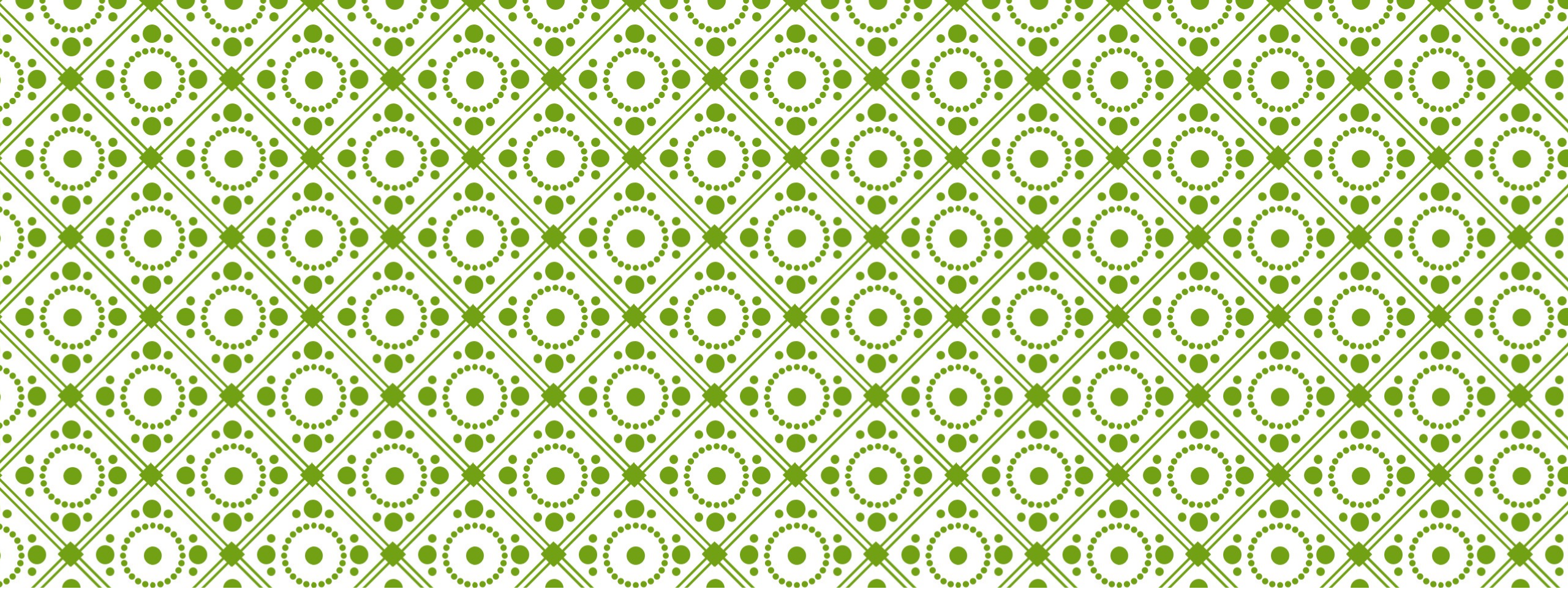




NONPARAMETRIC STATISTICS

Butron

Goal	Type of Data			
	Measurement (from Gaussian Population)	Rank, Score, or Measurement (from Non- Gaussian Population)	Binomial (Two Possible Outcomes)	Survival Time
Describe one group	Mean, SD	Median, interquartile range	Proportion	Kaplan Meier survival curve
Compare one group to a hypothetical value	One-sample <i>t</i> test	Wilcoxon test	Chi-square or Binomial test **	
Compare two unpaired groups	Unpaired <i>t</i> test	Mann-Whitney test	Fisher's test (chi-square for large samples)	Log-rank test or Mantel- Haenszel*
Compare two paired groups	Paired <i>t</i> test	Wilcoxon test	McNemar's test	Conditional proportional hazards regression*
Compare three or more unmatched groups	One-way ANOVA	Kruskal-Wallis test	Chi-square test	Cox proportional hazard regression**
Compare three or more matched groups	Repeated-measures ANOVA	Friedman test	Cochrane Q**	Conditional proportional hazards regression**
Quantify association between two variables	Pearson correlation	Spearman correlation	Contingency coefficients**	
Predict value from another measured variable	Simple linear regression or Nonlinear regression	Nonparametric regression**	Simple logistic regression*	Cox proportional hazard regression*
Predict value from several measured or binomial variables	Multiple linear regression* or Multiple nonlinear regression**		Multiple logistic regression*	Cox proportional hazard regression*

WILCOXON SIGNED-RANKS TEST FOR MATCHED PAIRS

Is a non parametric test that uses ranks for the following applications:

1. Test a null hypothesis that the population of matched pairs has differences with a median equal to zero.
2. Test a null hypothesis that a single population has a claimed value of the median.

WILCOXON SIGNED-RANKS TEST

Objective

Use the test for the following null and alternative hypothesis

H0: The matched pairs have differences that come from a population with a median equal to zero

H1: The matched pairs have differences that come from a population with a nonzero median

Notation:

T = the smaller of the following sums:

1. Sum of the positive ranks
2. Absolute value of the sum of the negative ranks

WILCOXON SIGNED-RANKS TEST

Requirements

1. The data consist of matched pairs that are a simple random sample
2. The population of differences (found from the pairs of data) has a distribution that is approximately symmetric

Test Statistic

If $n \leq 30$, the test Statistic is T

If $n > 30$, the test Statistic is $\frac{T - n(n-1)/4}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$

$$z = \frac{n(n+1)(2n+1)}{24}$$

WILCOXON SIGNED-RANKS TEST

Critical value

1. If $n \leq 30$, the critical T value is found in a table
2. If $n > 30$, the critical z values are also found in tables

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68
26	110	98	84	75
27	119	107	92	83
28	130	116	101	91
29	140	126	110	100
30	151	137	120	109

Source: Adapted from McComack, R. L. (1965). Extended tables of the Wilcoxon matched pair signed rank statistic. *Journal of the American Statistical Association*, 60, 864–871. Reprinted with permission from *The Journal of the American Statistical Association*. Copyright 1965 by the American Statistical Association. All rights reserved.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Note that the probabilities given in this table represent the area to the LEFT of the z-score.
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

For each of nine horses, a veterinary anatomist measured the density of nerve cells at specified sites in the intestine. The result for site I and site II are given in the table below. Each density value is the average of counts of nerve cells in five equal sections of tissue. The null hypothesis of interest is that in the population of all horses there is no difference between the two sites:

Table 1. Nerve Cell Density of Each of Two Sites

Animal	Site 1	Site 2
1	50.6	38
2	39.2	18.6
3	35.2	23.2
4	17	19
5	11.2	6.6
6	14.2	16.4
7	24.2	14.4
8	37.4	37.6
9	35.2	24.4

WILCOXON SIGNED-RANKS PROCEDURE

1. For each pairs of data, find the difference d by subtracting the second value from the first value. Disregard any pairs that have a difference of zero
2. Ignore the signs of the difference, then sort the differences from lowest to highest and replace the difference with the corresponding rank value.
3. Attach to each rank the sign of the difference from which it came.
4. Find the sum of the ranks that are positive. Find the absolute value of the sum of the ranks that are negative
5. Let T be the smaller of the two sum found in step 4
6. Let n be the number of pairs of data for which the difference d is not 0.
7. Determine the test Statistic and critical values based on the sample size.

WILCOXON RANK SUM TEST

Equivalent to Mann-Whitney U test

It is a nonparametric test that uses ranks of sample data from two independent population to test the null hypothesis that the two independent samples come from populations with equal medians.

WILCOXON RANK SUM TEST

Objective:

Use the Wilcoxon rank-sum test for the following null and alternative hypothesis

H0: The two samples come from populations with equal medians.

H1: The median of the first population is different from the median from the second population

Notation:

N_1 = size of sample 1 R = same as R_1

N_2 = size of sample 2 $\bar{R}$ = mean of the sample R values that is

R_1 = sum of ranks of sample 1 expected when the 2 pop. Have = medians

R_2 = sum of ranks of sample 2 σ_R = standard dev. Of the sample R

WILCOXON RANK SUM TEST

Requirement:

1. There are two independent simple random samples.
2. Each of the two samples has more than 10 values.

Test Statistic:

$$z = \frac{R - \mu_R}{\sigma_R}$$

where

R = sum of the ranks for the smaller sample,

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} \quad \text{and} \quad \sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

WILCOXON RANK SUM TEST

Procedure:

1. Temporarily combine the two samples into one big sample, then replace each sample value with its rank.
2. Find the sum of the ranks of the sample with smaller individual count
3. Calculate the value of the z test statistic.

Interpretation:

Common Critical values	Confidence level	Critical Value
0.90	1.645	
0.95	1.96	
0.99	2.575	

A	B
136	131
146	129
139	127
131	146
137	155
144	122
133	143
144	133
129	128
144	146
130	139
140	136
135	
n=13	n=12
R	R

MANN-WHITNEY U TEST

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

$$Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

Decision Rule: Reject H0 if $U <$ critical value from table

Critical values of the Mann-Whitney U Test

$n_1 \backslash n_2$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0	0
2	—	—	—	—	0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3	—	—	—	—	—	—	—	0	0	0	0	1	1	1	1	1	2	2	2	2
4	—	—	—	—	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
5	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	11	12	13	13
6	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	14
7	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
9	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
13	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	—	—	—	—	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

KRUSKAL WALLIS H TEST

Kruskal-Wallis Formula

$$H = \frac{12}{n(n+1)} \sum \frac{R_i^2}{n_i} - 3(n+1)$$

CHI-SQUARE TABLE

	$P(X \leq x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21

CHEST DECELERATION MEAS. FROM CAR CRASH TEST (IN GRAMS)

Small	Medium	Large
44	41	32
43	49	37
44	43	38
54	41	45
38	47	37
43	42	33
42	37	38
45	43	45
44	44	43
50	34	42
n= 10 R1=	10 R2=	10 R3=