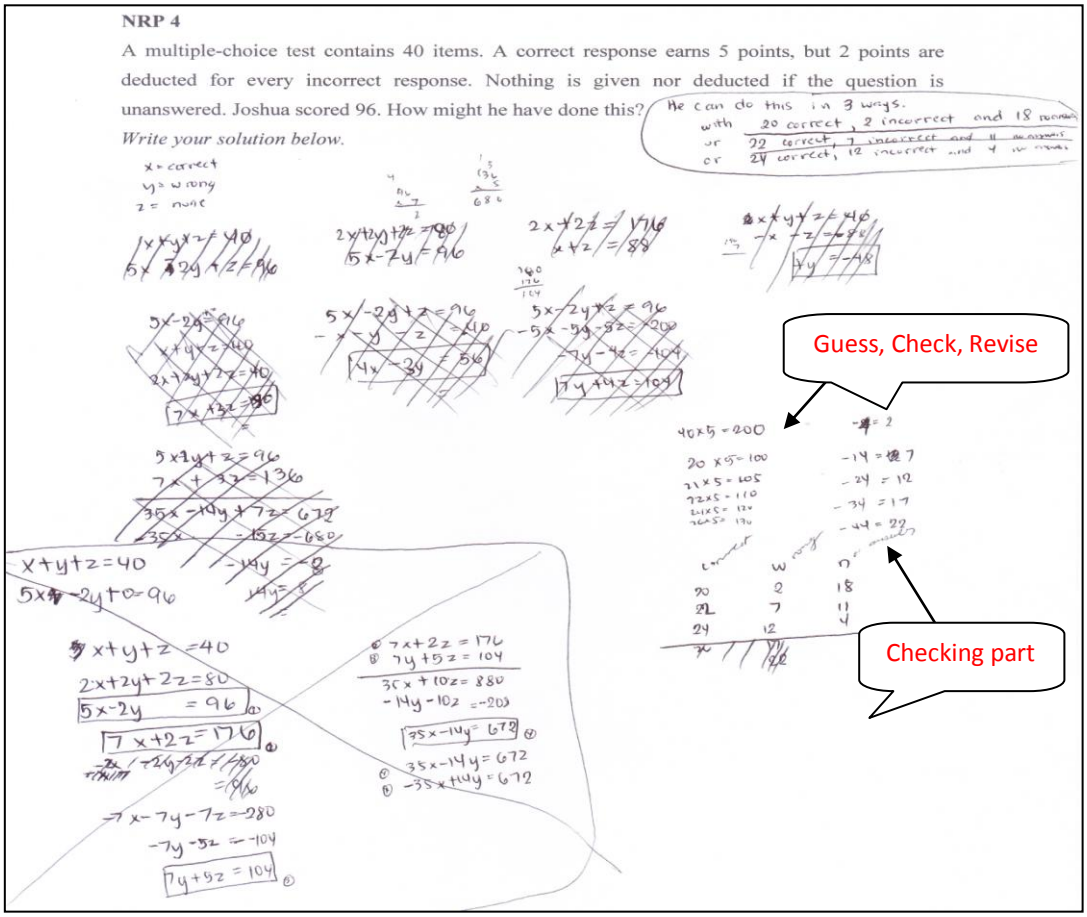
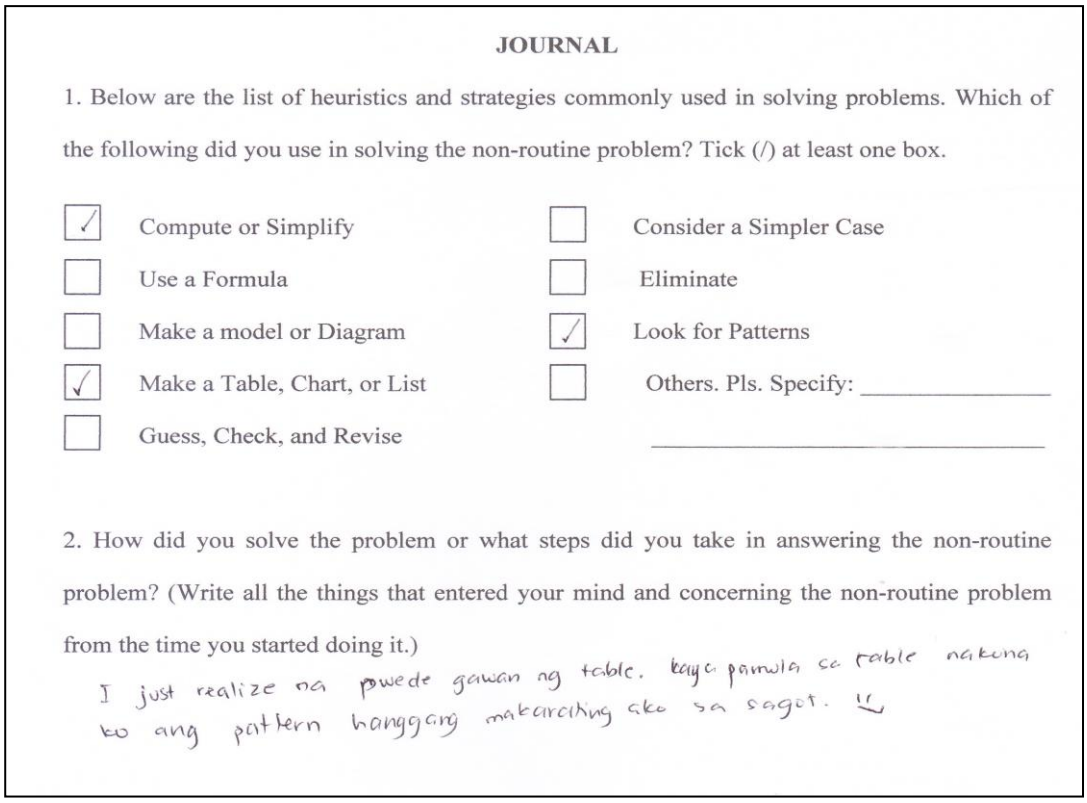


Figure 8. Student G8C's Solution to NRP 4.



JOURNAL

1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

- [x] Compute or Simplify
- [ ] Use a Formula
- [ ] Make a model or Diagram
- [x] Make a Table, Chart, or List
- [ ] Guess, Check, and Revise
- [ ] Consider a Simpler Case
- [ ] Eliminate
- [x] Look for Patterns
- [ ] Others. Pls. Specify: _____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.)

I just realize na pwede gawan ng table. kaya pinula sa table nakuna to ang pattern hanggang makarating ako sa sagot. ☺

Figure 9. Student G8C's journal page to NRP 4.

Figure 8 shows the solution of student C of the Grade 8 level to NRP 4 which shows consistency in the 5-Stage heuristic model. He started solving the problem by assigning variables for the unknown in the problem indicating the 1st stage (Read and Think) and translated the problem into mathematical sentences and equations signifying the 2nd stage (Explore and Plan). Student G8C selected CS to identify the possible combinations of the response (3rd stage – Select a Strategy). After several times of finding the answer through simplifying the equations, everything did not work out. He then used another strategy which is

GCR. He started guessing with $40 * 5 = 400$ then $20 * 5 = 100, 21 * 5 = 105, 26 * 5 = 130$ until he came up with an idea to make a table of possible number combinations. The 4th stage (Find an Answer) and 5th stage (Reflect and Extend) of his problem solving can be verified in his journal. He stated there that he just realized that after all the equations; he can make a table to verify the possible response combinations for the problem. Using GCR, he completed the table and arrived at the conclusion that among there are 3 ways to do the score of 96.

Figure 8 also shows that aside from the strategies that student G8C ticked (Figure 9), it shows that he also used GCR or the *trial and error* called by most students. You can see that he solved some addition and subtraction. Then after some computations (CS), he set aside all the equations (MD) and applied GCR and made a table (TCL). He ended up with 3 ways of doing the problem.

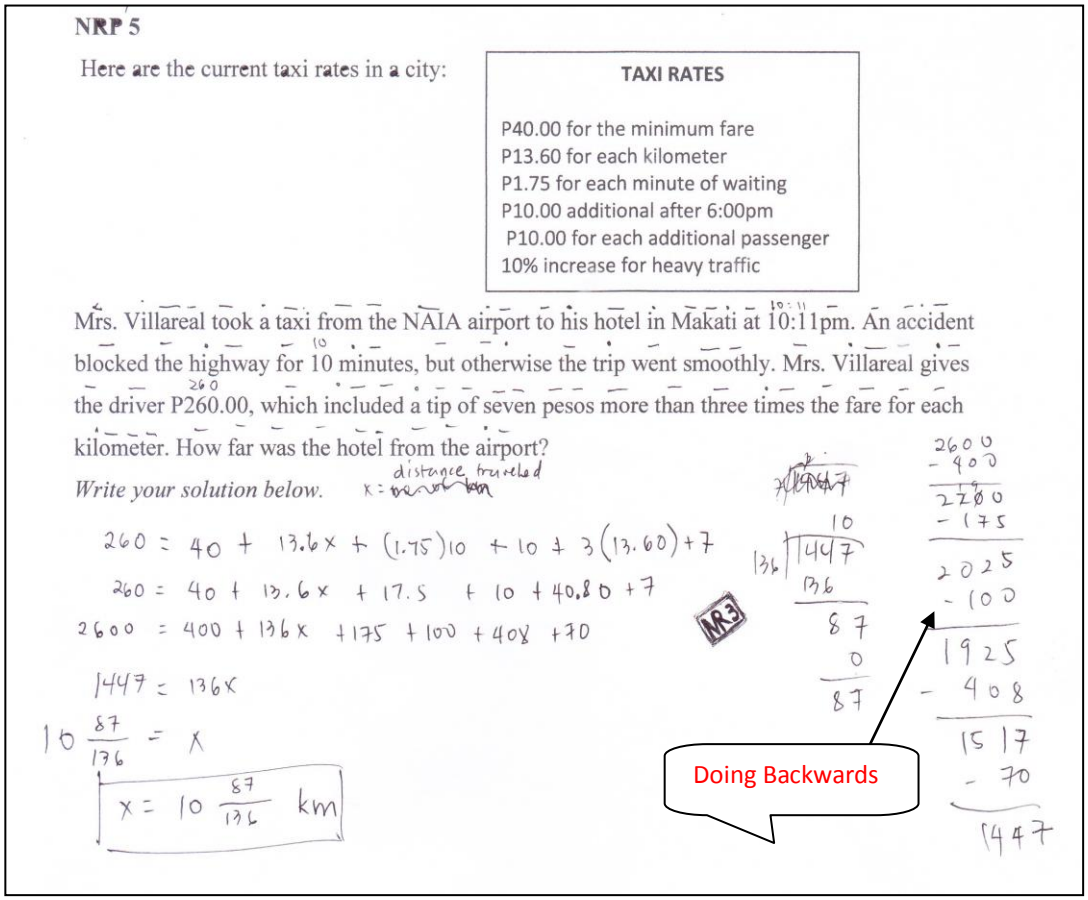
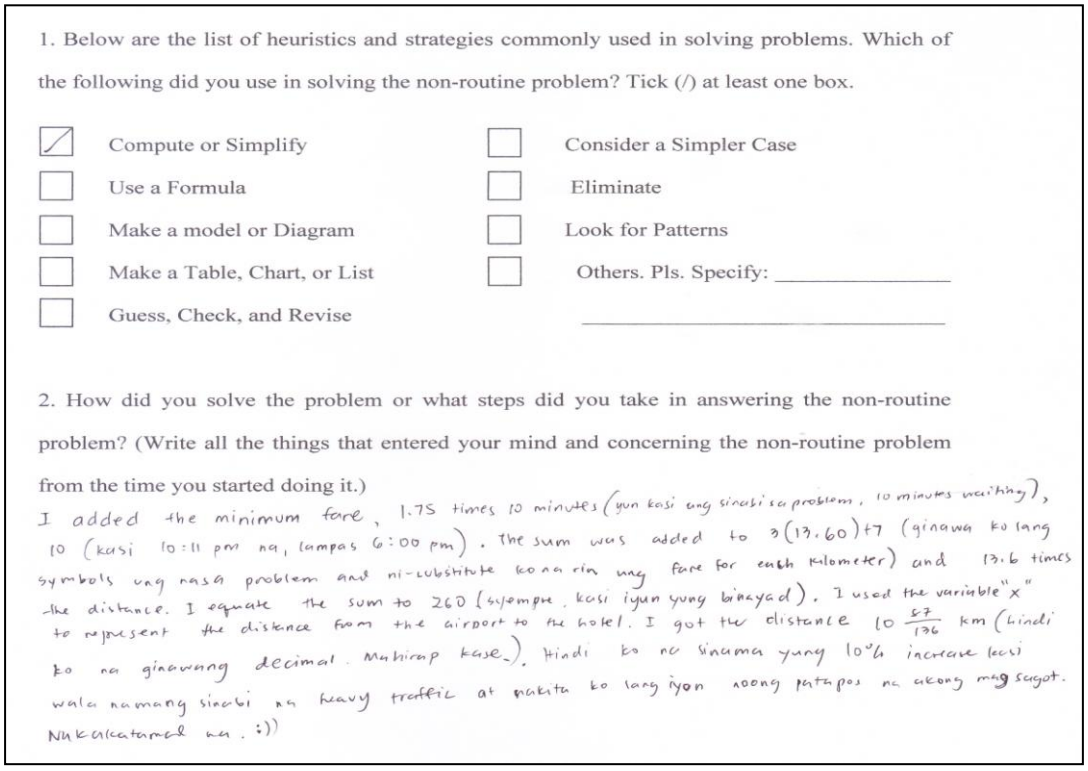


Figure 10. Student Y3C's Solution to NRP 5.

Figure 10 shows the solution of student C of the third year level to NRP 5 which shows consistency in the 5-Stage heuristic model. He started solving the problem by assigning *x* as the variable for the unknown, which is the distance in *km* of the airport to the hotel in Makati in the problem indicating the 1st stage (Read and Think) and translated the problem into a mathematical sentences and wrote an equation signifying the 2nd stage (Explore and Plan). Student Y3C

selected CS to solve for the possible value of *x* (3rd stage – Select a Strategy). After that he simplified the equation by Subtraction Property of Equality (SPE). He ended up with the equation $1447 = 136x$ and came up with an answer of $10\frac{87}{136}$ or 10.6 km. The 4th stage (Find an Answer) of his problem solving can be verified in his journal. He stated there that he just added the minimum fare, 1.75 times P10 for the 10 minutes waiting time, then another P10 because the travel was after 6:00 pm already and was added 3(13.60) + 7 as translated in the problem. All these add up to P260 so he equated the sum to 260. Using CS, he computed approximately 10.6 km.



1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

- [/] Compute or Simplify
- [ ] Use a Formula
- [ ] Make a model or Diagram
- [ ] Make a Table, Chart, or List
- [ ] Guess, Check, and Revise
- [ ] Consider a Simpler Case
- [ ] Eliminate
- [ ] Look for Patterns
- [ ] Others. Pls. Specify: _____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.)

I added the minimum fare, 1.75 times 10 minutes (yun kasi ung sinabi sa problem, 10 minutes waiting), 10 (kasi 10:11 pm na, lampas 6:00 pm). The sum was added to 3(13.60)+7 (ginawa ko lang symbols ung nasa problem and ni-substitute ko na rin ung fare for each kilometer) and 13.6 times the distance. I equate the sum to 260 (syempre, kasi iyun yung binayad). I used the variable "x" to represent the distance from the airport to the hotel. I got the distance to $\frac{87}{136}$ km (Hindi ko na ginawang decimal. Mahirap kase.). Hindi ko na sinama yung 10% increase (ecs) wala namang sinabi na heavy traffic at makita ko lang iyon noong patapos na akong mag sagot. Nakalkulate na. :))

Figure 11. Student Y3C's journal page to NRP 5.

In figure 10, aside from the strategy CS that student Y3C ticked (Figure 11), it shows that he unconsciously used E Eliminate and MD when he wrote an equation for the problem. Also he used SC Consider a Simpler Case when he applied doing backwards. One can verify this when he stated that he did not include the 10% increase since it was never stated in the problem that there was a heavy traffic and P10 for additional passenger. From here, one can say that 5th stage (Reflect and Extend) was verified that student Y3C checked his equation and answer.

Frequency of Strategies Used

All of the strategies given in the journal page were used by the students in solving the 5 NRPs. Students used a variety of strategies in solving the 5 NRPs. Student Y4C used 7 different strategies except MD while Y3B used only 2 strategies: CS and GCR . On the average, each student used 4 different strategies to solve the 5 NRPs. Problem solving strategies also varied depending on the problem.

Table 1. Frequency of each strategy used in each NRP

Table 2		NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY		**FREQUENCY**					
Compute or Simplify	CS	9	9	7	8	11	**44**
Use a Formula	F	11	0	2	0	0	**13**
Make a Model or Diagram	MD	1	6	5	6	4	**22**
Make a Table, Chart or List	TCL	2	2	5	6	2	**17**
Guess, Check, and Revise	GCR	2	9	7	10	6	**34**
Consider a Simpler Case	SC	2	2	2	0	11	**17**
Eliminate	E	0	2	2	3	3	**10**
Look for Patterns	LP	4	2	5	2	1	**14**

Table 1 shows that seven (7) strategies were used in NRP 1 namely: CS, F, TCL, GCR, SC, and LP. Use a formula (F) was the most used strategy for NRP 1 since it requires an algorithm and is all about counting. All seven (7) strategies were used in NRP 2 except F. CS and GCR was the most used strategy for NRP 2 which is about logic and requires *trial and error* and computations. All of the strategies were utilized in NRP 3. CS and GCR were the most used strategy since it requires logic and computations with the use of patterns and equations. Six (6) strategies were also used in solving NRP 4 which is about number properties and combinations. CS, MD, TCL, GCR, E and LP were the strategies used in NRP 4 and GCR was used most often. Lastly, Seven (7) strategies were used in NRP 5 which is all about logic and computation except F. CS and SC were the most suited strategy for NRP 5.

Table 1 also reveals the problem solving heuristics of the 12 students. Since these students were trained for the MTAP and quiz bee type contest which is mostly about routine problem based on the school competencies, they often use compute or simplify CS which is a typical characteristic of a math contestant. Likewise it shows that CS and GCR was the most helpful strategy for such type of problem. Eliminate E was the least used strategy as well as Use a Formula F and Look for Patterns LP

Table 2. Frequency of each strategy used in each grade/year level

Table 3		G7	G8	Y3	Y4	**TOTAL**
PROBLEM SOLVING STRATEGY		**FREQUENCY**				
Compute or Simplify	CS	**13**	**10**	**13**	**8**	**44**
Use a Formula	F	**4**	**4**	**2**	**3**	**13**
Make a Model or Diagram	MD	**6**	**7**	**5**	**4**	**22**
Make a Table, Chart or List	TCL	**3**	**2**	**5**	**7**	**17**
Guess, Check, and Revise	GCR	**11**	**7**	**7**	**9**	**34**
Consider a Simpler Case	SC	**5**	**4**	**3**	**5**	**17**
Eliminate	E	**3**	**2**	**3**	**2**	**10**
Look for Patterns	LP	**1**	**8**	**1**	**4**	**14**

Table 2 reveals that the most used strategy is Compute or Simplify (CS) with a frequency of 44 followed by Guess, Check and Revise (GCR) which is commonly known as *trial and error* among students with a frequency of 34. It was observed during the administration of the five NRPs that students noticed the different strategies enumerated in the journal page. Afterwards, the students

began to ask if they could refer to it as a guide to solve the problems. This gave them different strategies to choose from in order to arrive and get the correct answer. It also encouraged them to try others strategies especially when the strategy they used did not work. The students only used those strategies listed in their journal page even though there are some students who have written other strategies in the "others" item such as *Trial and Error* but basically mean the same as GCR.

Problem Solving Performance of Each Student

The over-all problem solving performance of the 12 selected students and their level of expertise in solving the 5 NRPs were also determined. The over-all problem solving performance of a student is the total score gathered from the 5 NRPs. The highest possible score for an NRP is 15 points and the lowest is 3 points. Thus, a range of points was made in this study to classify each student into three levels of expertise namely: Proficient (P) – 60 to 75 points, Apprentice (Ap) – 38 to 59 points, and Novice (N) – 15 to 37 points.

Table 3.1 Problem Solving Performance of Student G7A

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	5	5	5	15	Yes
2	5	5	5	15	Yes
3	5	5	5	15	Yes
4	5	5	5	15	Yes
5	4	3	2	9	No
Ave	**4.8**	**4.6**	**4.4**	**69**	

Table 3.1 shows the problem solving performance of Student A of Grade 7. He got a total of 69 points and answered 4 out of the 5 NRPs given correctly. He got the correct answers for NRPs 1 to 4 except for NRP 5. Student G7A is considered as a Proficient (P) problem solver and got the highest total score compared to the other students in this study.

Table 3.2 Problem Solving Performance of Student G7B

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	4	2	3	9	No
2	1	2	2	5	No
3	2	2	1	5	No
4	5	5	5	15	Yes
5	4	3	3	10	No
Ave	**3.2**	**2.8**	**2.8**	**44**	

Table 3.2 shows the problem solving performance of Student B of Grade 7. He got a total of 44 points and answered 1 out of the 5 NRPs given correctly. He got the correct answer for NRP 4 only. Student G7B is considered as an Apprentice (Ap) problem solver and got the second lowest score compared to the other students in this study. He almost got the correct answer for NRPs 1 and 5 but did not check his answers and committed some errors in his computations.

Table 3.3 Problem Solving Performance of Student G7C

	Levels of Understanding				
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	3	2	2	7	No
2	3	2	1	6	No
3	5	5	5	15	Yes
4	3	2	2	7	No
5	5	5	5	15	Yes
Ave	**3.8**	**3.2**	**3.0**	**50**	

Table 3.3 shows the problem solving performance of Student C of Grade 7. He got a total of 50 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 3 and 5. Student G7C is considered as an Apprentice (Ap) problem solver.

Table 4.1 Problem Solving Performance of Student G8A

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	2	2	1	5	No
2	1	3	2	6	No
3	2	1	2	5	No
4	5	5	5	15	Yes
5	2	1	2	5	No
Ave	**2.4**	**2.4**	**2.4**	**36**	

Table 4.1 shows the problem solving performance of Student A of Grade 8. He got a total of 36 points and answered 1 out of the 5 NRPs given correctly. He got the correct answer for NRP 4 only. Student G8A is considered as a Novice (N) problem solver and got the lowest score compared to the other students in this study.

Table 4.2 Problem Solving Performance of Student G8B

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	3	2	2	7	No
2	5	5	5	15	Yes
3	3	2	2	7	No
4	5	5	5	15	Yes
5	3	2	2	7	No
Ave	**3.8**	**3.2**	**3.2**	**51**	

Table 4.2 shows the problem solving performance of Student B of Grade 8. He got a total of 51 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 2 and 4. Student G8B is considered as an Apprentice (Ap) problem solver.

Table 4.3 Problem Solving Performance of Student G8C

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	5	5	5	15	Yes
2	3	2	3	8	No
3	2	2	2	6	No
4	5	5	5	15	Yes
5	3	2	3	8	No
Ave	**3.6**	**3.2**	**3.6**	**52**	

Table 4.3 shows the problem solving performance of Student C of Grade 8. He got a total of 52 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 2 and 4. Student G8C is considered as an Apprentice (Ap) problem solver.

Table 5.1 Problem Solving Performance of Student Y3A

Levels of Understanding					
NRP	Conceptual Understanding	Procedural Knowledge	Problem Solving Skills and Strategies	TOTAL	Correctness
1	5	5	5	15	Yes
2	5	5	5	15	Yes
3	3	3	2	8	No
4	5	5	5	15	Yes
5	5	5	5	15	Yes
Ave	4.6	4.6	4.4	**68**	

Table 5.1 shows the problem solving performance of Student A of the Third Year level. He got a total of 68 points and answered 4 out of the 5 NRPs given correctly. He got the correct answers for all the NRPs given except for NRP 5. Student Y3A is considered as an Apprentice (Ap) problem solver and got the second highest total score compared to the other students in this study.

Table 5.2 Problem Solving Performance of Student Y3B

Levels of Understanding					
NRP	Conceptual Understanding	Procedural Knowledge	Problem Solving Skills and Strategies	TOTAL	Correctness
1	3	2	2	7	No
2	5	5	5	15	Yes
3	3	2	2	7	No
4	5	5	5	15	Yes
5	3	2	1	6	No
Ave	**3.8**	**3.2**	**3**	**50**	

Table 5.2 shows the problem solving performance of Student B of the Third Year level. He got a total of 50 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 2 and 4. Student Y3B is considered as an Apprentice (Ap) problem solver.

Table 5.3 Problem Solving Performance of Student Y3C

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	3	2	2	7	No
2	5	5	5	15	Yes
3	3	2	1	6	No
4	5	5	5	15	Yes
5	5	5	5	15	Yes
Ave	**4.2**	**3.8**	**3.6**	**58**	

Table 5.3 shows the problem solving performance of Student C of the Third Year level. He got a total of 58 points and answered 3 out of the 5 NRPs given correctly. He got the correct answers for NRPs 2, 4 and 5.Hence, student Y3C is considered as an Apprentice (Ap) problem solver.

Table 6.1 Problem Solving Performance of Student Y4A

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	5	5	5	15	Yes
2	5	5	5	15	Yes
3	3	2	2	7	No
4	5	5	5	15	Yes
5	3	2	1	6	No
Ave	**4.2**	**3.8**	**3.6**	**58**	

Table 6.1 shows the problem solving performance of Student A of the Fourth Year level. He got a total of 58 points and answered 3 out of the 5 NRPs given correctly. He got the correct answers for NRPs 1, 2 and 4. Therefore, Student Y4A is considered as an Apprentice (Ap) problem solver.

Table 6.2 Problem Solving Performance of Student Y4B

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	5	5	5	15	Yes
2	3	3	2	8	No
3	3	3	2	8	No
4	5	5	5	15	Yes
5	3	2	1	6	No
Ave	**3.8**	**3.6**	**3.0**	**52**	

Table 6.2 shows the problem solving performance of Student B of the Fourth Year level. He got a total of 52 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 1 and 4. Student Y4B is considered as an Apprentice (Ap) problem solver.

Table 6.3 Problem Solving Performance of Student Y4C

Levels of Understanding					
NRP	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
1	3	3	2	8	No
2	5	5	5	15	Yes
3	3	2	2	7	No
4	5	5	5	15	Yes
5	3	2	1	6	No
Ave	**3.8**	**3.4**	**3.0**	**51**	

Table 6.3 shows the problem solving performance of Student C of the Fourth Year level. He got a total of 51 points and answered 2 out of the 5 NRPs given correctly. He got the correct answers for NRPs 2 and 4. Student Y4C is considered as an Apprentice (Ap) problem solver.

To sum it all, there were 2 Proficient (P) Problem Solvers which is the highest level of expertise in this study. One from the Grade 7 level and the other one was from the Third year level. There was no Proficient (P) Problem Solver in

both the Grade 8 and Fourth year level. Also, there were 9 Apprentice (Ap) Problem Solvers. Two Apprentice (Ap) Problem Solvers from the Grade 7 and Grade 8 and Third Year level and three came from the Fourth Year level. Lastly, there was 1 Novice (N) Problem Solver that came from the Grade 8 level. There was no Novice (N) Problem Solver from the Grade 8, Third and Fourth Year level.

Analysis on the Problem Solving Performance

Student G7A got the highest score of 69 points as shown in table 4.1 and answered 4 out the 5 NRPs given correctly. Student G7A got the correct answer for NRPs 1 to 4 except NRP 5. It only shows that this student really loves mathematics and enjoys solving problems. He also added that he enjoys challenging problems in Math and loves to memorize formulas. He also like reading books and once had a tutor way back 2008. He also competed during his elementary days in the prestigious International Math and Science Olympiad (IMSO) Division level, Math-rathon and Speed Math Competition. Student A of the third year level got the second highest score of 68 points as shown in table 6.1 and also answered 4 out the 5 NRPs given correctly just like Student G7A. While Student G7A didn't get the correct answer for NRP 5, student Y3A got the correct answer for NRP 5 but failed to get the correct answer for NRP 3. Unlike Student G7A, Student Y3A did not compete in any other competition except the

MTAP Math Challenge but had trainings in MTAP Talented 7-Saturday sessions. Also Student Y3A believed that Math makes complicated things simpler. The common denominator among the two top scorers is that both like to read and they really enjoy doing math and solving problems. Obviously, it shows that one's experiences in different competitions, trainings and loving what one is doing especially one's problem solving, add to having Advanced (Ad) Problem Solvers.

On the other hand, the lowest scorer among students was Student A of the Grade 8 level who got a score of 36 points. Student G8A got the correct answer for NRP 3 only. This year was Student G8A's first time to join and be included in a Math competition like the MTAP Math Challenge. Surely trainings and experience in competitions are factors for his performance in the 5 NRPs.

Table 7 Average Score of Students per Grade/Year Level

Levels of Understanding	
Grade/Year Level	**Average Score**
G7	54.33
G8	46.33
Y3	58.67
Y4	53.33

Table 7 shows the average score of the students in each grade/year level. The Third year level got the highest average score since one of them was a

Proficient (P) Problem solver who got the second highest score of 68 points and two were Proficient (P) Problem solvers with 50 and 58 points respectively. The Grade 7 level got the second highest average score since one of them was also Proficient (P) Problem solver who got the highest individual score of 69 points. However, they placed second because of the score one of their two Apprentice (A) Problem solvers who got a score of 44 points and the other 50 points. The fourth year level placed third with an average of 53.33 while the Grade 8 level ranked fourth since one them was a Novice (N) Problem solver who got a score of 36 points. The 5.33 point difference between the third year and fourth year level's average scores showed that the amount of time spent in high school does not affect the problem solving performance of the students in solving NRPs. This was also the case of the grades 7 and 8 levels. Again, some factors can be considered. One of which is that in general, these students were not exposed to these kinds of questions specially NRPs but some may have exposures due to their previous experiences in joining other Math competitions and in terms of reading habits of these students. Among the four levels, the third year level students performed best in solving the 5 NRPs.

Table 8 Performance of each Grade/Year level
in the Three Levels of Understanding

Levels of Understanding					
Grade/Year Level	**Conceptual Understanding**	**Procedural Knowledge**	**Problem Solving Skills and Strategies**	**TOTAL**	**Correctness**
G7	3.93	3.53	3.40	10.86	8 out of 15
G8	3.27	2.93	3.07	9.27	5 out of 15
Y3	4.2	3.87	3.67	11.74	10 out of 15
Y4	3.93	3.60	3.20	10.73	7 out of 15

Table 8 shows the problem solving performance of each grade/year level in the three levels of understanding. A score of 5 points is advanced, 3 points is proficient and 1 point is apprentice. The third year level earned the highest average in all the three levels of understanding and answered 10 out of 15 correct responses. They were between the advanced and proficient levels in Conceptual Understanding and proficient in the Procedural Knowledge and Problem Solving Skills and Strategies. The Grade 7 level got the second average in all the levels of understanding and answered 8 out of the 15 correct responses. The third year and grade 7 levels were followed by the fourth year level while the grade 8 level earned the lowest average in all the three levels of understanding in this study. The grade 8 students were between the apprentice and proficient levels in Conceptual Understanding and Procedural Knowledge and Problem Solving Skills and Strategies while got the lowest average and level

in the Procedural Knowledge. The third year level garnered the highest level of expertise in solving NRPs in this study.

CHAPTER V

SUMMARY, CONCLUSIONS, and RECOMMENDATIONS

This chapter represents the summary of findings, conclusions derived from the study and recommendations based on the results of the study.

Summary

The study was conducted to identify the nonroutine problem solving heuristics of selected students in San Pablo City Science High School (SPCSHS) with the intention to compare and analyze the consistency of Krulik and Rudnick's 5-Stage Heuristic Model (1996) against the solutions and answers of the students in the NRPs. Likewise, the study sought to classify the students into three level of expertise and compare the students' problem solving performance both individually and as a group (by grade/year level). The participants involved were the top 3 students who have represented the school in various mathematics contests per grade/year level. The study is descriptive in nature because the study requires an analysis of heuristics in solving nonroutine problems.

Data needed in this study were gathered by conducting an examination. A preliminary questionnaire was given to each student to get their profile. Five nonroutine problems were solved by each student. After solving each nonroutine

problem, each student was asked to make a journal of his/her reflection on how he/she solved the problem. The journal includes guide questions to help the students focus on their reflection. Analysis of data was made using the Oregon Mathematics Problem Solving Rubrics, (Oregon Department of Education 1991).

Findings

Based on the presentation and interpretation of data, the following findings were drawn:

1. All the respondents are within the ages twelve (12) to sixteen (16) years old. All students were in good condition upon solving the 5 NRPs.

2. Thirty (30) out of the 60 solutions to the 5 NRPs of 12 students or 50% showed consistency with Krulik and Rudnick's 5-Stage Heuristic Model (1996) except that the *e*xtending *part* of the 5th stage was not fully or clearly seen in their solutions but shows evidence of the *checking part*. On the other hand, 30 solutions followed the first four stages only and did not show the checking of answer(s) which is part of the 5th stage Reflect and Extend.

3. Seven out of the eight possible strategies were used solve the 5 NRPs with an average of four strategies per student. Compute and Simplify (CS) was

the most frequently used strategy followed by Guess, Check and Revise (GCR). Eliminate (E) was the least used strategy followed by Use a Formula (F).

4. After using the Oregon Mathematics Problem Solving Rubric in evaluating the student's problem solving skills, the result showed that out of the twelve (12) students, two (2) were considered as a proficient problem solver, nine (9) were apprentice problem solver and one (1) was a novice problem solver. Among the four year levels, the third year level performed the best.

Conclusions

In the light of the given findings of the study, the following conclusions were drawn:

1. One student from the grade 7 and another from the third year level were considered proficient problem solvers which indicate that the number of years in school or year level is not directly proportional to the performance or level of expertise of a student in this study. It doesn't necessarily mean that a higher year level will have more expert problem solver against the lower year level whether individually or as a group. This was also verified since the third year level performed best in the 5 NRPs given followed by the grade 7 level.

2. Factors such as (a) attitude towards Math just like in the case of student G7A and Y3A since they really enjoyed challenging math problems; (b) trainings and (c) exposures to various Math competitions like in the case of G8A who only competed this year and student G7A who had experienced competing in the IMSO and others; and (d) even hobbies and (e) parents' support like G7A and Y3A who loves reading books and are given tutors and awards for their performance in school should be considered which in one way or another may have an effect on the problem solving performance of these students.

Recommendations

In view of the findings and conclusions obtained in this study, the researcher proposed the following recommendations:

1. First, in the problem solving process, checking of answer should be given more emphasis. In this study, only 50% of the solutions showed the *checking part* of the 5th stage Reflect and Extend (Krulik and Rudnick 1996). According to Krulik and Rudnick (1996), problem solving is regarded as one of the primary skills that students must take with them when they leave the classrooms and enter the real world. Henceforth, checking of answer should be included in that process and is recommended to be given more emphasis among

students. Also, the *extending part* is also necessary for further enrichment. As a result, this will not only affect the student's problem solving skills but more so in developing their creative thinking.

2. Second, the heuristics or strategies used in solving problems specifically nonroutine vary from one student to another depending on the kind of situation that particular NRP will give and impress to the students. What is essential is that whatever strategy a student used, at the end of the problem solving process, he should have arrived at the correct answer. The heuristics or strategies used in solving the type of problem in this study are really helpful in the problem solving process of the students. Hence, it is also recommended that students should be taught about these strategies and when to apply them. If students are aware and were trained in using these strategies, problem solving can be fun and interesting and not a boring part of Mathematics learning.

3. The third recommendation is that in this study, it was seen that one nonroutine problem (NRP) elicits an average of four strategies among students. Hence, NRPs not only develops the problem solving skills of the students but also promotes critical thinking in analyzing the situation in a given problem. This also helps the students in their problem solving as whole. Unlike routine problems, NRPs promotes higher order thinking skills and not just an application

of an algorithm and develops critical thinking among students. The SPCSHS doesn't explicitly require the use of NRPs during discussions so students are not exposed to this kind of problems. Also the mathematics curriculum of the SPCSHS focuses more on routine problems as prescribed in the learning competencies (PSLC) and may have little or zero use of NRPs. Thus, it is recommended that there should be the inclusion of NRPs in the curriculum or one that opens opportunities to students to be exposed in nonroutine problems. Also, students should not be confined to the notion that there is one right way to solve a problem. Thinking beyond is the keyword. It helps students develop their creative thinking and allows them to discover even more. Another recommendation to expose students to NRPs and to problem solving in general is through creating an elective subject that will cater and develop knowledge and skills of students in problem solving specifically nonroutine problems. In this case, math teachers and instructors should be also exposed to this kind of math problems for they are the ones to teach their students. Putting it in simple words, we as teachers, cannot give what we do not have and thus trainings are also required and needed.

4. In line with the third recommendation for the inclusion of NRPs in the curriculum, it is also recommended that the use of NRPs be included in the public secondary schools. Since changes have been made in the national curriculum

because of the K to 12 program, intentional use of NRPs can be made since it will be in line with the twin goals of Mathematics (problem solving and critical thinking) in the basic education levels which includes the secondary school.

5. Lastly, since the students who participated in this study have above average skills in Mathematics and are among the top students who represent the school in various mathematics contests which means they all belong to high-performing group, it is also suggested that a similar study be done with students belonging to a low-performing group. Comparison between heuristics in solving nonroutine problems of high-performing and low-performing students can also be done. Moreover, it is recommended that this study be given ample amount of time, number of participants and use of other instruments to describe the heuristics in solving nonroutine problem to give us a more reliable result. An interview to the respondents can also be done to verify the 5th stage *extending part* against their solutions in the NRPs.

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APPENDICES

Appendix A

STUDENT'S PROFILE

Name: _____ Year Level and Section: _____
Age: _____ Birthday: _____
Address: _____
Father's Name: _____ Occupation: _____
Mother's Name: _____ Occupation: _____
Rate your math ability (10-highest, 1-lowest)
What are your hobbies? _____
How many hours on the average do you sleep? _____
Do you exercise regularly? _____
How many hours (average) do you spend watching TV/movies in a day? _____
Do you play computer games? If so, how many hours (average) in a day? _____
Do you surf the internet? If so, how many hours (average) in a day? _____
Do you enjoy doing math? _____
Are you able to apply math in everyday life? If so how?
_____
_____
Will math be useful to you in the future? How?
_____
_____
Do your parents help/tutor you in your studies? _____
Do you have a tutor in math or in other subjects? If yes, who?
_____
Do you get rewards for high grades? If so, what rewards? _____
Is math one of your favorite subjects? _____
What do you like about math? _____
Do you enjoy solving problems? _____
What type of problems do you find easy to solve? _____
Do you have special trainings on how to solve math problems? _____
What training(s)? _____
Are you an MTAP contestant? _____
Aside from MTAP competition, what other math contest have you participated?
_____

Appendix B

NONROUTINE PROBLEMS

NRP 1

Madel invited her closest friends in her 18^{th} birthday. In a game at her party, her friends are forming equal teams to play a game. When her friends form groups of 2, 3, 4, 5, or 6, there is always exactly one person left. Given the smallest possible number of Madel's friends that could be at the party, how many handshakes would there be if after the game everyone shakes hands with everyone else?

NRP 2

Fernando Amorsolo was born several years after Jose P. Rizal published his novel Noli Me Tangere. He became the first National Artist in Painting in our country in the year 1972. If he was *m* years old in the year m^2, *h*ow old was he in 1960?

NRP 3

A new airline is beginning flights next week. In the preliminary instructions, all flight attendants are told that they must wear a different outfit every day. Cristina, one of the new flight attendants, has three times as many blouses as pairs of slacks, and twice as many colorful scarves as blouses. How many blouses, scarves, and pairs of slacks must Cristina own in order to be able to wear a different outfit every day for at least three years?

NRP 4

A multiple-choice test contains 40 items. A correct response earns 5 points, but 2 points are deducted for every incorrect response. Nothing is given nor deducted if the question is unanswered. Joshua scored 96. How might he have done this?

NRP 5

Here are the current taxi rates in a city:

TAXI RATES
P40.00 for the minimum fare
P13.60 for each kilometer
P1.75 for each minute of waiting
P10.00 additional after 6:00pm
P10.00 for each additional passenger
10% increase for heavy traffic

Mrs. Villareal took a taxi from the NAIA airport to his hotel in Makati at 10:11pm. An accident blocked the highway for 10 minutes, but otherwise the trip went smoothly. Mrs. Villareal gives the driver P260.00, which included a tip of seven pesos more than three times the fare for each kilometer. How far was the hotel from the airport?

Appendix C
JOURNAL

1. Below are the list of heuristics and strategies commonly used in solving problems. Which of the following did you use in solving the non-routine problem? Tick (/) at least one box.

☐ Compute or Simplify	☐ Consider a Simpler Case
☐ Use a Formula	☐ Eliminate
☐ Make a model or Diagram	☐ Look for Patterns
☐ Make a Table, Chart, or List	☐ Others. Pls. Specify:
☐ Guess, Check, and Revise	_____

2. How did you solve the problem or what steps did you take in answering the non-routine problem? (Write all the things that entered your mind and concerning the non-routine problem from the time you started doing it.)

Appendix D

INSTRUMENT VALIDATION FORM

Please check (/) the column that indicates your level of agreement for each statement:

SA - Strongly Agree

A - Agree

DA - Disagree

SD - Strongly Disagree

	SA (4)	A (3)	DA (2)	SD (1)
1. The problem is free from grammatical error(s).				
2. The problem uses vocabulary appropriate to students' reading level.				
3. The problem promotes HOTS (higher order thinking skills)				
4. The problem requires heuristics (strategies) in order to arrive at the correct answer.				
5. The problem requires critical thinking and tests multiple skills				
6. The problem requires more time for analyzing and solving				
7. The problem allows multiple solutions				
8. The problem presents a real-life scenario/situation				
9. The problem is not usually found in textbooks				
10. The problem is definitely a non routine problem.				

Validated by:

_____

Appendix E Frequency of the Problem Solving Strategy of each Student						
GRADE 7A	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1	1	1	1	**5**
Use a Formula F	1		1			**2**
Make a Model or Diagram MD		1			1	**2**
Make a Table, Chart or List TCL			1	1		**2**
Guess, Check, and Revise GCR		1	1	1	1	**4**
Consider a Simpler Case SC			1		1	**2**
Eliminate E		1	1	1		**3**
Look for Patterns LP						**0**
GRADE 7B	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS		1	1	1	1	**4**
Use a Formula F	1					**1**
Make a Model or Diagram MD				1		**1**
Make a Table, Chart or List TCL				1		**1**
Guess, Check, and Revise GCR		1	1	1	1	**4**
Consider a Simpler Case SC	1				1	**2**
Eliminate E						**0**
Look for Patterns LP						**0**

GRADE 7C	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1	1		1	**4**
Use a Formula F	1					**1**
Make a Model or Diagram MD		1	1	1		**3**
Make a Table, Chart or List TCL						**0**
Guess, Check, and Revise GCR			1	1	1	**3**
Consider a Simpler Case SC					1	**1**
Eliminate E						**0**
Look for Patterns LP				1		**1**
GRADE 7 SUMMARY	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	2	3	3	2	3	**13**
Use a Formula F	3	0	1	0	0	**4**
Make a Model or Diagram MD	0	2	1	2	1	**6**
Make a Table, Chart or List TCL	0	0	1	2	0	**3**
Guess, Check, and Revise GCR	0	2	3	3	3	**11**
Consider a Simpler Case SC	1	0	1	0	3	**5**
Eliminate E	0	1	1	1	0	**3**
Look for Patterns LP	0	0	0	1	0	**1**

GRADE 8A	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1		1	1	**4**
Use a Formula F	1					**1**
Make a Model or Diagram MD		1	1			**2**
Make a Table, Chart or List TCL						**0**
Guess, Check, and Revise GCR	1		1	1	1	**4**
Consider a Simpler Case SC		1			1	**2**
Eliminate E			1			**1**
Look for Patterns LP	1	1	1		1	**4**
GRADE 8B	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1			1	**3**
Use a Formula F	1					**1**
Make a Model or Diagram MD			1	1		**2**
Make a Table, Chart or List TCL						**0**
Guess, Check, and Revise GCR		1	1	1		**3**
Consider a Simpler Case SC					1	**1**
Eliminate E						**0**
Look for Patterns LP						**0**

GRADE 8C	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS		1		1	1	**3**
Use a Formula F	1		1			**2**
Make a Model or Diagram MD			1	1	1	**3**
Make a Table, Chart or List TCL	1			1		**2**
Guess, Check, and Revise GCR						**0**
Consider a Simpler Case SC	1					**1**
Eliminate E					1	**1**
Look for Patterns LP	1	1	1	1		**4**
GRADE 8 SUMMARY	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	2	3	0	2	3	**10**
Use a Formula F	3	0	1	0	0	**4**
Make a Model or Diagram MD	0	1	3	2	1	**7**
Make a Table, Chart or List TCL	1	0	0	1	0	**2**
Guess, Check, and Revise GCR	1	1	2	2	1	**7**
Consider a Simpler Case SC	1	1	0	0	2	**4**
Eliminate E	0	0	1	0	1	**2**
Look for Patterns LP	2	2	2	1	1	**8**

THIRD YEAR A	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1		1	1	**4**
Use a Formula F	1					**1**
Make a Model or Diagram MD	1				1	**2**
Make a Table, Chart or List TCL			1	1		**2**
Guess, Check, and Revise GCR		1		1		**2**
Consider a Simpler Case SC					1	**1**
Eliminate E						**0**
Look for Patterns LP						**0**
THIRD YEAR B	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1		1	1	1	**4**
Use a Formula F						**0**
Make a Model or Diagram MD		1				**1**
Make a Table, Chart or List TCL						**0**
Guess, Check, and Revise GCR	1	1			1	**3**
Consider a Simpler Case SC					1	**1**
Eliminate E						**0**
Look for Patterns LP						**0**

THIRD YEAR C	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1	1	1	1	**5**
Use a Formula F	1					**1**
Make a Model or Diagram MD		1	1			**2**
Make a Table, Chart or List TCL		1		1	1	**3**
Guess, Check, and Revise GCR		1		1		**2**
Consider a Simpler Case SC					1	**1**
Eliminate E		1		1	1	**3**
Look for Patterns LP	1					**1**
THIRD YEAR SUMMARY	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	3	2	2	3	3	**13**
Use a Formula F	2	0	0	0	0	**2**
Make a Model or Diagram MD	1	2	1	0	1	**5**
Make a Table, Chart or List TCL	0	1	1	2	1	**5**
Guess, Check, and Revise GCR	1	3	0	2	1	**7**
Consider a Simpler Case SC	0	0	0	0	3	**3**
Eliminate E	0	1	0	1	1	**3**
Look for Patterns LP	1	0	0	0	0	**1**

FOURTH YEAR A	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS			1			**1**
Use a Formula F	1					**1**
Make a Model or Diagram MD						**0**
Make a Table, Chart or List TCL		1	1	1		**3**
Guess, Check, and Revise GCR		1	1	1	1	**4**
Consider a Simpler Case SC					1	**1**
Eliminate E						**0**
Look for Patterns LP	1		1			**2**
FOURTH YEAR B	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1	1			1	**3**
Use a Formula F	1					**1**
Make a Model or Diagram MD		1		1		**2**
Make a Table, Chart or List TCL	1		1		1	**3**
Guess, Check, and Revise GCR		1	1	1		**3**
Consider a Simpler Case SC					1	**1**
Eliminate E					1	**1**
Look for Patterns LP			1			**1**

FOURTH YEAR C	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	1		1	1	1	**4**
Use a Formula F	1					**1**
Make a Model or Diagram MD				1	1	**2**
Make a Table, Chart or List TCL			1			**1**
Guess, Check, and Revise GCR		1		1		**2**
Consider a Simpler Case SC		1	1		1	**3**
Eliminate E				1		**1**
Look for Patterns LP			1			**1**
FOURTH YEAR SUMMARY	NRP 1	NRP 2	NRP 3	NRP 4	NRP 5	**TOTAL**
PROBLEM SOLVING STRATEGY	**FREQUENCY**					
Compute or Simplify CS	2	1	2	1	2	**8**
Use a Formula F	3	0	0	0	0	**3**
Make a Model or Diagram MD	0	1	0	2	1	**4**
Make a Table, Chart or List TCL	1	1	3	1	1	**7**
Guess, Check, and Revise GCR	0	3	2	3	1	**9**
Consider a Simpler Case SC	0	1	1	0	3	**5**
Eliminate E	0	0	0	1	1	**2**
Look for Patterns LP	1	0	3	0	0	**4**

Appendix F

CERTIFICATION OF EDITOR

Philippine Normal University
The National Center for Teacher Education
Faculty of Arts and Languages

11 March 2014

CERTIFICATION

This is to certify that I have read and edited in its entirety the Seminar paper of Ian A. Villareal entitled " **HEURISTICS IN SOLVING NONROUTINE PROBLEMS OF SELECTED STUDENTS IN SAN PABLO CITY SCIENCE HIGH SCHOOL.**"

Prof. Ma. Concepcion Y. Raymundo

Name and Signature of Editor

Appendix G

CERTIFICATION OF ANTI-PLAGIARISM

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CERTIFICATION

This is to certify the special project of IAN A. VILLAREAL entitled "*Heuristics in Solving Nonroutine Problems of Selected Students in San Pablo City Science High School*" **passed** the authenticity test performed by the Office on March 13, 2014, using Anti-Plagiarism software of the University.

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