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Error Pattern Analysis of Solving Problems in Probability as Basis for Remedial Intervention

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ERROR PATTERN ANALYSIS OF SOLVING PROBLEMS IN PROBABILITY AS BASIS FOR REMEDIAL INTERVENTION

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by

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APPROVAL SHEET

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ABSTRACT

Title of Thesis	**:**	**"ERROR PATTERN ANALYSIS OF SOLVING PROBLEMS IN PROBABILITY AS BASIS FOR REMEDIAL INTERVENTION"**
Name of Researcher	**:**	**ANDREW J. SIDUCON**
Degree	**:**	**Master of Arts in Education**
Specialization	**:**	**Mathematics Education**
Key Concepts	**:**	**Error Analysis, Newman Analysis of Error, Transformation Error, Probability, Remedial Intervention**
Advisers	**:**	**Dr. Rene R. Belecina** **Dr. Gladys C. Nivera**

The main purpose of the study was to describe and analyze the errors encountered by the students in solving open-ended problems in Probability using Newman Error Analysis. This study used descriptive-qualitative method to analyze the solution given by the students. The researcher developed and validated a diagnostic test in Probability which is composed of 16 items and divided into four subtopics namely: (1) Addition Rule, (2) Complement Rule, (3) Multiplication Rule, and (4) Conditional Probability. The participants of the study are the 219 freshmen students taking up Bachelor of Elementary Education and Bachelor of

Secondary Education in Cavite State University who are enrolled in the subject MATH 6 – Contemporary Mathematics. To determine the representative samples, simple random sampling was used.

The study revealed that out of 64 solutions analyzed using Newman procedure, forty-four (68.75%) could be classified as Transformation errors (T), eleven (17.19%) were Process skill errors (P), six (9.37%) belonged to Carelessness errors (X), two (3.13%) appeared Comprehension errors and the last one (1.56%) was classified as Motivation error (M).

Most of the students showed evidence of Transformation errors in solving problems in probability. It means that the students generally understood the problem but failed to apply the appropriate method to solve it. However, some students knew the appropriate method to solve the problem but could not correctly perform the mathematical procedures of applying Addition Rule, Complement Rule, Multiplication Rule, or Conditional. Most of the students failed to add and subtract fractions correctly. Some showed evidence of misusing data and applying inappropriate formula.

Based on the major findings and conclusions, a three-day remedial intervention was made. This remedial intervention was the product of the writer’s careful analysis of students’ errors using the Newman Error Analysis (NEA) procedure in solving open-ended problems in Probability. This remedial intervention aims to correct Transformation and Process Skill errors which were found to be the most common error of the students in solving problems in Probability. In general, the main goals of this remedial intervention are (1) to give one-on-one/group intensive remedial instruction in the application of rules of probability and (2) to improve the problem-solving performance of students with learning difficulties in the rules of probability. The remedial intervention is composed of the following parts: (1) Focus of discussion, (2) Students’ expected outcomes, (3) Resources, materials and supplies needed, (4) The procedure, and lastly (5) Evaluation.

For recommendation, a definite need for a more comprehensive study of error patterns in probability with larger sample should be made for more conclusive and accurate result. In addition, an experimental research on effective teaching method on how to minimize students’ error in solving probability problems and testing the hypothesis that a student will commit less carelessness error if the exam uses a longer period of time are highly recommended. A case study on students who commit comprehension error in solving problems in probability is also recommended to further explain why they commit such kind of error. An error analysis on addition and subtraction of fraction is also highly suggested. Lastly, further research on the students’ Response to Intervention (RTI) should be made to determine the effectiveness of the remedial intervention.

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CHAPTER I

THE PROBLEM AND ITS BACKGROUND

Introduction

Throughout history, societies have sought to educate their people to produce goods and services, to respond efficiently and productively to their world, and to satisfy their curiosity and aesthetic impulse. To achieve these objectives, people need to acquire knowledge and to think systematically.

Education is defined as those experiences that train, discipline, and develop the mental and physical potentials of the maturing persons. According to Paul Horton, the two most important functions of education are to prepare people to make a living and to help people reach their potentials for personal fulfillment and social contribution. Furthermore, sociologist Emile Durkheim believes that education functions to serve the needs of the society.

Diagnostic assessment is an integral part of decision making process in education. It gives the teacher a picture of his students' strengths and weaknesses in the subject area. It also reveals how much they know about the subject and how much more they still need to know before they pass the course. Furthermore, the results of diagnostic assessment will reveal the sources and nature of students' cognitive difficulty in order to facilitate correction or remediation.

The study of mathematical probability as a subject is very important because of its applicability to real-life scenario. Probability may be traced back in time in a game of chance which we associate with gambling. Today, the study of probability has gone beyond gambling or game of chance. The main goal of studying probability is for the decision-making process in business, in economics, in weather forecasting, and even in education. Thus, proper solving of real-life problems should be one of the main focuses in teaching probability in school.

Noraini and Shiney (2005) suggested that the study of probability is often deceptive; on the surface, it seems close to everyday experience where terms such as *randomness* and *chance* are used by laypeople, the media, rating boards, and athletes to justify one action over another. They further explain that solutions to probability problems are shaped by the interaction between a general intellectual schema, accepted intuitively by the student, and certain specific constrain to the problem. Probability does not consist of mere technical information and procedures leading to solutions. Rather, it requires a way of thinking that is genuinely different from that required by the most school mathematics.

In 2000, the National Council of Teachers of Mathematics recommended that learning statistics with understanding should be the vision of school mathematics. To develop learning environments conducive to understanding of probability, teachers need to be aware of students' difficulties in learning probability concepts. Parallel to the previous statement, Heibert and Carpenter (1992) found that since students learn new concepts by building on what they already know, learning with understanding can be viewed as making connections either within existing knowledge or between existing knowledge and new information.

Noraini and Shiney (2005) advanced that errors and difficulties in probability learning do not arise in a random way. Frequently, it is possible to discover regularities in them, or to find some association with other variables of the task, the subjects, or the present and past circumstances. As Kataoka (2008) revealed, not all probability concepts are easily understood at first, as many of them are rather abstract, requiring caution and skill from the teacher in the conduct of the teaching – learning process, in order to provide pupils with the proper development of their probability reasoning.

The present study tried to find out the errors committed by the students in solving problems in probability and propose a remedial interventions that can help eliminate the said errors. Thus using Newman Error Analysis (NEA) is helpful to answer the research query.

Conceptual Framework

Perhaps the most important goal of mathematics is for the learners to be independent in solving both abstract and real-life numerical problems. Thus, problem solving should be an integral part and focus of the mathematics curriculum to develop the students' understanding of and independence to solve a variety of problems.

According to Kataoko (2008), basic knowledge of probability is essentially important in the foundation of citizen, as it allows them to understand many random facts that happen in their daily lives, in addition to acting together with statistics in the inferential context for making decision.

Noraini and Shiney (2005) discovered that over the last three decades much has been written about students' inability to learn the concepts surrounding judgment under uncertainty but the solution has been scarce. Moreover, they suggested that it is crucial to investigate college students' probabilistic reasoning strategies. They further added that by determining the inaccurate procedures students use for solving problems, the teacher can provide specific instruction besides assisting them in using effective procedures for solving specific types of computations.

Many instructors and teachers of mathematics are aware that their students commit the same, if not, almost similar errors when they try to solve mathematical problems. With this notion, several theories were established to explain the cause and occurrence of errors in mathematics. Two of these theories are the Repair and the Misgeneralization theories.

The Repair Theory was established by Brown and VanLhen (1980). The theory explains that error occurs when students are faced with a difficult situation or unfamiliar feature of a task that leads them to impasse. This impasse or difficulty is resolved by modifying a known procedure and incorrectly applying it to the task. It can also be inferred that students associate a given problem to a previously solved or discussed similar problem but apply the wrong procedure, rule, or principle.

After four years, the Misgeneralization Theory of Sleeman (1984) as quoted by Blando, et al (1989) suggests that some errors are due to the students' inference or associations with several rules which are consistent with a given example or task, but are actually not just the correct rule for the problem at hand. Usually these errors occur during the encoding stage, at which the students is developing hypothesis or analyzing the problem.

Rico and Castro (1983) revealed that error analysis and diagnosis have been under the influence of the predominant ideas in pedagogy and psychology. Thus, this diagnosis could lead to a more effective way of eliminating procedural error committed by the students in solving mathematical problems. Moreover, Noraini and Shiney (2005) confirmed that by determining the inaccurate procedures students use for solving problems, teachers can provide specific instruction besides assisting them in using effective procedures for solving specific type of computations. They further discussed that knowledge about difficulties students face when solving problems will provide vital information to teachers who can then develop better teaching procedures to overcome the students' weaknesses.

Several studies have tried to classify errors in solving different problems in mathematics. Few of these were the studies made by Roberts (1969), West (1971), Knifong and Holtman (1976), Engelhart (1977), Newman (1977), Radatz (1979), Brown and Burton (1980), Fajemidagba (1986), Hadar, Zaslavsky, and Imabr (1987), and Mac Gregor and Stacey (1993).

Clements and Ellerton (1996) offered that students' ability in solving mathematical problems can be observed through Newman's Error Analysis (NEA). This analysis is used to investigate whether a student can apply the required skills in the procedures of solving mathematical problems. This method of analyzing students' error in written mathematical task has been published in many countries in which data from many countries have been reported and analyzed along lines suggested by Newman.

According to Newman (1977), a person wishing to obtain a correct solution to a one-step word problem must ultimately proceed according to the following hierarchy: (1) Read the problem, (2) Comprehend what is read, (3) Carry out a mental transformation from the words of question to the selection of an appropriate mathematical strategy, (4) Apply the process skills demanded by the selected strategy, and (5) Encode the answer in an acceptable form.

Newman used the word "hierarchy" because, according to her, failure at any level of the above sequence prevents problem solvers from obtaining satisfactory solutions (unless by chance they arrive at correct solutions by faulty reasoning).

In 1980, Clements illustrated the Newman procedure with a diagram shown in **Figure 1**. According to Clements, errors due to the form of the question are essentially different from those in the other categories because the source of difficulty resides fundamentally in the question itself rather than in the interaction between the problem solver and the question. This distinction is represented in **Figure 1** under the label "Question Form" placed beside the five-stage hierarchy. Two other categories, namely "carelessness" and "motivation", have also been shown as separate from the hierarchy, although, as indicated, these types of errors can occur at any stage of the problem – solving process. A carelessness error, for example, could be a reading error, a comprehension error, and so on. Similarly, someone who had read, comprehended, and worked out an appropriate strategy for solving problem might decline to proceed further in the hierarchy because of a lack of motivation.

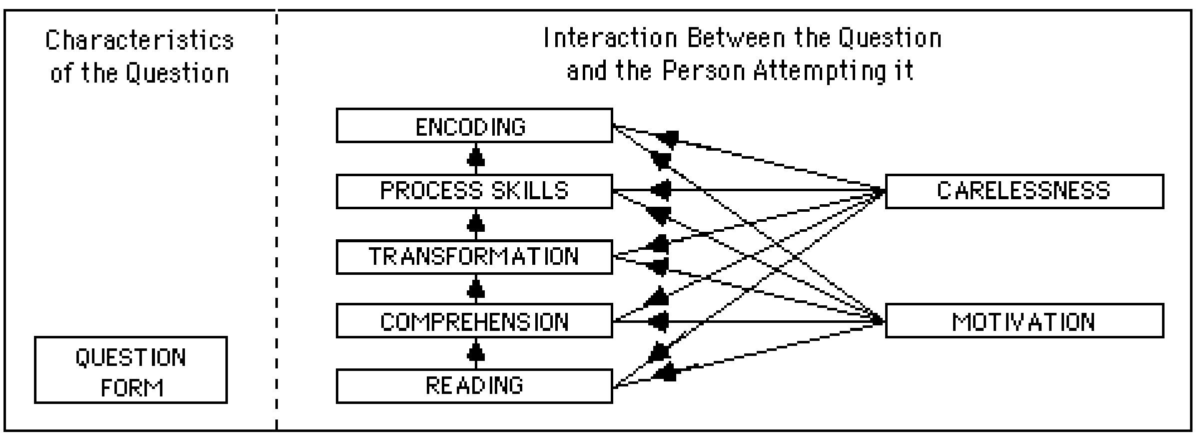


Figure 1: The Newman Hierarchy of Error Causes (Modified by Clements, 1980)

Clarke (1989) pointed out that analyses of data based on the Newman procedure have drawn special attention to (a) the influence of language factor on mathematics learning, and (b) the inappropriateness of many "remedial" mathematics programs in schools in which there is an overemphasis on the revision of standard algorithm.

White (2010) discussed that NEA was designed as a simple diagnostic procedure. He further explained that NEA provides a framework for considering the reasons that underlay the difficulties students experience with mathematical word problems and is a process that assists teachers to determine where misunderstanding occurs. NEA also provides directions for teachers to target effective teaching strategies in overcoming the computational error.

Furthermore, White (2008) confirmed in his paper that the inclusion of NEA has been extremely beneficial in providing teachers with new insights into where and why the students break down in solving word number problems.

This theoretical base was supported by increasing researches anchored in NEA. Some of these studies are those conducted by Casey (1978), Clements (1980), Watson (1980), and Clarkson (1980), Ellerton and Clements (1992), Prakitipong and Nakamura (2006) and, perhaps the most recent, Jha (2012).

Newman Error Analysis is done by means of a structured interview. Noraini and Shiney (2005) found out that examining written solutions to mathematical task to identify different types of errors does not entirely grant the researcher with necessary information on students' thought process and learning strategies. A structured interview will provide more reliable information about their abilities or difficulties in learning and solving given tasks. Similarly, Konold (1991) suggested that such structured interviews provide the opportunity to add further insights into how students think about a particular topic. This information can be used to plan future interventions and monitor conceptual development. Hence using NEA is very useful in determining the patterns of errors the student commit in solving written mathematical problem like those in Probability.

On 2005, White presented a summary of a workshop from a very successful primary teacher professional learning program conducted in Brunei Darussalam. In the paper, he discussed several steps in conducting the Newman Interview. These are the steps. (1) Talk to child in a friendly way. (2) Tell the child you want him/her to do once again some of the problems that he/she did before. (3) Give the child a new answer sheet, and ask him/her to answer a question for which he/she originally gave an incorrect answer. Encourage the child to go on working. Say nothing more until he/she is finished. (4) Ask the child some or all of the five key Newman question or request for the particular question being considered (see below). Do not help the child in any stage, but note any of the child's answers which is especially revealing. (5) Decide on the Newman error classification.

The Newman Error/Mistake Analysis Procedure suggested by White (2005) based on Newman (1977) is as follows:

1. To identify reading error (R): "Read the question to me."
2. To identify comprehension error (C): "Tell me what the problem asked you to do."
3. To identify transformation error (T): "Now tell me what method you used to solve/find the problem/answer."
4. To identify process skill error (P): "Now go over your step of your working, and tell me what you were thinking."
5. To identify encoding errors (E) – "Tell me, what is the answer to the question? Point to your answer."

In addition, White (2005) also discussed the Newman classification of Error which is based on Newman (1977) and Clement (1980):

Reading Error (coded as R) – an error is classified as Reading if the child cannot read a key word or symbol in the written problem to the extent that this prevents him/her from proceeding further along an appropriate problem – solving path.

Comprehension Error (coded as C) – an error is classified as Comprehension if the child is able to read all the words in the question, but cannot grasp the overall meaning of the words and, therefore, is unable to proceed further along an appropriate problem-solving path.

Transformation Error (coded as T) – an error is classified as Transformation if the child understands what the questions want him/her to find out but is unable to identify the operation, or sequence of operations, needed to solve the problem.

Process Skill Error (coded as P) – an error is classified as Process Skill if the child identifies an appropriate operation, or sequence of operations, but does not know the procedures necessary to carry out these operations accurately.

Encoding (coded as E) – an error is classified as Encoding if the child correctly works out the solution to the problem, but cannot express this solution in an acceptable written form.

Carelessness Error (coded as X) – an error is classified as Carelessness if the child attempts to answer the question for the second time and manages to get the correct answer.

Motivation Error (coded as M) – an error is classified as Motivation if the child declines to proceed further in the hierarchy because of a lack of motivation. (For example, a problem-solver may exclaim: "What a trivial problem. It's not worth going any further.")

With these theoretical bases, findings, and thorough review of related literature, the researcher chose probability as the main focus of this paper and the use of NEA that the present study is primarily anchored.

The conceptual paradigm (Figure 2) serves as the basis for analyzing the students' error pattern in solving problems in Probability and some remedial intervention that can be prepared to address these errors.

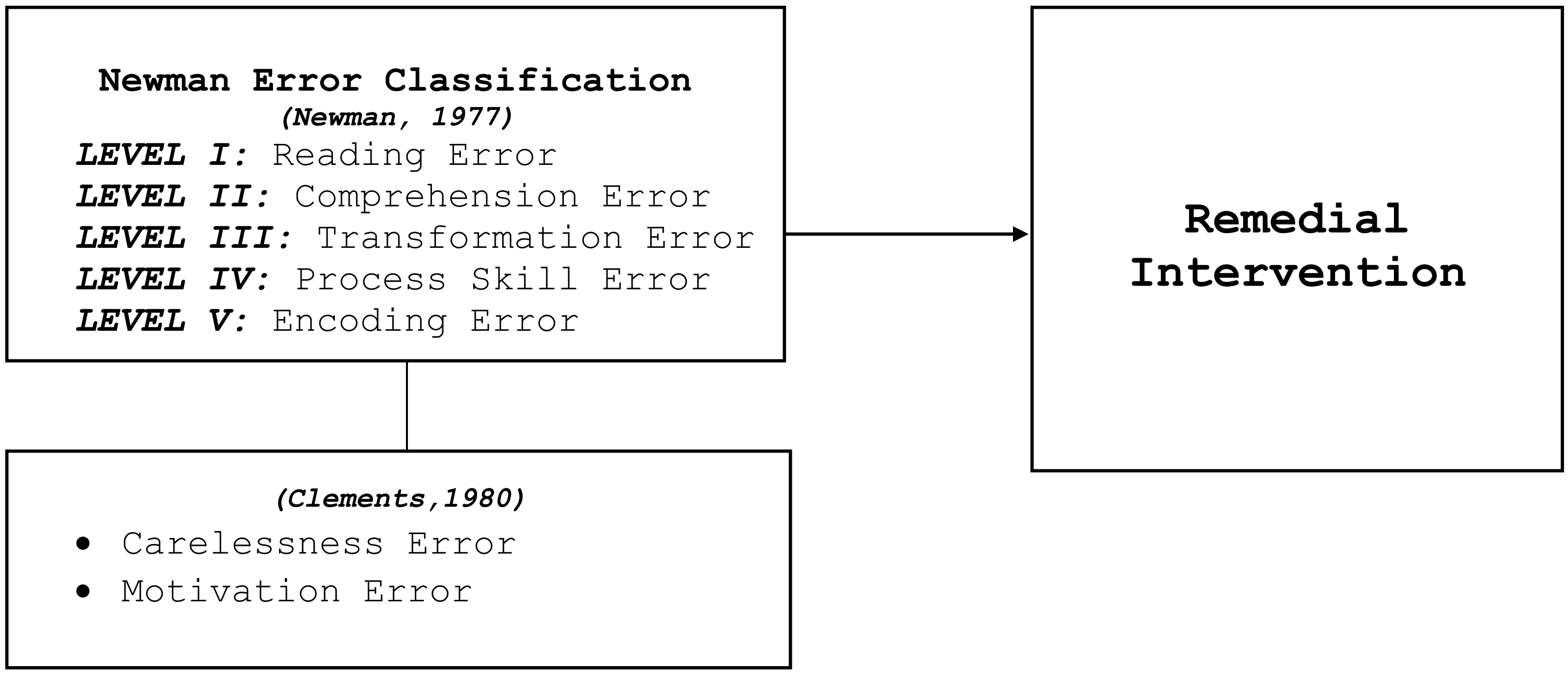


Figure 2. Conceptual Paradigm

Figure 2 illustrates the focus of this study. The students' written solutions and the responses to the interview were analyzed and classified based on the Newman classification of error. However, carelessness and motivation errors were separated because they are not part of the hierarchy of Newman's error but could occur at any stage during the interview. After identifying to what particular errors the students commit in solving-problem in Probability, the researcher prepared a remedial intervention based on this findings.

Statement of the Problem

This study aimed to describe and analyze the errors encountered by the students in solving open-ended problems in Probability.

Specifically, the study tried to answer the following questions:

1. How are the errors of students in solving problems in Probability be described in terms of:

    1.1 Reading Error;

    1.2 Comprehension Error;

    1.3 Transformation Error;

    1.4 Process Skill Error;

1.5 Encoding Error;

1.6 Carelessness Error; and

1.7 Motivation Error?

2. What remedial intervention can be prepared to address errors committed by the students based on Newman's Error Analysis?

Significance of the Study

Nowadays, schools basically function to promote the total development of the students. The administrators, guidance counselors and teacher should exert cooperative efforts to bolster each student's normal development, and to improve their adjustment as they learn formally in schools. This responsibility includes the wholesome development of every student, a better understanding of their behavior, and knowledge of their strengths and weaknesses in their academic subjects such as Mathematics. The outcome of the study will serve as the basis for a remedial intervention.

Hopefully, it will be useful to the following:

Cavite State University administrators. The results of the study may serve as basis of information for the administrators of Cavite State University in designing and implementing intervention program in response to the patterns of error committed by the students in solving problems in Probability.

Mathematics and Statistics instructors. The outcomes of this study may provide Mathematics and Statistics instructors a basis for innovating techniques to minimize students' error in Probability. Plus, by determining the patterns of errors, the instructor can design individualized instruction that will directly address the students' weaknesses and work to solve them.

Students. The findings will benefit the students as they will become aware of the common errors that students like them may encounter when they try to solve probability related problems, thereby reducing the chance that they will commit the same or similar mistake again.

Future researchers. The result of this study can be a basis for other researchers in the conduct of future researches with error patterns in solving problems as their subject matter or modifying teaching strategies in teaching Probability as their main focus.

Scope and Delimitation of the Study

This study focused on describing and analyzing the errors encountered by students of Cavite State University – Don Severino de las Alas Campus in solving open-ended problems in Probability using Newman's Error Analysis. The errors were classified as Reading, Comprehension, Transformation, Process Skill, Encoding, Carelessness, and Motivation.

The topics in Probability that were included in the analysis of data were the Complement Rule, Addition Rule, Multiplication Rule, and Conditional Probability.

The participants of the study were the 219 freshmen in Bachelor of Secondary Education (BSE) and Bachelor of Elementary Education (BEE) of Cavite State University who were enrolled in the subject MATH 6 – Contemporary Mathematics for the second semester of the academic year 2012-2013.

This study was conducted in March 2013, a month after the proposal was approved by the Dean of the College of Science, Dr. Rosemarivic Villena-Diaz.

Definition of Terms

The following terms were used in the study and were operationally defined to serve as reference for the reader.

- **Carelessness error –** An error that is defined with respect to the responses for the twice-administered test (Clements, 1982). In this study, carelessness error referred to a case when a student attempts to answer the same question for the second time and solves it correctly after previously answering it incorrectly.
- **Comprehension error –** An error that occurs if the student succeeds in reading all the words in the question, but fails to grasp the overall meaning of the words and therefore, was unable to proceed further along an appropriate problem-solving path (Newman, 1977). In this study, comprehension error refers to a case when the student can read the problem but cannot explain what the question is about.
- **Encoding error -** An error that occurs if the student works out the solution to the problem, but cannot express the solution in an acceptable written form (Newman, 1977). In this study,

encoding error refers to a case when the student gets the correct answer but fails to label the answer correctly.

- **Error pattern analysis** – An assessment strategy that allows the teacher to understand the students' procedure for solving computation and determine the consistency of their mistake (Assessment Strategy, 2004). In this study, error pattern analysis refers to a diagnostic procedure of identifying the type of error committed by the students in solving open-ended problems in probability using Newman Error Analysis (NEA).
- **Motivation error -** An error that occurs after a student has read, comprehended, and worked out an appropriate strategy for solving a problem but declines to proceed further because of lack of motivation. In this study, motivation error happens when the student does not want to answer or to continue to answer the question which he has previously answered incorrectly.
- **Open-ended questions** – These are questions which require students to show their solutions in order to arrive at an answer. This type may also refer

to a free-response test item. In this study, open-ended questions refer to problems/questions in Probability where the students need to apply the concept of Complement Rule, Addition Rule, Multiplication Rule and Conditional Probability and which require them to show a complete solution.

- **Process Skill error -** An error that occurs when the student can identify an appropriate operation, or series of operations, but does not know the necessary measures to carry out these operations perfectly (Newman, 1977). In this study, process skill error refers to a case when the student cannot correctly perform mathematical process of the application of Addition Rule, Compliment Rule, Multiplication Rule, or Conditional Probability to solve the given problem.
- **Reading error –** An error that occurs when the student fails to read a key word or symbol in the written problem to the extent that it prevents him/her from proceeding further along an appropriate problem-solving path (Newman, 1977).

In this study, reading error refers to a case when the student cannot read a word(s) in the problem.

- **Remedial intervention –** In this study, remedial intervention refers to a structured plan for providing instructional activities to remediate the identified errors in solving problems in probability.
- **Transformation error -** An error that occurs when the student is unable to identify the operation, or series of operation (Newman, 1977). In this study, transformation error refers to a case when the student had understand what the questions want him/her to find out but is unable to identify whether he/she will apply the method of Addition Rule, Compliment Rule, Multiplication Rule, or Conditional Probability.

CHAPTER II

REVIEW OF RELATED LITERATURE

This chapter presents the review of related literature both local and foreign that is relevant to this study. The studies included here pertain to the nature and underlying principle of error analysis in Mathematics education: the error causes, taxonomies and classification, the Newman analysis and some related studies, and the last part is about Probability.

Theoretical Basis of Error Analysis in Mathematics Education

Error in mathematical computation is always an issue among teachers and students and is the likely reason why researches in computational error add up over the past years and continue to grow until the present.

Several theories were established to explain the cause and occurrence of errors in mathematics. Two of these are the Repair and the Misgeneralization theories.

The Repair Theory was established by Brown and VanLhen (1980). The theory explains that error occurs when a student is faced with a difficult situation or unfamiliar feature of a task that leads him to impasse. This impasse or difficulty is resolved by modifying a known procedure and incorrectly applying it to the task. It can also be inferred that students associate a given problem to a previously solved or similarly discussed problem and apply the wrong procedure, rule, or principle.

The Misgeneralization Theory of Sleeman (1984) as quoted by Blando, et al (1989) suggests that some errors are due to the students' inference or associations with several rules which are consistent with a given example or task, but are actually not just the correct rule for the problem at hand. Usually these errors occur during the encoding stage, at which the students is developing hypothesis or analyzing the problem.

In the latter scenario, analysis of errors emerges. White (2005) believes that mistakes can become entrenched, so error analysis is the first step towards doing something relevant which will remove the cause of the mistakes.

Lumpas (1997) notes that error analysis has been widely used by researchers as a means of making inferences about the nature of mental processes in mathematics. Therefore, it is believed that studying mathematics error will greatly help teachers re-align and re-adjust their teaching concern in order to help pupils improve not only their computational abilities but also the problem – solving ability. She further recognizes that a categorization of students' error will truly guide teachers understanding of the computational errors that hinder a students' ability to benefit from instruction.

Ashlock (1986) as cited by Cohen and Spenciner (1998) upholds that the purposes of error analysis in Mathematics are (1) to identify the patterns of errors or mistakes that students make in their work, (2) to understand why students make errors, and (3) to provide targeted instruction to correct errors. When conducting an error analysis, the teacher checks the student's mathematics problems and categorize the errors.

Taxonomy of Errors in Mathematics

West (1971) as cited by Valencia (1981) diagnostically analyzed children's error in addition and subtraction of whole numbers. He identified errors made by children as carelessness and conceptual ones. The carelessness errors are generally spotted as the ones that consistently appear in a child's paper, which means he has an incorrect understanding of the procedure.

However, Roberts (1969) as cited by Ashlock (1986) observed that errors in computation are not just the result of carelessness alone or insufficient drill. He added that some errors are based on faulty or incompletely learned mathematical concepts. The errors of many students follow a pattern because although they learned something, what they have actually learned is incorrect. The patterns include wrong operation, obvious computational error, defective algorithm, and random responses.

Similar to Robert's error patterns (1969), Ashlock (1986) also included Engelhart's 8 (eight) classification of errors (1977) in his paper namely: basic error, defective algorithm, grouping error, inappropriate inversion, incorrect operation, incomplete algorithm, identity error, and zero error.

Meanwhile, Brown and Burton (1980) as cited by Blando (1989) found out "stable errors" among children which they called bugs. The theoretical model for bugs relied on the Repair theory of Brown and Van Lehn (1980).

Blando (1989) further stressed that "bugs" in the context of repair theory were seen as errors that occur at the production stage. He called these errors a Mal-rule. Mal-rules in Mathematics are violations of legal mathematical rules which correspond to Sleeman's Misgeneralization theory.

In 1979, a German educator name Hendrik Radatz argued that if one is to classify errors according to a pupil's individual difficulty, one should acknowledge that errors are also functions of other variables in the educational process – like the teacher, the curriculum, the environment, and the possible interaction among these variables. He explained that errors in the learning of Mathematics are the result of very complex processes, one of which could be the thought process of the child. He gave at least five errors which pupils usually commit in Mathematics problems. They are: (1) error due to difficulties in obtaining spatial information; (2) error due to lack of mastery of prerequisite skills, facts, and concept; (3) error due to incorrect association or rigidity of thinking; and 4) errors due to the application of irrelevant rules or strategies; and (5) errors due to language difficulty.

Language difficulty is also what Mac Gregor and Stacey (1993) focused on in their paper titled "Cognitive Models Underlying Students' Formulation of Simple Linear Equations". They found out that many students have difficulty in formulating algebraic equations from information presented in words. Several students attempt to translate a statement of English into an equation by replacing key words by mathematical symbols sequentially from left to right or by trying to express the meaning of the problem. This finding also confirms the finding of Clarkson (1991) which showed that comprehension errors make up a high proportion of the errors made when the Papua New Guinea students attempted to solve mathematical word problems.

In addition, Mac Gregor (1993) explained that student may consider syntactic translation to be valid, since some textbooks teach students to form number sentences on algebraic equation by matching specific verbal cues to mathematical symbols from left to right.

Aside from language difficulty, reading difficulty is also one of the sources of errors in performing or solving mathematical word problems. In the study of Knifong and Holtsman (1976) as cited by Lumpas (1997), reading difficulty was figured out to be a source of error to which they attribute the 48% non-clerical and non-computational errors. They also hypothesized that reading skills slowed down students so they were unable to complete the test within the given time. This did not relegate reading to a minor role as a source of difficulty. Poor reading skills might have been a factor in 48% of the erred problems of their subjects. Ironically, however, they did not claim that the data confirm or deny that improvement of reading skills would lead to word problem success.

Another study conducted by Hadar, Zaslavsky, and Imbar (1987) which was modeled after the research of Radatz (1979), attempted to classify and group errors into six categories, a popular model for classifying errors particularly in high school Mathematics. The six categories are misused data, misinterpreted language, logically invalid inference, distorted theorem or definition, unverified solution, and technical error.

Clerical and other errors are the categories used by Knifong and Holtman (1976) as they analyzed children's written solution to word problems. They explained that clerical errors had little to do with understanding or word problem solving ability. Moreover, they affirmed that student must have read and understood the problem well enough to succeed in his or her computation.

Meanwhile, Garcia (1994) cited the findings of Lockhead, et. al. (1981) and Clements (1980) in her study on translating expression for verbal problems. Lockhead (1980) discovered that difficulties encountered by the students are quite robust in the sense that they show up consistently in different symbol systems. That is, high percentages of reversal are also observed in translations from pictures to equations, data tables to equation, and equations to sentences. These demonstrate that the reversal error is not simply a result of ordering the words in a problem in a particular way or out of carelessness.

Fajemidagba (1986) in Salman (2002) reported that semantic errors were committed as a result of inadequate understanding of the language of the problem. He further concluded in his study that students at the secondary or university level have great difficulties in solving-word problems in mathematics, and such difficulties resort to poor performance in the subject.

In addition to the above categories of error, Brousseeau in Mulhern (1989) suggested four ways in which error may occur: (1) Errors are result of major misconception about fundamental aspects of mathematics; (2) Frequently, errors occur as a result of a correct and faithful application of a systematically flawed procedure; (3) Errors may occur when pupils used flawed procedure and have misconceptions which are quite unrecognized by the teacher and (4) Pupil may invent their own highly original, non-formal method of solving problems. These are usually the result of "inductive reasoning" processes based on "special cases" of more general problem type and, while often reliable, such methods sometimes give rise to serious errors.

Error Analysis on Specific topics in Mathematics

In the paper presented by Mazungwa and Chimba (2012), they analyzed errors and misconceptions in an undergraduate course in Calculus. The analysis of the result showed that the majority of errors were due to knowledge gaps in basic algebra. They further discussed that errors in Calculus are related to learners' lack of advance mathematical thinking since concept in Calculus are intertwined.

Rulh, Belward and Balati (2011) explored the students' error in mathematics regarding simplifying rational expression. They have classified the errors in simplifying rational expression as (1) simple cancellation error, (2) cancellation by subtraction error, and (3) cancellation by division of coefficients retaining the variable error. The results revealed that the most commonly observed error was simple cancellation error.

Bush (2001) examined the common algebra-related misconception and errors of middle school students. The results showed that basic computational errors were found consistently throughout the responses of the students.

Idris and Narayanan (2011) discussed the types and pattern of systematic errors often made by students in the operation of addition and subtraction of fraction. The findings indicated that errors in addition operation are careless errors, negligence errors, and systematic random error. As for subtraction of fraction, systematic, careless and random errors were manifested.

In a similar graduate study conducted by Ravina (2001) entitled "Computational Errors and Misconceptions in Mathematics of First Year High School Students", she classified student's errors according to patterns, such as basic errors, procedural errors or systematic errors, and evaluation errors. She found out that procedural error is the most common error committed by the students in solving problems in percentage of number, ratio, percentages, and proportion.

Garcia (1999) in his master’s thesis tried to identify and classify the errors committed by freshman students in College Algebra. The findings showed that the errors were due to deficiency in conceptual knowledge, procedural knowledge, problem-structure recognition, a priori conception of facts and rules, and non-structured errors.

A Systematic Exploration of Inaccuracy and Miscalculation in Mathematics: The Newman’s Error Analysis

Australian educator and researcher Sid Bourke affirmed in his paper “Error Patterns in Computation” that computational difficulties experienced by the students have been studied comprehensively in the recent years. He cited the works of Cox (1975), Ashlock (1972) and Lankford (1974) which all focused on classifying, identifying, and analyzing students’ error in performing mathematical task. The last research that Bourke mentioned is the work of another Australian educator named Ann Newman (1977). Newman analyzed both systematic and non-systematic errors made by sixth-grade students in reading, comprehending, processing, and answering a range of mathematical task, and from here, Newman Error Analysis (NEA) came into being.

As Clements and Ellerton (1996) affirmed, students' ability in solving mathematical problems can be observed through NEA. This analysis is used to investigate whether a student can apply the required skills in the procedures of solving mathematical problems. This method of analyzing students' error in written mathematical task has been published in many countries in which data from many countries have been reported and analyzed along lines suggested by Newman.

According to Newman (1983), a person wishing to obtain a correct solution to a one-step word problem must ultimately proceed according to the following hierarchy: (1) Read the problem, (2) Comprehend what is read, (3) Carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy, (4) Apply the process skills demanded by the selected strategy, and (5) Encode the answer in an acceptable form.

Newman used the word "hierarchy" because failure at any level of the above sequence prevents problem solvers from obtaining satisfactory solutions (unless by chance they arrive at correct solutions by faulty reasoning).

Clarke (1989) pointed out that analyses of data based on the Newman procedure have drawn special attention to (a) the influence of language factor on mathematics learning; and (b) the inappropriateness of many "remedial" mathematics programs in schools in which there is an overemphasis on the revision of standard algorithm.

Of course, Casey (1978) as mentioned by Clements and Ellerton (1995) pointed out that problem solvers often return to lower stages of the hierarchy when attempting to solve problems. However, even if some of the steps are revisited during the problem – solving process, the Newman hierarchy provides a fundamental framework for the sequencing of essential steps.

White (2010) stressed that NEA was designed as a simple diagnostic procedure. He further observed that NEA provides a framework for considering the reasons that underlay the difficulties students experience with mathematical word problems and a process that assists teachers to determine where misunderstanding occurs. NEA also provides directions for teachers to target effective teaching strategies to overcome the computational error.

In addition, White (2005) presented a summary of a workshop from a very successful primary teacher professional learning program conducted in Brunei Darussalam. In the paper, he discussed several steps to remember in conducting the Newman Interview. These are: (1) Talk to child in a friendly way; (2) Tell the child you want him/her to do once again some of the problems that he/she did before; (3) Give the child a new answer sheet, and ask him/her to answer a question for which he originally gave an incorrect answer. Encourage the child to go on working. Say nothing more until he/she is finished; (4) Ask the child some or all of the five key Newman questions or requests for the particular question being considered (see below). Do not help the child in any stage, but note any of the child's answers which is especially revealing; and (5) Decide on the Newman error classification.

The Newman Error/Mistake Analysis Procedure suggested by White (2005) based on Newman (1977) include the following:

1. To identify reading error (R): "Read the question to me."
2. To identify comprehension error (C): "Tell me what the problem asked you to do."

3. To identify transformation error (T): "Now tell me what method you used to solve/find the problem/answer."
4. To identify process skill error (P): "Now go over your step of your working, and tell me what you were thinking."
5. To identify encoding errors (E) – "Tell me, what is the answer to the question? Point to your answer."

White (2005) also discussed the Newman classification of Error which is based on Newman (1977) and Clement (1980):

<u>Reading Error</u> (coded as R) – an error is classified as Reading if the child cannot read a key word or symbol in the written problem to the extent that this prevents him/her from proceeding further along an appropriate problem – solving path.

<u>Comprehension Error</u> (coded as C) – an error is classified as Comprehension if the child is able to read all the words in the question, but fails to grasp the overall meaning of the words and, therefore, is unable to proceed further along an appropriate problem-solving path.

Transformation Error (coded as T) – an error is classified as Transformation if the child understand what the questions ask him/her to find out but is unable to identify the operation, or sequence of operations, needed to solve the problem.

Process Skill Error (coded as P) – an error is classified as Process Skill if the child identifies an appropriate operation, or sequence of operations, but does not know the procedures necessary to carry out these operations accurately.

Encoding (coded as E) – an error is classified as Encoding if the child correctly works out a solution to the problem, but cannot express this solution in an acceptable written form.

Carelessnes Error (coded as X) – an error is classified as Carelessness if the child attempts to answer the question for the second time and manages to get the correct answer.

Motivation Error (coded as M) – an error is classified as Motivation if the child declines to proceed further in the hierarchy because of a lack of motivation. (For example, a problem-solver may exclaim: "What a trivial problem. It's not worth going any further.")

White (2005) cited the advice of Clements (2004) in his paper for inexperienced user of Newman Procedure. According to Clements, the interviewer should not help the student in answering the problem. This could lead to wrong classification of error which will fall under “Carelessness Error”.

Clements (2004) also added that sometime during the interview process, a student would know how to do something but would be unable to express that knowledge verbally in an interview. And further he emphasized that in doing the Newman research, it is normal for the student to have answered the written questions twice before being asked the sequence of Newman questions. The first administration of the test usually occurs in a whole-class test situation.

Furthermore, Clements (2004) suggested that the interview should take place as soon as possible after the first test. The second time the student attempts to answer the written questions is usually in the interview situation, before the Newman questions are asked.

Studies on Mathematical Problem-Solving using NEA

The paper presented by Ellerton and Clements (1992) during the Fifteenth Annual Conference of the Mathematics Education Research Group of Australia tried to review and summarize some initial study of errors committed by the students in written mathematical task in Australia, Asia and Papua New Guinea. In Australia, the initial study of Newman (1977) found out that Reading, Comprehension, and Transformation errors made by 124 low-achieving Grade 6 pupils accounted for 13%, 22% and 12% respectively of all the errors made. Thus, almost half of the errors occurred before the application of Process Skills.

Furthermore, studies carried out with primary and junior secondary school in Melbourne, Australia, by Casey (1978), Clements (1980), Watson (1980), and Clarkson (1983) obtained similar results, with about 50% of errors first occurring at the Reading, Comprehension, or Transformation stages. In each study all students were individually interviewed and with the exception of Casey, who helped interviewees over early break-down points to see if they were then able to proceed towards satisfactory solutions, error classification was based on the first break-down point on the Newman hierarchy.

Almost the same results were obtained in the studies made in Asia and Papua New Guinea. It can be said that in each of the studies, over 50% of the initial errors made were in one of the Reading, Comprehension, and Transformation categories. This means that for most errors, students had either not been able to understand the word problems or, if understanding had been present, they had not devised appropriate strategies for solving the given problems. Moreover, since the results show a high proportion of Comprehension and Transformation error, it also suggests that many Asian and Papua New Guinea children have considerable difficulty in understanding and developing appropriate representation of word problems.

Prakitipong and Nakamura (2006) presented a similar study of analysis of the mathematics performance of grade five students in Thailand by using Newman Procedure. And the study revealed that there are many stages that students need to go through in order to arrive at correct answers in mathematical problems. What they found out is similar to the findings of previous researches regarding students' error patterns. They discovered that students' error in multiple choice questions were mostly made at the stage of mathematical processing which is comprised of transformation, process skills, and encoding level, while those in structured questions were made at the stage of the comprehension level. Thus, the researchers called for more attention to the importance of language factors in mathematics learning.

These studies validated the report of Ellerton and Clement (1992) which raises the difficult issue of what educators can do to improve a learner's comprehension of mathematical text or ability to transform, that is to say, to identify an appropriate way to assist learners to construct sequences of operations that will solve a given word problem.

Another study made by Jha (2012) in India attempted to disclose the fundamental reasons of students' poor achievement through analysis of levels of their ability using Newman's Procedure. The data recommended that most of students' error occurred at comprehension as well as the transformation level. He further discussed that good achievers and fair achievers did not make mistake at the reading level, but poor achievers errors were observed at the reading level and mostly at the comprehension level.

On Probability

Noraini and Shiney (2005) pointed out that over the last three decades much has been written about people's inability to learn the concepts surrounding judgment under uncertainty but the solution has been scarce. They also suggested that it is crucial to investigate college students' probabilistic reasoning strategies. They further added that by determining the inaccurate procedures students use for solving problems, a teacher can provide specific instruction besides assisting them in using effective procedures for solving specific types of computation.

Castro (1998) argued that mistakes in calculating probability are due to failures in conceptual understanding and to a lack of adequate cognitive and metacognitive learning strategies. Consequently, Noraini and Shiney (2005) believed that the students lack a solid conceptual understanding of the basic probability concept always ends up in mindless substitution in formulas.

Konold (1991) in his study used the interview method to explain college students’ beliefs in probability. He said that everyday reasoning often relies on reasoning about the probabilities, and that people use a variety of heuristic to judge probability.

Gordon (1995) as cited by Noraini and Shiney (2005) in his study revealed that students make more errors and in particular often apply the wrong formula when the key words *or* and *and* are missing from the probability problems requiring the additive and multiplicative rules, respectively.

Synthesis of Related Literature

Researches in computational error have added up over the past years and continue to grow until the present. Several theories like the Repair (1980) and the Misgeneralization (1984) were established to explain the occurrence of such errors in Mathematics. As a result, authorities and researchers in mathematics education believe that analysis of error in mathematics computation is essential to help the teachers and students understand the nature and probable causes of such inaccuracy in computation (Ashlock, 1986; Lumpas, 1997; White, 2005).

Ashlock (1986) advised that in conducting an error analysis, the teacher should check the student’s mathematics problems and categorize these errors. Some of the local and international studies which tried to categorize errors in solving different problems in mathematics are the studies made by Knifong and Holtman (1976), Engelhart (1977), Newman (1977), Radatz (1979), Brown and Burton (1980), Fajemidagba (1986), Hadar, Zaslavsky, and Imabr (1987), and Mac Gregor and Stacey (1993), Garcia (1999), Bush (2001), Ravina (2001), Idris and Narayanan (2011), Rulh, Belward, and Balati (2011), Mazungwa and Chimba (2012).

White (2009) acknowledge that while there are many other theoretical approaches available for the researchers whose interest is on error analysis, Newman Error Analysis is probably the easiest to use and adapt and has proven popular among teachers for both diagnostic features and classroom pedagogical strategy for remediation. Furthermore, Clements and Ellerton (1996) believed that this analysis is used to investigate whether a student can apply the required skills in the procedures of solving mathematical problems. This method of analyzing students’ error in written mathematical task has been published in many countries in which data have been reported and analyzed along lines suggested by Newman.

This theoretical base was supported by an increasing research anchored on NEA. Some of these studies were those of Casey (1978), Clements (1980), Watson (1980), and Clarkson (1980), Ellerton and, Clements (1992), Prakitipong and Nakamura (2006) and perhaps the most recent, Jha (2012).

The goal of all these cited literature and studies are to identify all the theoretical bases of which this paper is anchored. The present study utilized descriptive and qualitative approach, the same method used by Ellerton and Clements (1992), Prakitipong and Nakamura (2006) and Jha (2012) who all tried to explore the error patterns in solving mathematical problems using NEA. The data analysis and procedure were patterned after the paper of White (2005) and Yeo (2009).

CHAPTER III

METHODOLOGY

This chapter presents the research design, the selection of participants of the study, the instruments for data gathering, the Newman Procedure, and the statistical treatment of data as the basis for determining the error patterns committed by the students in solving problems in Probability.

Research Design

The study used the descriptive method of research as it intended to describe and classify the errors of the students in solving problems in Probability. According to Adanza (1995) descriptive method is designed for the investigator to gather information about the present condition. The main objective of the descriptive study is to describe the nature of the situation as it exists at the time of the study and to explore the causes of a particular phenomenon.

The research is also qualitative in nature because open-ended questions were used to gather data (Key, 1997 as cited by Borja, 2011).

The Research Instrument

The developed and validated diagnostic test in Probability with 20 open-ended problems was used to identify the types of students' error. The final instrument (Appendix A) with 16 items is divided into four subtopics namely: (1) Addition Rule, (2) Complement Rule, (3) Multiplication Rule, and (4) Conditional Probability. The distribution of each topic is shown in the table below:

Table 1
Distribution of Topics

Topic	Item Placement
Complement Rule	1, 2, 4, 5
Addition Rule	6, 7, 8, 9
Multiplication Rule	3, 10, 12, 13
Conditional Probability Rule	11, 14, 15, 16

The instrument was referred to experts who are at least master's degree holder in the field of Mathematics education or Statistics in order to check the validity of the instrument. All inappropriate items were modified as suggested by the experts to suit the purpose of the study. The experts also rated each open-ended item as to the level of agreement using a Likert – scale in Appendix B.

The weighted mean of the responses given by the experts for each item was obtained and interpreted as follows:

Interpretation	**Mean Score**
• Very good item • Very acceptable item	3.26 – 4.00
• Good item • Acceptable item • Consider minor revision	2.51 – 3.25
• Poor item • Unacceptable item • Consider major revision	1.76 – 2.50
• Very poor item • Disregard the item	1.00 – 1.75

To establish reliability, the Test-Retest Method was used. The diagnostic test was administered twice with one week interval to twenty-five (25) freshmen students of Bachelor of Science in Medical Technology who were enrolled in STAT 1 (Elementary Statistics). The correlation coefficient was statistically computed and interpreted using Pearson Product Moment Correlation. The coefficient of $r = 0.852$ suggested that there was a strong correlation and thus, making the test reliable.

The open-ended problems were subjected to a ***Scoring Rubric for Diagnostic Test in Probability*** (Appendix C) which was developed by the researcher. The perfect score for this test is 48 points. The rubric was validated by the three experts who have been teaching Mathematics and Statistics for more than four years. The reliability of the rubric was established using the inter-rater reliability method. The same experts who validated the rubric were asked to independently score the papers of ten students who are included in the first try-out of the instrument. These scores were statistically interpreted using the Kendall Coefficient of Concordance ($p=0.0014$). Since the computed p-value was less than 0.05, then the rubric is said to be very reliable.

Participants of the Study

The participants of the study were the 219 freshmen students taking up Bachelor of Elementary Education and Bachelor of Secondary Education of Cavite State University who were enrolled in the subject MATH 6 – Contemporary Mathematics during the second semester of the Academic Year 2012-2013.

In this study, the students were divided into four groups namely Complement Group, Addition Group, Multiplication Group and Condition Probability Group. The groupings were done for the purpose of sub-analysis which was based on the sub-topics of Probability. To determine the representative samples, sixteen (16) students from each group were randomly chosen using simple-random sampling. The Fish-bowl method had been used to determine the students who underwent the Newman's Procedure. Table 2 shows the distribution of the students who committed an error per item.

Table 2
Distribution of Students Who Committed an Error per Item

Group	**Item number**	***N***
Complement	1	45
	2	60
	4	53
	5	41
Addition	6	39
	7	54
	8	76
	9	106
Multiplication	3	82
	10	106
	12	78
	13	119
Conditional	11	34
	14	67
	15	47
	16	54

Where:

N = Total number of student who committed error per item

The Newman Procedure: Interview and Analysis of Error

Prakitipong and Nakamura (2006) defined the Newman Procedure as a method that analyzes errors in sentence problem. In the process of problem solving, many factors support the students to arrive at a correct answer. This method supposes that in the process of problem solving there are two kinds of obstacles that hinder students from arriving at the correct answers:

1. Problems in linguistic fluency and conceptual understanding that correspond with level of simple reading and understanding meaning of problems, and
2. Problems in mathematical processing that consist of transformation, process skills, and encoding answers.

The two further added that this classification implies that the students have to interpret the meaning of the question in mathematical context before they proceed to mathematical processing to obtain appropriate answer. This procedure can be conducted thru an interview. To identify at which level students' error occur in problem solving. The levels are:

I. **Reading level:** Can the student read the question? (Simple recognition of words and symbols)

II. **Comprehension level:** Can the student understand the meaning of the question? (Linguistic understanding of problems)

III. **Transformation level:** Can the student identify the mathematical operation or procedure? (Transformation from linguistic understanding to mathematical interpretation)

IV. **Process Skill level:** Can the student perform the mathematical calculation or the procedure accurately? (Execution of mathematical processing)

V. <u>**Encoding level:**</u> Can the student interpret the answer appropriately? (Representation of result from mathematical processing)

For instance, the following conversation to describe this method is given in Figure 3. In the transcript, "***I***" stands for interviewer and "***S***" stands for student.

(Problem)
A pizza of twelve slices, cost five hundred twenty eight yen. How much will one piece of pizza cost?

(Process of interview)
I: "Can you read the question?" (Reading level)
S: (Student reads the whole question.)
I: "What does the question ask you to do?" (Comprehension level)
S:"It's asking me to find a piece of pizza, and how much?"
I: "Then, what operation do you work out to find the answer?" (Transformation level)
S: "Using subtraction." ← **(Error occurred at this level.)**
I: "Can you show me your calculation or write it on this paper?" (Process skills)
S: "There are 12 pieces of pizza, so a piece of pizza is 528 – 12 = 516

Figure 3. Sample of Transcribed Interview using Newman Analysis of Error

Based on the interview, the error occurred at the transformation level because the student comprehended what the question was after, but failed in developing an appropriate operation. Thus, the interviewer identified the student's error by using the Newman Procedure.

Clements (1980) added two more categories of error and used them as modified model of Newman's Hierarchy of Error. He made it clear that these two additional errors are outside the hierarchy but they may occur in any of the stages. These two are carelessness error and motivation error. In his study, he defined careless error with respect to responses for the twice-administered MAMP test. The following procedure was adopted for identifying carelessness error:

1. Attention was confined to students who gave correct answers to questions on one occasion but wrong answers on the other; these students were interviewed and the Newman method was employed to probe their thinking about those questions for which they had given one right and one wrong response.

2. If during the Newman interview, a student once again obtained an incorrect answer to a question, then the original error was not classified as caused by carelessness.

3. If, during the Newman interview, a student obtained the correct answer to a question without any assistance from the interviewer, then the error was classified as cause by carelessness provided further probing did not suggest the student was unsure whether the answer was right or wrong. As part of the probing

procedure the student was shown the two original answers and asked which was correct.

Similarly, motivation error is manifested if someone who had read, comprehended, and worked out an appropriate strategy for solving a problem might decline to proceed further in the hierarchy because of a lack of motivation. For example, a problem –solver might exclaim: "What a trivial problem. It's not worth going further".

In this study, the following interview questions/ requests were asked to the interviewee/student:

1. To identify reading error (R), "Please, read the problem to me."

2. To identify comprehension error (C), "Tell me what the problem/question asked you to do/find."

3. To identify transformation error (T), "Tell me what method you used to solve the problem."

4. To identify process skill error (P), "Please, explain to me your solution"

5. To identify encoding error (E), "Tell me your answer to the problem. Please box your final answer."

Data Analysis Procedure

Percentages were used to summarize the data obtained from the interview regarding the students' error in solving problems in probability.

The interviews that were conducted with the 64 students were audio-taped. The following two-point categorizations in behavioral terms for each stage of Newman's Hierarchy of Error were used to analyze the interview protocol which was patterned in the study of Yeo (2009):

Level I. **Reading**

1. The student was able to read the problem.
2. The student was ***not*** able to read the problem.

Level II. **Comprehension**

1. The student was able to comprehend the problem. This was confirmed by the response to the question/request "Tell me what the problem/question asked you to do/find."
2. The student was ***not*** able to comprehend the problem. This was confirmed by the response to

the question/request "Tell me what the problem/question asked you to do/find."

Level III. **Transformation**

1. The student was able to identify the appropriate method (Complement, Addition, Multiplication, or Conditional Probability) to solve the problem.

2. The student was ***not*** able to identify the appropriate method (Complement, Addition, Multiplication, or Conditional Probability) to solve the problem.

Level IV. **Process Skill**

1. The student was able to correctly perform the mathematical procedure of the application of Complement Rule, Addition Rule, Multiplication Rule or Conditional Probability.

2. The student was ***not*** able to correctly perform the mathematical procedure of the application of Complement rule, Addition Rule, Multiplication Rule or Conditional Probability.

Level V. **Encoding**

1. The student was able to identify and label the correct answer to the problem.
2. The student was ***not*** able to identify and label the correct answer to the problem.

Separate from the Newman's hierarchy of errors was Carelessness and Motivation Errors.

Carelessness error: The student correctly answered the same problem in which he/she is previously incorrect.

Motivation error: The student does not want to answer the problem or to continue to answer the problem in which he is previously incorrect.

CHAPTER IV

Presentation, Analysis, and Interpretation of Data

This chapter presents the analysis and interpretation of the gathered data. It also presents the answers to the research questions of this study.

Research Question # 1. How can the errors of students in solving problems in Probability be described in terms of:

1.1 Reading Error;
1.2 Comprehension Error;
1.3 Transformation Error;
1.4 Process Skill Error;
1.5 Encoding Error;
1.6 Carelessness Error; and
1.7 Motivation Error?

a. Complement Group

The Complement Group was composed of 16 students who committed errors in answering problems no. 1, 2, 4 and 5. Table 3 shows the types of error committed by the students in answering problems under the complement rule.

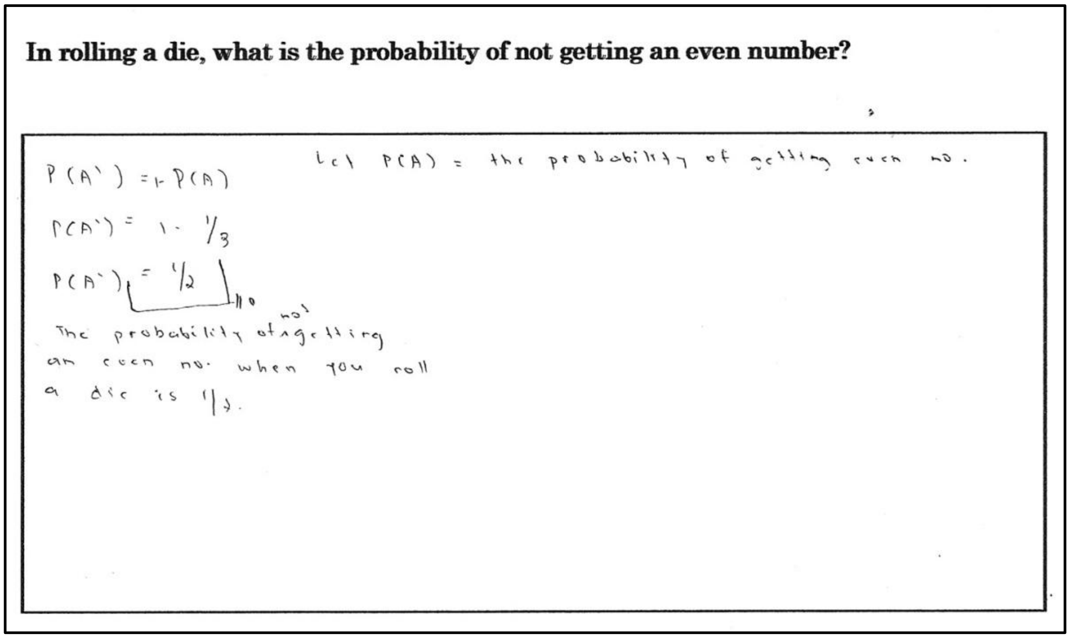
Table 3

Types of Error Committed by the Students in Answering Problems under the Complement Rule

Item Number	**Newman Classification of Error**							Total
	R	**C**	**T**	**P**	**E**	**X**	**M**	
1	0	0	0	3	0	1	0	**4**
2	0	0	3	1	0	0	0	**4**
4	0	0	2	0	0	2	0	**4**
5	0	0	2	2	0	0	0	**4**
Total	**0**	**0**	**7**	**6**	**0**	**3**	**0**	**16**

The data shows that out of sixteen errors comitted by the students, seven (43.75%) were classified as Transformation (T), six (37.5%) were classified as Process Skill (P) while the remaining three (18.75%) were Carelessness (X). A transcript of the interview was given after the student's solution. In the transcript, ***I*** stand for the interviewer and ***S*** for the student.

Figure 4 shows an example of Process Skill error under item number 1.


In rolling a die, what is the probability of not getting an even number?

$P(A') = 1 - P(A)$ Let $P(A)$ = the probability of getting even no.

$P(A') = 1 - 1/3$

$P(A') = \boxed{1/2}$

The probability of not getting an even no. when you roll a die is 1/2.

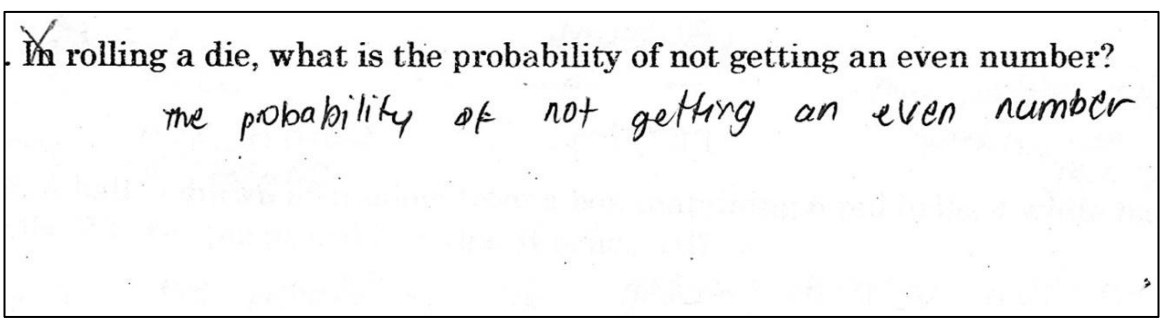
Figure 4. Sample of Process Skill Error in Item no. 1.

The Process Skill error in this solution was verified in the transcript below.

I: Miss ____, please read the problem to me.
S: [The student read Question no. 1 correctly]
I: Tell me what the problem asked you to do.
S: Ano… Hanapin po yung ano… yung kapag ni-roll po yung die, hanapin po yung probability na hindi ka makakakuha ng even number.
I: Tell me the method you used to solve the problem.
S: Complement
I: Please, explain your solution.
S: ***Kasi po yung binigay nyo pong formula is probability of A prime is 1 minus the probability of A. So, yung even number po sa die tatlo, 2,4,6. So base po sa formula, 1 minus 1/3 is ½. So, the probability of not getting an even number in a die is ½ (Error occurred at this stage)***
I: Please box your final answer.
S: [The student boxed her final answer]

As indicated in the transcript, it was evident that the error occured at the Process Skill level. The student was able to read, comprehend, and determine the appropriate method that she needs use for her to answer the problem correctly. Though the correct answer is actually ½, it was not considered as correct since there was an inconsistency in her solution. When the student subtracted 1/3 from 1, her answer was ½ which was mathemetically incorrect. This solution is supported by the statement of Newman (1983) that some student by chance might arrive at correct solutions by faulty reasoning.

The next two figure shows an example of a Carelessness error in item number 1 when the student tried to answer the same problem for the second time under the Complement rule.



. In rolling a die, what is the probability of not getting an even number?

the probability of not getting an even number

Figure 5. Sample of an Incorrect Student's Solution in Item no. 1 during the First Administration of Diagnostic Test

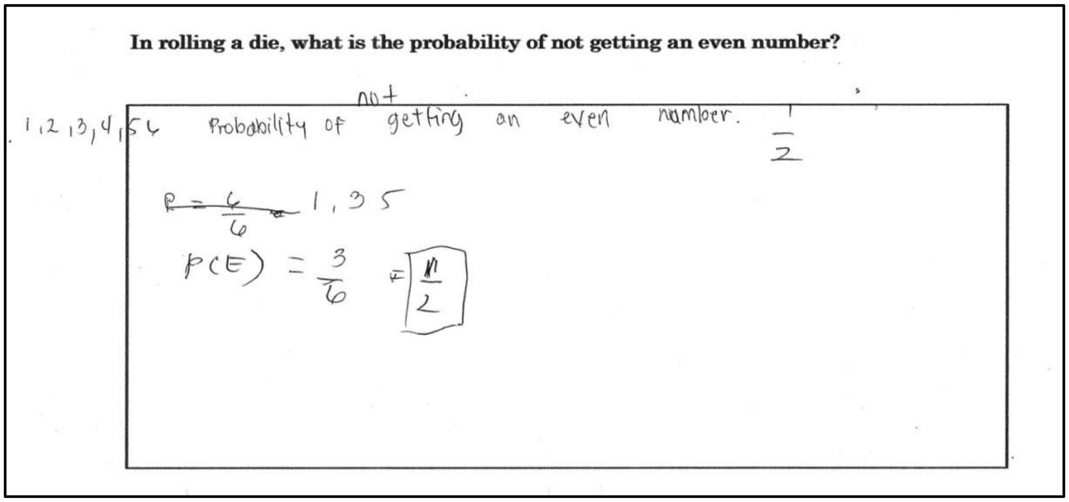


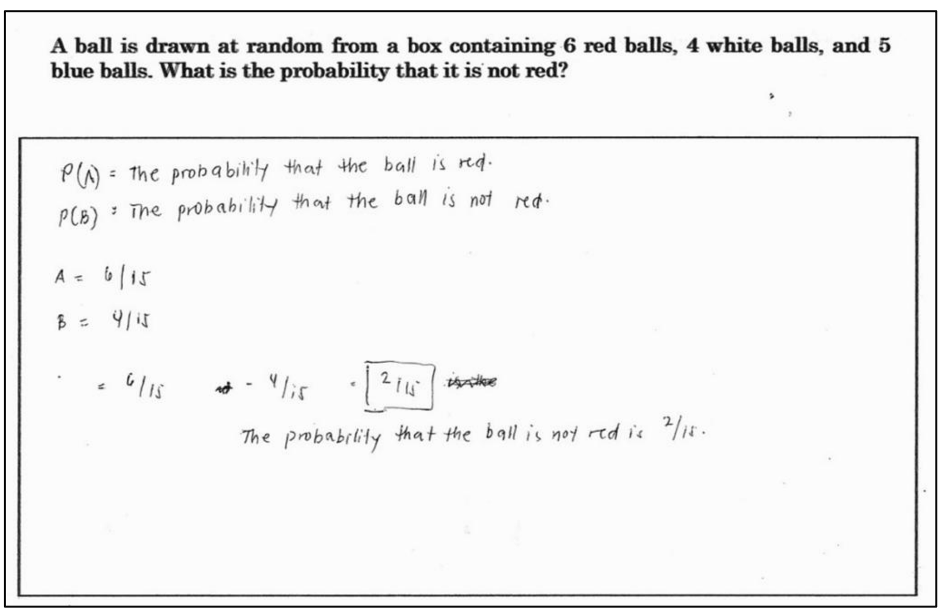
Figure 6. Sample of a Carelessness Error in Item no. 1

In figure 5, there was no solution shown but merely a restatement of the given. The incorrect answer corresponded to a Motivation Error. On the other hand, a correct solution was given by the student in figure 6 when he attempted to answer the same question for the second time. This Carelessness error was confirmed in the transcript below.

I: Please read the problem to me.
S: [The student read Question no. 1 correctly]
I: Tell me what the problem asked you to do.
S: Yung kukunin ko po yung ano… yung not getting an even number po.
I: Tell me the method you used to solve the problem.
S: Multiplication
I: Please, explain your solution.
S: Kinuha ko po yung tatlong not even number po. Tapos po… ayun po i-nover ko po sa 6 tapos ni-lowest term ko po sa ½
I: Tell me your answer to the problem.
S: one-half po.
I: Please box your final answer.
S: [The student boxed her final answer]

In the interview, the student was able to read and comprehend the problem. However, when asked about the method she used to solve the problem, she responded that it was multiplication. Her response was not considered as an error for she was able to explain and got the answer correctly. Hence, the error is classified as Carelessness. This also confirms the statement of Clements (2004) as cited by White (2005) which states that sometimes a student knows how to do something but is unable to express that knowledge verbally in an interview.

Figure 7 shows an example of a Transformation error in item number 2 under Complement Rule of Probability.



A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. What is the probability that it is not red?

P(A) = The probability that the ball is red.
P(B) = The probability that the ball is not red.

A = 6/15
B = 4/15

$= 6/15 - 4/15 = \boxed{2/15}$

The probability that the ball is not red is 2/15.

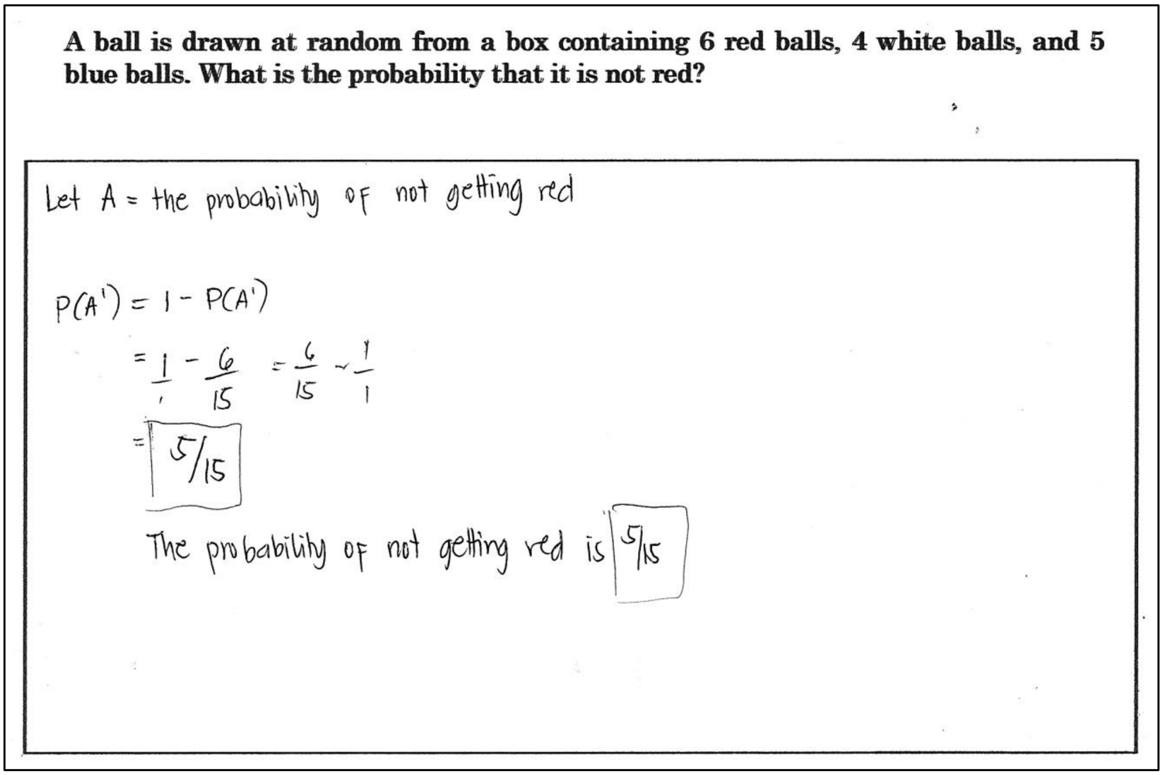
Figure 7. Sample of a Transformation Error in Item no. 2

A transformation error on the previous solution is confirmed in the transcript below.

I: Miss ____, please read the problem to me.
S: [The student reads question number 2 correctly]
I: Tell me what the problem asked you to find.
S: The probability that the ball is not red.
I: Tell me the method you used to solve the problem.
*S: **Addition… nagminus po ako… nag subtract po ako… (Error occurred at this stage)***
I: Can you explain to me your solution?
S: Ito pong A yung 6… yung red balls. Tapos i-nadd ko yung 6, 4 at 5. Yung total ng balls ay 15. Yung B, 4 which is white. Tapos, sinubtract ko po yung 6 sa 4… 2/16
I: What do you think is the answer to the problem?
S: 2/16
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the transcribed interview, the error occurred at the transformation level where the student was asked about the method she used to solve the problem. The student was able to read and comprehend the problem but failed to identify the correct method she used to solve the problem. The student answered “Addition” instead of using Complement rule to solve the problem.

The next figure shows us a Process Skill error under item number 2.



A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. What is the probability that it is not red?

Let A = the probability of not getting red

$P(A') = 1 - P(A')$

$= \frac{1}{1} - \frac{6}{15} = \frac{6}{15} - \frac{1}{1}$

$= \boxed{5/15}$

The probability of not getting red is $\boxed{5/15}$

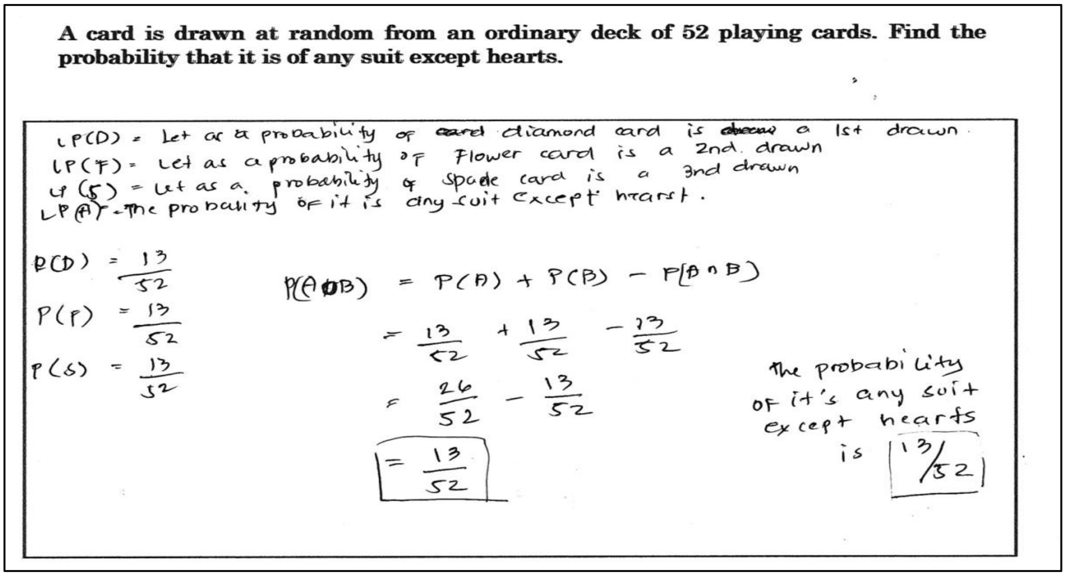
Figure 8. Sample of a Process Skill Error in Item no. 2

This Process Skill error was confirmed during the interview using NEA. The transcript of the interview is given below.

I: Miss _____, please read the problem to me.
S: [The student read Question no. 2]
I: Tell me what the problem asked you to find.
S: The probability that it is not red.
I: Tell me the method you used to solve the problem.
S: The complement of A
I: Please, explain your solution.
S: Let A be equal to the probability of not getting red. Then the formula is the probability of A complement is one minus complement of A. so, 1 minus 6/15 is equal to 5/15. ***(Error occurred at this stage)***
I: Tell me your answer to the problem.
S: The probability of not getting red in 5/15.
I: Okay, please box your final answer.
S: [The student boxed her final answer]

As indicated in the transcript, the student read and understood what the problem is asking her to find. She even gave the right method to use to find the answer. However, when she was asked to explain her solution, it was evident that she committed an error in subtracting fraction since subtracting 6/15 from 1 will give us 9/15 or 3/5 instead of 5/15. Hence, the error was in process skill.

Another example of a Transformation error in Complement Rule of Probability (item number 4) is shown in Figure 9.


A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit except hearts.

LP(D) = Let as a probability of ~~card~~ diamond card is ~~drawn~~ a 1st drawn.
LP(F) = Let as a probability of Flower card is a 2nd. drawn
LP(S) = Let as a probability of Spade card is a 3rd drawn
LP(A) = The probality of it is any suit except heart.

$P(D) = \frac{13}{52}$

$P(F) = \frac{13}{52}$

$P(S) = \frac{13}{52}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{13}{52} - \frac{13}{52}$

$= \frac{26}{52} - \frac{13}{52}$

$\boxed{= \frac{13}{52}}$

The probability of it's any suit except hearts is $\boxed{13/52}$

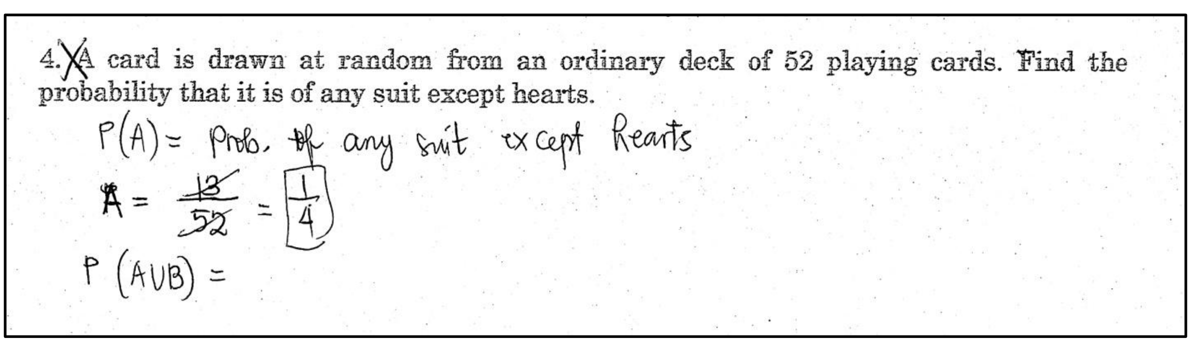
Figure 9. Sample of Transformation Error in Item no. 4

This transformation error is verified in the next transcript.

I: Please read the question to me.
S: [The student reads question number 4 correctly]
I: Tell me what the problem asked you to find.
S: Find the probability that it is of any suit except hearts.
I: Tell me the method you used to answer the problem.
S: ***Addition Rule (Error occurred at this stage)***
I: Can you explain to me your answer?
S: Let the probability of D – diamond in the first draw. And let F be the probability of flower card sa second draw. The probability of S – the probability of spade card in third draw. The probability of A is it is any suit except heart.
I: Can you go over your solution here?
S: The probability of A union B is equal P of A plus P of B minus P of A intersection B. 13/52 plus 13/52 – 13/ 52 is equak to 26/52 minus 13/52 is 13/52 so the final answer is 13/52.
I: What is your answer to the problem?
S: Sir eto po. [The student pointer his finger to his answer.]

As indicated in the transcribed interview, the error also occurred at the transformation level when the student was asked about the method she used to solve the problem. The student was also able to read and comprehend the problem but failed to use the correct method. The student answered “Addition Rule” instead of using Complement Rule to solve the problem.

The next two figures show how a Carelessness error is committed when the student attempted to answer the same question for the second time and answered it correctly after previously giving an incorrect answer. Figure 10 is the student’s solution to item number 4 during the first administration of the diagnostic test and Figure 11 is the solution on the second time the student answered the same problem.

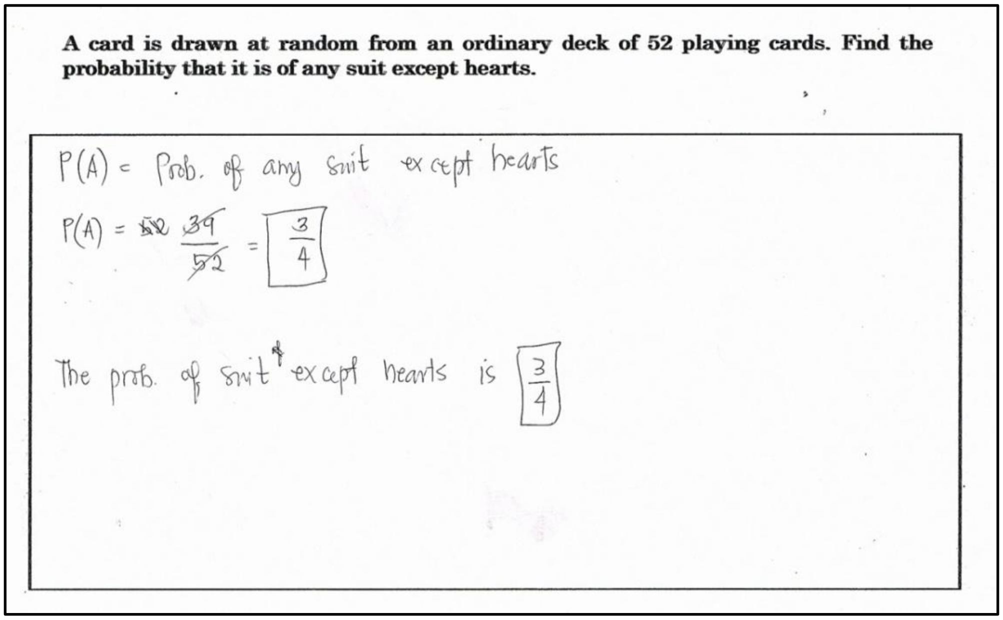

4. A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit except hearts.

P(A) = Prob. ~~of~~ any suit except Hearts

$A = \frac{13}{52} = \boxed{\frac{1}{4}}$

$P(A \cup B) =$

Figure 10. Sample of Incorrect Student’s Solution in Item no. 4 during the First Administration of Diagnostic Test


A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit except hearts.

P(A) = Prob. of any suit except hearts

$P(A) = \cancel{52}\ \frac{39}{\cancel{52}} = \boxed{\frac{3}{4}}$

The prob. of suit except hearts is $\boxed{\frac{3}{4}}$

Figure 11. Sample of Careless Error in item no. 4

This Carelessness error was confirmed during the interview using NEA. The transcript of the interview is given below.

I: Please read the question to me.
S: [The student reads question number 4 correctly.]
I: Tell me what the problem asked you to do.
S: Ilan po yung probability n'ya na makakuha, randomly, na maliban dun sa heart na card.
I: Tell me the method you used to answer the problem.
S: Yung… hindi naman siya addition… pero eto po ang ginawa ko sir… [the student look at her solution]
I: Ano ang ginawa mo? Can you explain to me you solution?
S: So bale kasi sir ang card ay 52 lahat. Ang naka lagay po dito kung kukuhanin nya isa-isa maliban dun sa heart, ilan yung probability. So, i-leless natin yung 13 na hearts na cards. So 39 yung natira. So 39/52 then i-ano s'ya sa 13 kaya ¾.
I: So what is your answer to the problem?
S: ¾
I: Please box your final answer.
S: [The student boxed his final answer]

During the interview, the student was able to read and comprehend the problem. However, when asked about the method she used to solve the problem, she responded that it could not be addition rule but she was able to explain and arrived at the correct answer. Therefore, it was not a Transformation error but rather a clear case of a Carelessness error. This confirms Clements (2003) as cited by White (2005 that sometimes a student knows how to do something but is unable to express that knowledge verbally in an interview.

The next figure will show us an example of a Transformation error in item number 5.

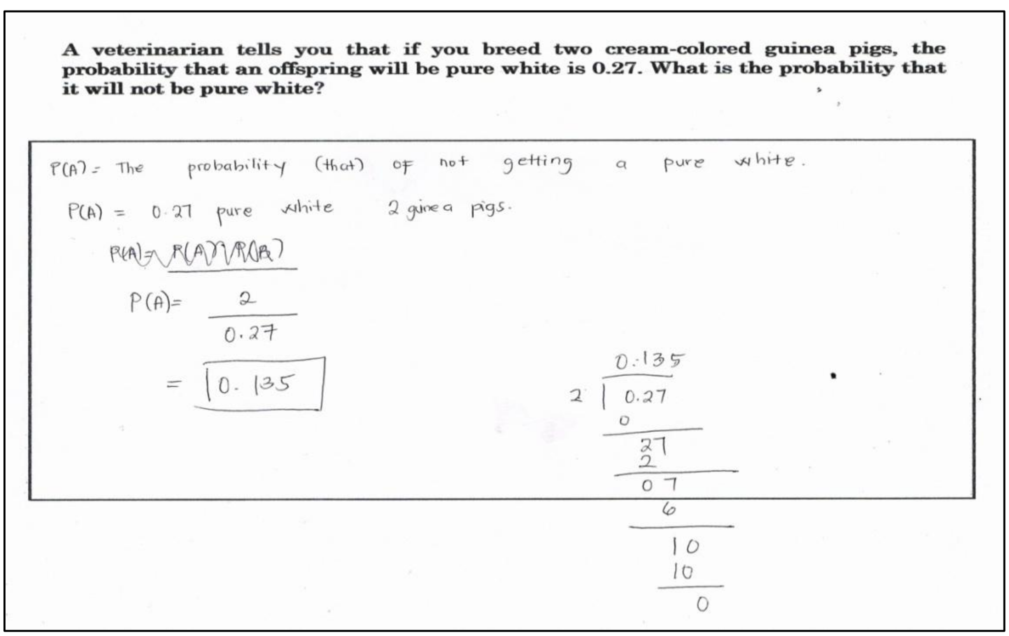


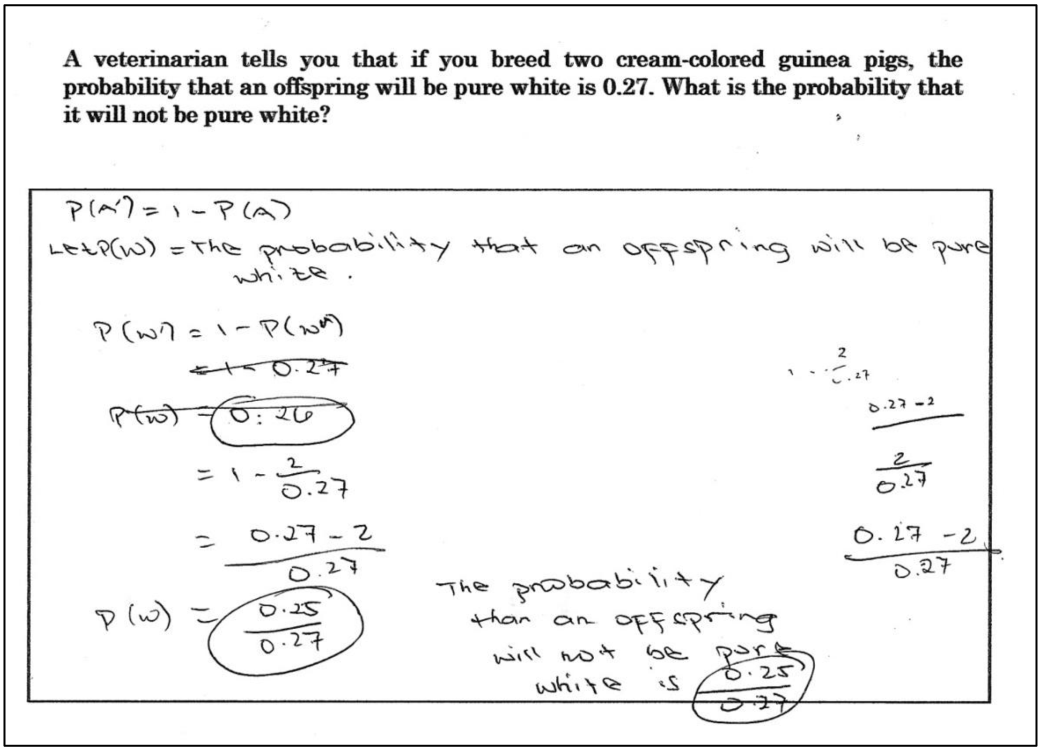
Figure 12. Sample of Transformation Error in Item no. 5

Transformation error on the previous solution is confirmed in the transcript below.

I: Please read the problem to me.
S: [The student reads Question no. 5]
I: Tell me what the problem asked you to do.
S: Sir, yung probability po na hindi makukuha yung pure white na guinea pig.
I: Tell me the method you used to answer the problem.
S: Conditional po. ***(Error occurred at this stage)***
I: Please, explain to me your solution.
S: Bale po dinivide ko lang po yung 0.27 sa 2
I: So what is your final answer?
S: 0.135?
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the transcribed interview, the error occurred at the transformation level when the student was asked about the method she used to solve the problem. The interviewee responded "Conditional Rule" instead of Compliment Rule thus, her error was in transformation.

Another Process Skill error in complement rule is presented in the next figure.



A veterinarian tells you that if you breed two cream-colored guinea pigs, the probability that an offspring will be pure white is 0.27. What is the probability that it will not be pure white?

$P(A') = 1 - P(A)$

Let P(W) = The probability that an offspring will be pure white.

$P(W') = 1 - P(W)$

$\cancel{= 1 - 0.27}$

$\cancel{P(W) = 0.26}$

$= 1 - \frac{2}{0.27}$

$= \frac{0.27 - 2}{0.27}$

$P(W) = \frac{0.25}{0.27}$

The probability than an offspring will not be pure white is $\frac{0.25}{0.27}$

$1 - \frac{2}{0.27}$

$\frac{0.27 - 2}{}$

$\frac{2}{0.27}$

$\frac{0.27 - 2}{0.27}$

Figure 13. Sample of Process Skill Error in Item no. 5

This Process Skill error is verified through the next transcribed interview.

I: Please read the problem to me.
S: [The student read Question no. 5]
I: Tell me what the question asked you to do.
S: The probability that it will not be pure white
I: Tell me the method you used to solve the problem.
S: Complement of Rule
I: Please explain to me your solution.
S: Let P of W equal to the probability that an offspring will be pure white. The solution is: The probability of W complement is equal to 1 minus the probability of W. This one sir (the student pointed her hands to 2/0.27) is for the 2 cream colored and yung 0.27 mininus ko po gaya po ng ginawa natin.
(Error occurred at this stage)
I: So what is your final answer?
S: [the student pointed her fingers to 0.25/0.27]

In the interview, the student was able to read and comprehend the problem. He was also able to give the appropriate method to solve the problem which is the complement rule of probability. However, he failed to apply this rule correctly because there was a misused data in his solution. The word "two" in the problem should not be a part of the correct solution. However, he divided this "two" by 0.27 then subtracted it from 1 thus rendering his answer erroneous. Based on this, his error was in Process Skill.

b. Addition Group

The Addition Group was composed of 16 students who committed errors in answering problems no. 6, 7, 8, and 9. Table 4 shows the types of error committed by the students in answering problems under the addition rule of probability.

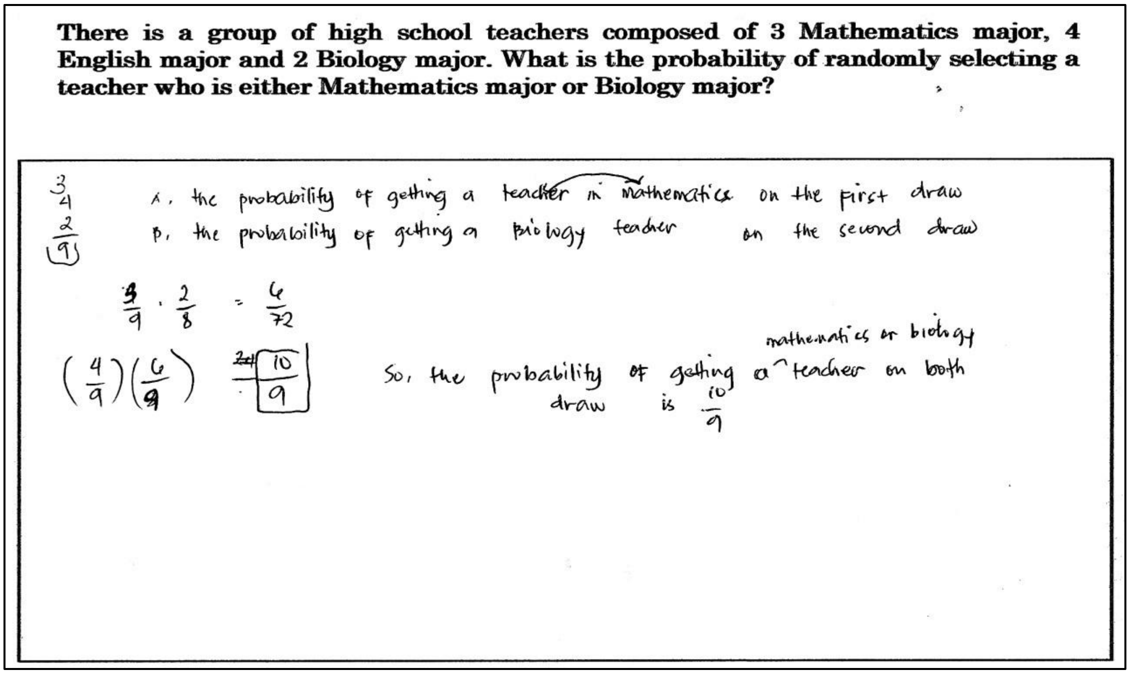
Table 4

Types of Error Committed by the Students in Answering Problem under the Addition Rule

Item Number	**Newman Classification of Error**							Total
	R	**C**	**T**	**P**	**E**	**X**	**M**	
6	0	0	2	0	0	2	0	**4**
7	0	0	1	2	0	1	0	**4**
8	0	0	4	0	0	0	0	**4**
9	0	0	4	0	0	0	0	**4**
Total	**0**	**0**	**11**	**2**	**0**	**3**	**0**	**16**

The results revealed that out of sixteen errors shown by the students, eleven (68.75%) were classified as Transformation (T), two (12.5%) were Process Skill (P) while the remaining three (18.75%) were is Carelessness (X).

Figure 14 shows an example of a Transformation Error in item number 6.


There is a group of high school teachers composed of 3 Mathematics major, 4 English major and 2 Biology major. What is the probability of randomly selecting a teacher who is either Mathematics major or Biology major?

$\frac{3}{4}$ A, the probability of getting a teacher in mathematics on the first draw

$\frac{2}{9}$ B, the probability of getting a biology teacher on the second draw

$\frac{3}{9} \cdot \frac{2}{8} = \frac{6}{72}$

$\left(\frac{4}{9}\right)\left(\frac{6}{9}\right) \quad \boxed{\frac{10}{9}}$

So, the probability of getting a mathematics or biology teacher on both draw is $\frac{10}{9}$

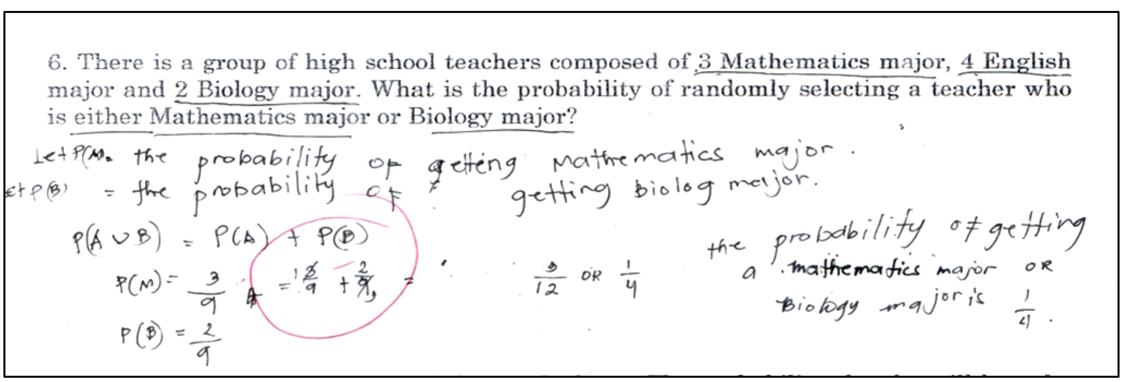
Figure 14. Sample of Transformation Error in Item no. 6

The Transformation error in item number 6 was confirmed in the interview using NEA. The transcript of the interview is given below.

I: Please read the problem to me.
S: [The student read Question no. 6]
I: Tell me what the problem asked you to do.
S: Ano daw po yung probability na makakuha ng teacher na Mathematics o Biology major.
I: Tell me the method you used to answer the problem.
S: Sir nakalimutan ko na po yung tawag. ***(Error occurred at this stage)***
I: Okay, so just explain to me your solution.
S: Yung A po yung probability of getting a mathematics teacher in the first draw. Tapos po yung probability of getting a biology teacher in the second draw. Ang ginawa ko po kinom-pute ko po muna lahat tapos ang kinalabasan po ay yung 3 dinivide ko sa 9. Tapos pinag-times ko po yung dalawa.
I: So what is your answer?
S: Sir eto pa po yung isa: yung sa English po dinivide ko rin po sa 9 yung total. Tapos pinag plus ko lang po.
I: So what do you think is the answer to the problem?
S: 10/9

Based on the interview, the student read the problem and was able to understand what the problem asked him to find. However, when he was asked about the appropriate method to use to get the answer, he responded that he could not remember how to call it. According to Clements (2003) as cited by White (2005), sometimes a student knows how to do something but is unable to express that knowledge verbally in an interview. So the interviewer asked the student to explain his solution. At this stage, it was evident that the student failed to apply the appropriate method to solve the problem, the Additions Rule. Instead, he applied the Multiplication Rule of probability which was not appropriate for the problem. Hence, his answer belonged to the Transformation stage.

The next two figures together show an example of Carelessness error in item number 6.



6. There is a group of high school teachers composed of 3 Mathematics major, 4 English major and 2 Biology major. What is the probability of randomly selecting a teacher who is either Mathematics major or Biology major?

Let P(M) = the probability of getteng mathematics major.
Let P(B) = the probability of getting biolog major.

$P(A \cup B) = P(A) + P(B)$

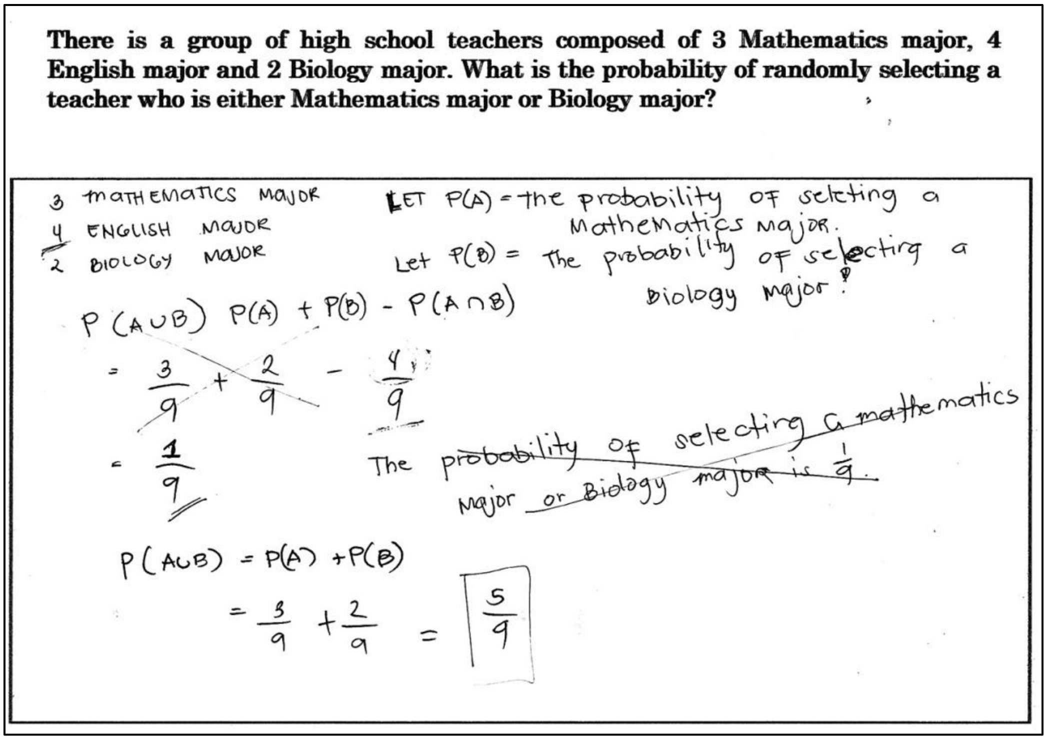
$P(M) = \frac{3}{9}$

$P(B) = \frac{2}{9}$

$\frac{3}{12}$ OR $\frac{1}{4}$

the probability of getting a mathematics major OR Biology major is $\frac{1}{4}$.

Figure 15. Sample of an Incorrect Student's Solution in Item no. 6 during the First Administration of Diagnostic Test



There is a group of high school teachers composed of 3 Mathematics major, 4 English major and 2 Biology major. What is the probability of randomly selecting a teacher who is either Mathematics major or Biology major?

3 MATHEMATICS MAJOR
4 ENGLISH MAJOR
2 BIOLOGY MAJOR

LET P(A) = the probability of selecting a Mathematics major.
Let P(B) = The probability of selecting a Biology major.

$P(A \cup B)\ \cancel{P(A) + P(B) - P(A \cap B)}$

$\cancel{= \frac{3}{9} + \frac{2}{9} - \frac{4}{9}}$

$= \frac{1}{9}$

~~The probability of selecting a mathematics major or Biology major is $\frac{1}{9}$.~~

$P(A \cup B) = P(A) + P(B)$

$= \frac{3}{9} + \frac{2}{9} = \boxed{\frac{5}{9}}$

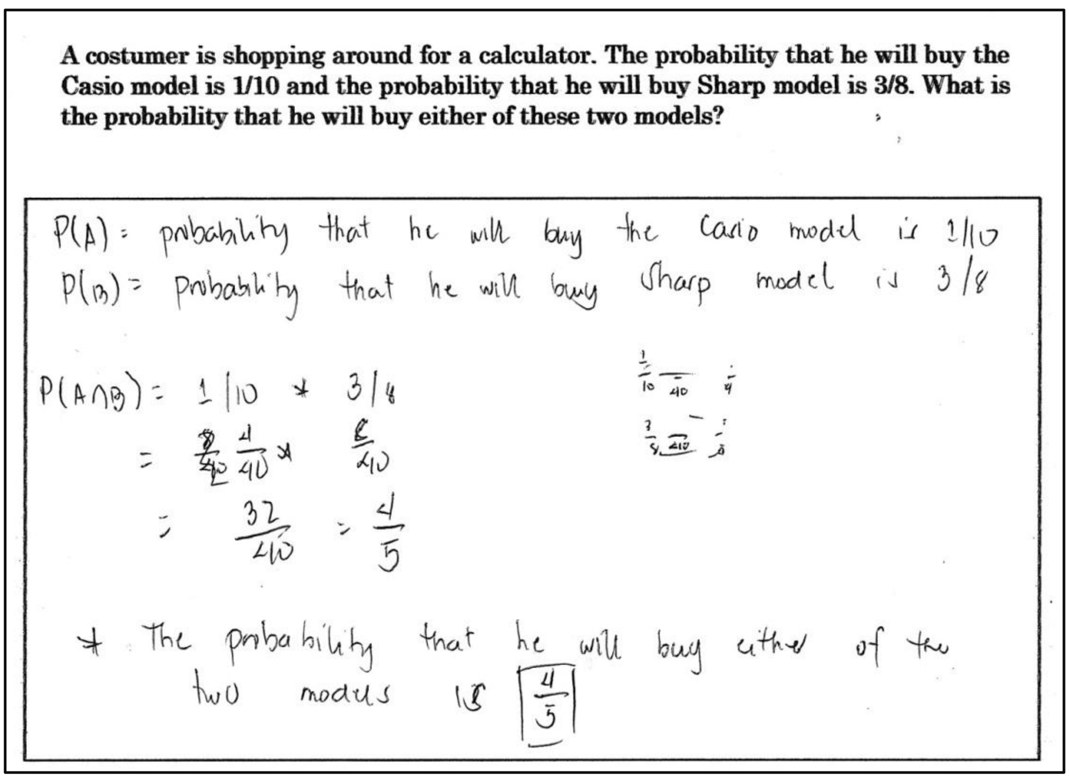
Figure 16. Sample of Careless Error in Item no. 6

This Careless error in item number 6 was confirmed in the transcript of the interview below:

I: Please read the problem to me.
S: [The student read Question no. 6]
I: Tell me what the problem asked you to find.
S: The probability of getting a Mathematics major or Biology major.
I: Tell me the method you used to solve the problem.
S: Sir, MEE po sya.
I: So Mutually Exclusive Event. Please explain to me your solution.
S: The probability of A union B is equal to P of A plus P of B. It is equal to 3/9 plus 2/9 equals 5/9.
I: So what do you think is the answer to the problem?
S: The probability of selecting a Mathematics major or Biology major is 5/9
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the interview, the student got the answer correctly when he tried to answer the problem for the second time in which he was previously incorrect hence, his error was in Carelessness.

The next figure shows an example of a Transformation error in answering problem number 7 under the Addition Rule of Probability.


A costumer is shopping around for a calculator. The probability that he will buy the Casio model is 1/10 and the probability that he will buy Sharp model is 3/8. What is the probability that he will buy either of these two models?

P(A) = probability that he will buy the Casio model is 1/10
P(B) = probability that he will buy Sharp model is 3/8

$P(A \cap B) = 1/10 * 3/8$
$= \frac{4}{40} * \frac{8}{40}$
$= \frac{32}{40} = \frac{4}{5}$

* The probability that he will buy either of the two models is $\boxed{\frac{4}{5}}$

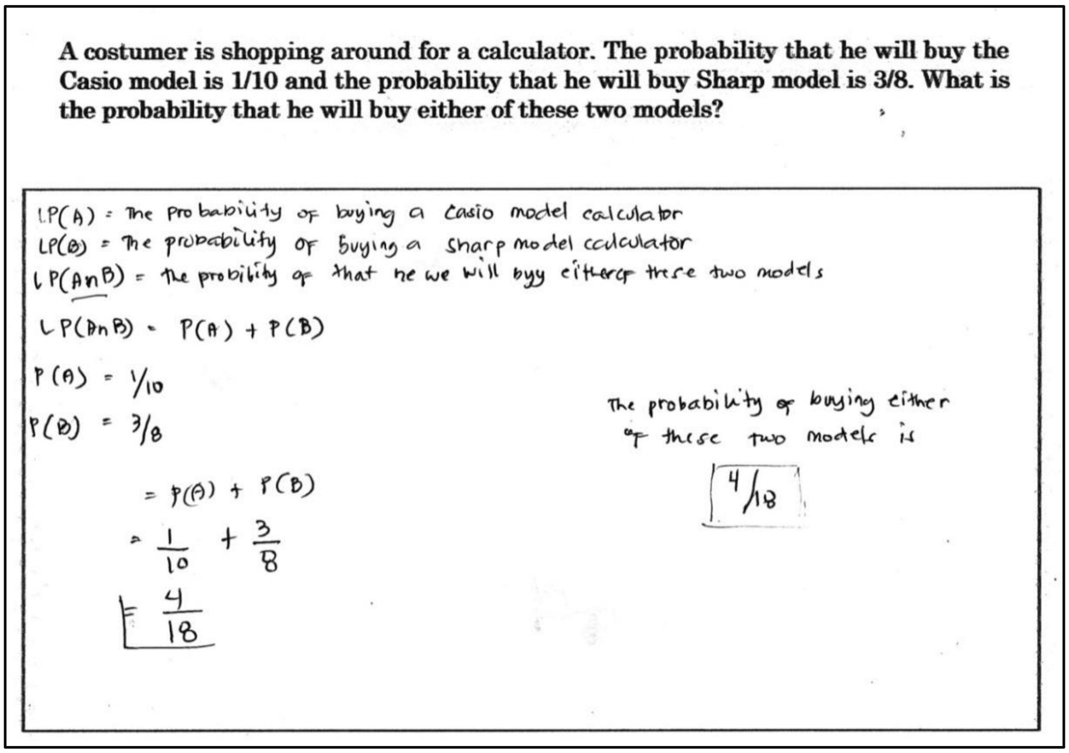
Figure 17. Sample of Transformation Error in Item n0. 7

The transformation error above is confirmed through the next transcript.

I: Please read the problem to me.
S: [The student read Question no. 7 correctly]
I: Tell me what the problem asked you to do.
S: The problem asked me to find the probability that he will buy either of the two models.
I: Tell me the method you used to solve the problem.
S: ***[The student is quite for 10 seconds] Sir di ko po maalala. (Error occurred at this stage)***
I: But you have a solution here. Can you explain to me your solution?
S: The probability of A is the probability that he will buy the casio model is 1/10. The probability of B is the probability that he will buy the Sharp model is 3/8. Then I multiply 1/10 (the probability of A) and 3/8 (the probability of B). Then I find the common denominator of each and this is the answer (the student pointed to her answer which is 4/5).
I: So tell me the answer to the problem.
S: The probability that he will buy either of the two models is 4/5.
I: Please box your final answer.
S: [The student boxed her final answer]

In the above transcript, the student was able to read and comprehend the problem. However, when asked about the method she used to solve the problem, she responded that she could not remember which method to use. A Transformation error could not be established yet since there was a solution given. It was only when the researcher asked the student to explain further her solution that a Transformation error was clearly committed error since she multiplied the probability instead of using Addition rule for not Mutually Exlusive Events.

The next figure is an example of a Process Skill error committed by the student when he answered problem number 7.



A costumer is shopping around for a calculator. The probability that he will buy the Casio model is 1/10 and the probability that he will buy Sharp model is 3/8. What is the probability that he will buy either of these two models?

LP(A) = The probability of buying a Casio model calculator
LP(B) = The probability of buying a Sharp model calculator
LP(AnB) = the probility of that he we will byy eitherr there two models

LP(AnB) = P(A) + P(B)

P(A) = 1/10

P(B) = 3/8

$= P(A) + P(B)$

$= \frac{1}{10} + \frac{3}{8}$

$= \frac{4}{18}$

The probability of buying either of these two models is $\boxed{4/18}$

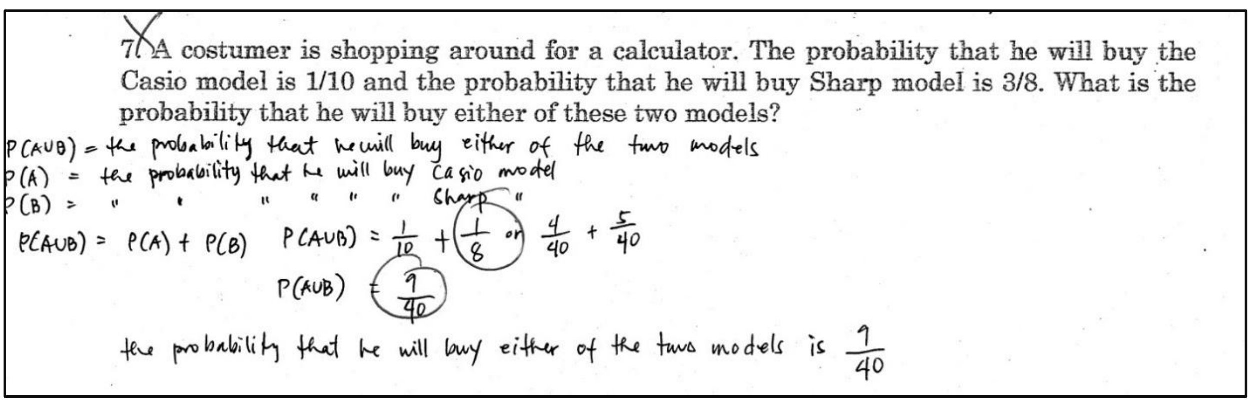
Figure 18. Sample of Process Skill Error in Item no. 7

Analyzing the student's written solution was not enough in this case, so an interview was conducted to confirm that a Process Skill error was committed by the student. The transcribed interview is given below.

I: Please read the problem to me.
S: [The student read Question no. 7 correctly]
I: Tell me what the problem asked you to find.
S: The probability that he will buy either of these two models.
I: Tell me the method you used to solve the problem.
S: Addition rule.
I: Can you explain to me your solution?
S: ***The probability of A is the probability of buying a Casio model calculator. The probability of B is the probability of buying a Sharp model calculator. Let the probability of A intersection B is the probability the he will buy either of these two models. So formula is the probability of A intersection B is probability of A plus the probability of B. (Error occurred at this stage)***
I: So tell me the answer to the problem.
S: The probability of buying either of these two models is 4/18
I: Please box your final answer.
S: [The student boxed his final answer]

In the interview, the student was able to read and comprehend the problem. He was also able to give the appropriate method to solve the problem which was addition rule. However, when he added 1/10 to 3/8, his answer was 4/18 which was mathematically incorrect. Based on his answer, he failed to follow the correct procedure in adding two fractions with different denominators thus rendering his answer erroneous. Furthermore, the student used intersection of events instead of union of events. This error only confirms the study of Idris and Narayanan (2011) that an error in addition of fraction is systematic random error.

The next figure is an example of a Carelessness error when the student tried to answer item no. 7 for the second time and got it correctly in which he was previously incorrect. Figure 19 is the student's solution during the first administration of diagnostic test with incorrect answer and Figure 20 shows the solution the second time the student answered the same problem.


7. A costumer is shopping around for a calculator. The probability that he will buy the Casio model is 1/10 and the probability that he will buy Sharp model is 3/8. What is the probability that he will buy either of these two models?

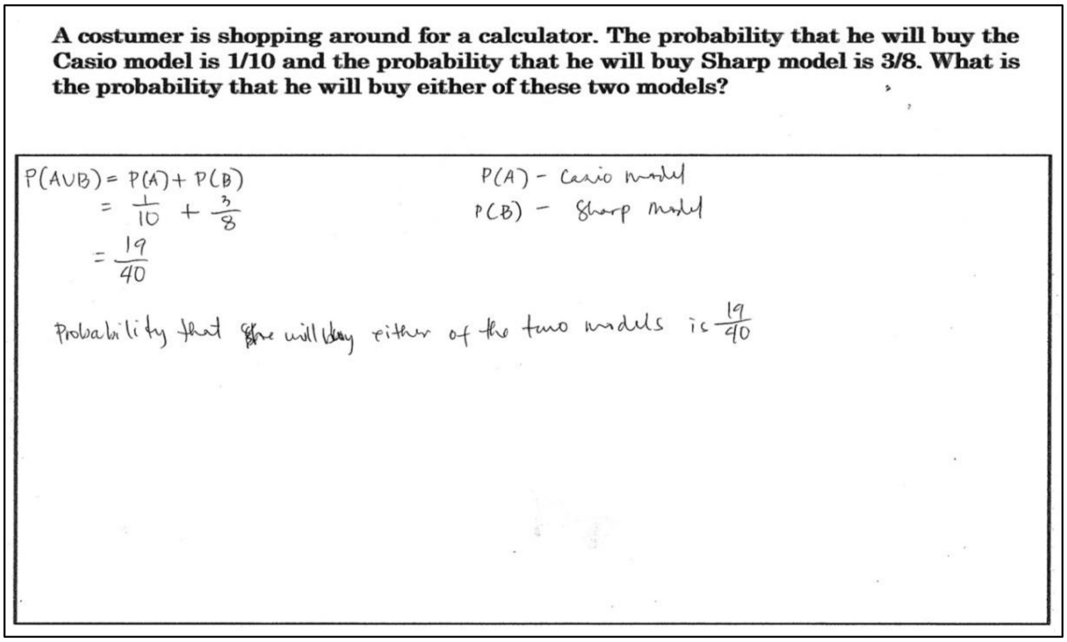
$P(A \cup B)$ = the probability that he will buy either of the two models
$P(A)$ = the probability that he will buy Casio model
$P(B)$ = " " " " " " Sharp "

$P(A \cup B) = P(A) + P(B)$ $\quad P(A \cup B) = \frac{1}{10} + \frac{1}{8}$ or $\frac{4}{40} + \frac{5}{40}$

$P(A \cup B) = \frac{9}{40}$

the probability that he will buy either of the two models is $\frac{9}{40}$

Figure 19. Sample of an Incorrect Student's Solution in Item no. 7 during the First Administration of Diagnostic Test



A costumer is shopping around for a calculator. The probability that he will buy the Casio model is 1/10 and the probability that he will buy Sharp model is 3/8. What is the probability that he will buy either of these two models?

$P(A\cup B) = P(A) + P(B)$

$= \frac{1}{10} + \frac{3}{8}$

$= \frac{19}{40}$

P(A) - Casio model

P(B) - Sharp model

Probability that she will buy either of the two models is $\frac{19}{40}$

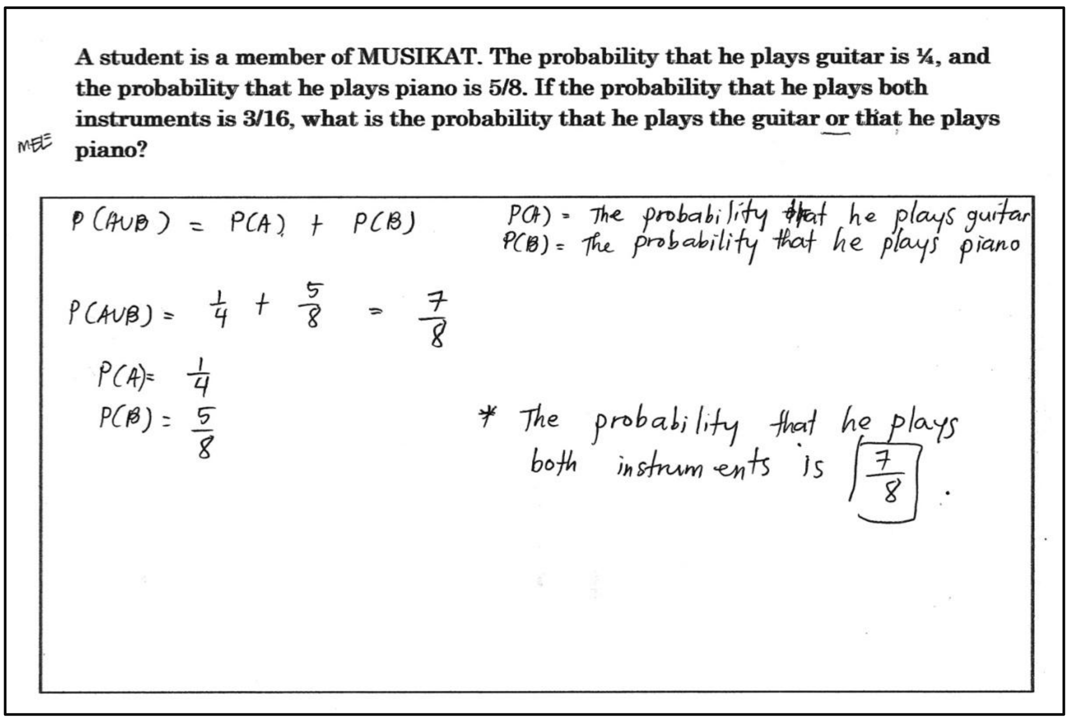
Figure 20. Sample of Careless Error in Item no. 7

This Carelessness error was verified during the interview using NEA. The transcribed interview is given below.

I: Please read the problem to me.
S: [The student read Question no. 7 correctly]
I: Tell me what the problem asked you to do.
S: Yung union po ng probability ng Casio model at ng Sharp model
I: Tell me the method you used to solve the problem.
S: Addition po.
I: Please explain to me your solution.
S: Since ang formula po pagnag-aadd ng probability ay probability of A union B is equal to P of A plus P of B. Ang value po ng P of A is 1/10 at yung value po ng P of B is 3/8 so ayun po pinag add lang po sila.
I: So what do you think is the answer to the problem?
S: 19/40 po
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the interview, the student got the answer correctly when he tried to answer problem no. 7 for the second time in which he was previously incorrect hence, it was considered as Carelessness error.

An example of Transformation error in item no. 8 is given in the next figure.



A student is a member of MUSIKAT. The probability that he plays guitar is ¼, and the probability that he plays piano is 5/8. If the probability that he plays both instruments is 3/16, what is the probability that he plays the guitar or that he plays piano?

MEE

$P(A \cup B) = P(A) + P(B)$

P(A) = The probability ~~that~~ that he plays guitar

P(B) = The probability that he plays piano

$P(A \cup B) = \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$

$P(A) = \frac{1}{4}$

$P(B) = \frac{5}{8}$

* The probability that he plays both instrum-ents is $\boxed{\frac{7}{8}}$.

Figure 21. Sample of Transformation Error in Item no. 8

The Transformation error committed by the student was verified in the next transcript.

I: Please read the problem to me.
S: [The student read Question no. 8 correctly]
I: Tell me what the problem asked you to find.
S: What is the probability that he plays the guitar or that he plays the piano.
I: Tell me the method you used to solve the problem.
S: ***Addition Rule. (Error occurred at this stage)***
I: Can you go over your solution and explain it to me step by step.
S: I used mutually exclusive event. The probability of A union B is equal to the probability of A plus the probability of B. The probability of A is the probability that he plays the guitar and the probability of B is that he plays the piano. I just add the probability of A to the probability of B. SO ¼ plus 5/8 is equal to 7/8.
I: So tell me the answer to the problem.
S: 7/8

As indicated in the transcript, the student was able to read and comprehend the problem. On the contrary, when asked about the method he used to solve the problem, he answered that he used Addition rule. This answer was partially correct because, indeed the problem calls for the application of Addition rule of probability. However, the student used the addition rule for mutually exclusive events instead of using it for not mutually exclusive event. The student failed to subtract 3/16 which is the probability of the intersection of the two events given in the problem. Therefore, the error was classified as a Transformation error.

Furthermore, in the written solution given by the student, it is noticeable that the student underlined the word "or" in the problem. The probable reason for underlining word "or" was for this student to use a context clue since "or" suggested that the problem calls for the union of the probability of the events. Thus, he needed to add these probabilities.

Figure 22 illustrates another example of transformation in Addition Rule of probability (Item no. 9)

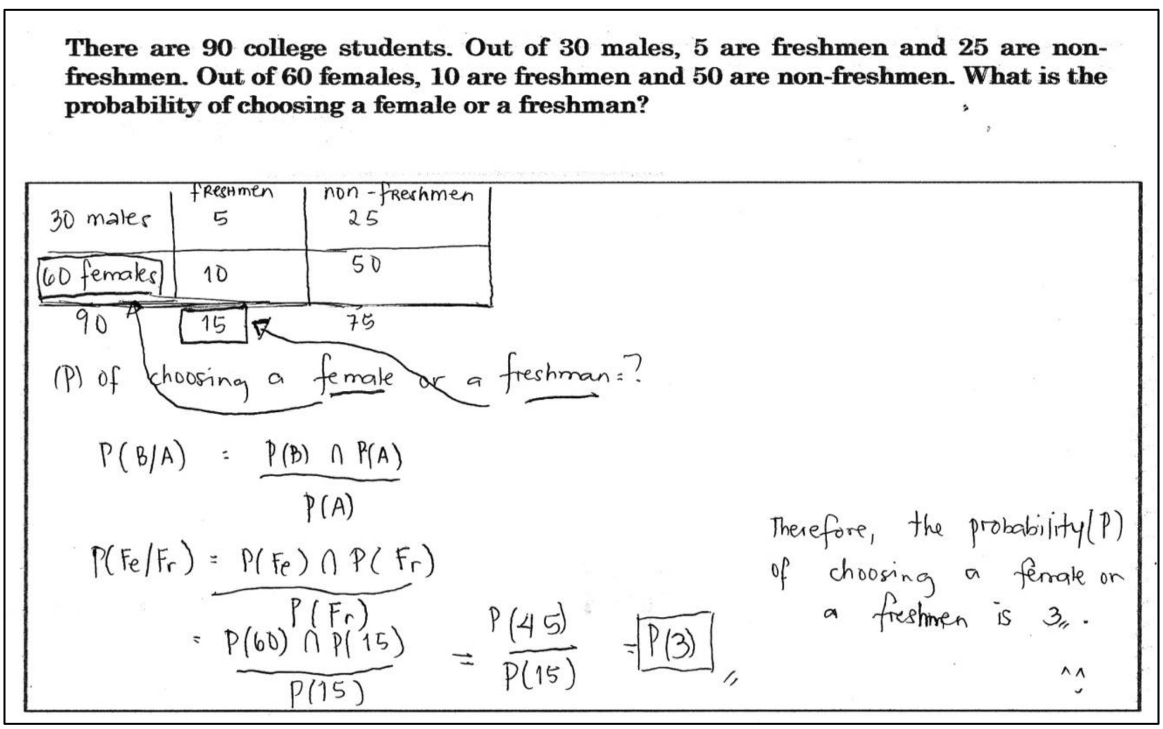


Figure 22. Sample of Transformation Error in Item no. 9

The Transformation error above was verified in the transcribed interview below.

I: Can you read the problem to me? *S: [The student read question number 9]* *I: Tell me what the problem asked you to do.* *S: Sir yung probability of choosing a female or a freshman.* *I: What method did you use to solve the problem?* ***S: Sir eto po, the probability of B over A is equal to the probability of B intersection of the Probability of A over Probability of A.(Error occurred atthis stage.*** *I: Can you explain to me your solution?* *S: Sir nilagay ko po mun yung mga given para po hindi ako masyadong malito. Tapos sir nakakalito yung ganyang pagkakalagay kung ilan sir yun male tapos kung ilan yung female. Yung pagkakahilaway ng freshmen sa non-freshmen. Tapos sir i-nanalyse hinahanap sir yun probability of choosing a female or a freshman. Tapos ang ginawa ko sice female yung hinahanap yung buong total ng female ang nilagay ko. Tapos sa freshmen po, pinaghalo ko lang yung freshmen ng male at freshmen ng female. Tapos eto na nga po yung ginamit ko na formula. Yung pong probability of B over A.* *I: So what is the answer to the problem?* *S: Sir 3.* *I: Can you box your final answer.* *S: [The student boxed her final answer]*

In the transcript above, the students was able to read and comprehend the problem clearly. However, the student failed to apply the appropriate method to solve the problem when she was asked about the method she used to solve item no. 9. She mentioned "intersection of probability" which was not appropriate for the problem. Hence, the student's error was in transformation.

c. Multiplication Group

The Multiplication Group was composed of 16 students who committed errors in answering problems no. 3, 10, 12 and 13. Table 5 shows the types of error committed by the students in answering problem under the multiplication rule.

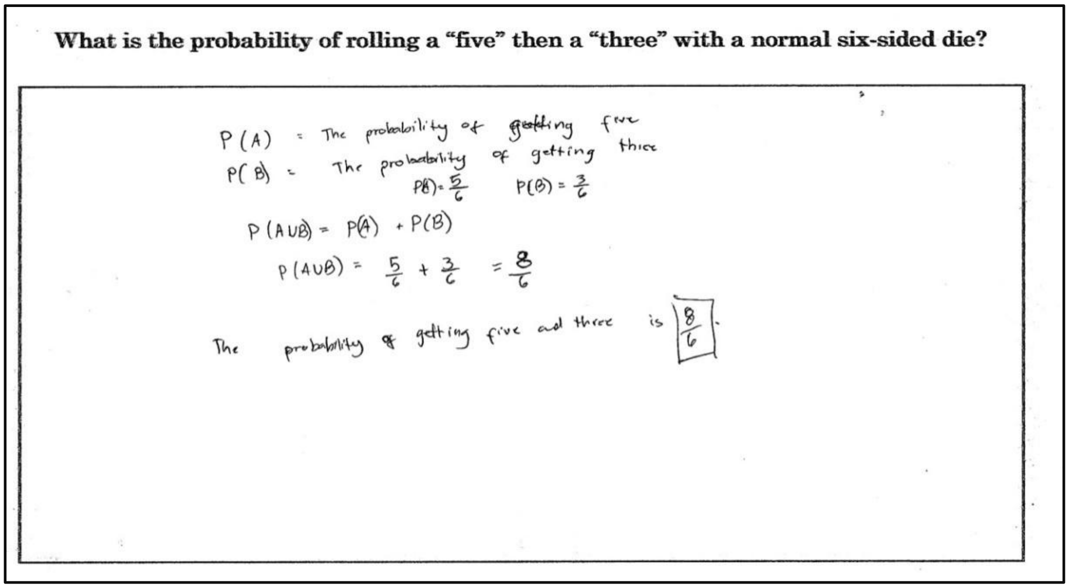
Table 5

Types of Error Committed by the Students in Answering Problem under the Multiplication Rule

Item Number	**Newman Classification of Error**							Total
	R	**C**	**T**	**P**	**E**	**X**	**M**	
3	0	2	1	1	0	0	0	**4**
10	0	0	4	0	0	0	0	**4**
12	0	0	4	0	0	0	0	**4**
13	0	0	4	0	0	0	0	**4**
Total	**0**	**2**	**13**	**1**	**0**	**0**	**0**	**16**

The data shows that out of sixteen errors committed by the students, two (12.5%) were classified as Comprehension (C), thirteen (81.25%) were Transformation (T) while a single student (6.25%) had an error in Process Skill (P).

Figure 23 shows an example of a Comprehension error in answering item no. 3.


What is the probability of rolling a "five" then a "three" with a normal six-sided die?

$P(A)$ = The probability of getting five

$P(B)$ = The probability of getting three

$P(B) = \frac{5}{6}$ $P(B) = \frac{3}{6}$

$P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = \frac{5}{6} + \frac{3}{6} = \frac{8}{6}$

The probability of getting five and three is $\boxed{\frac{8}{6}}$.

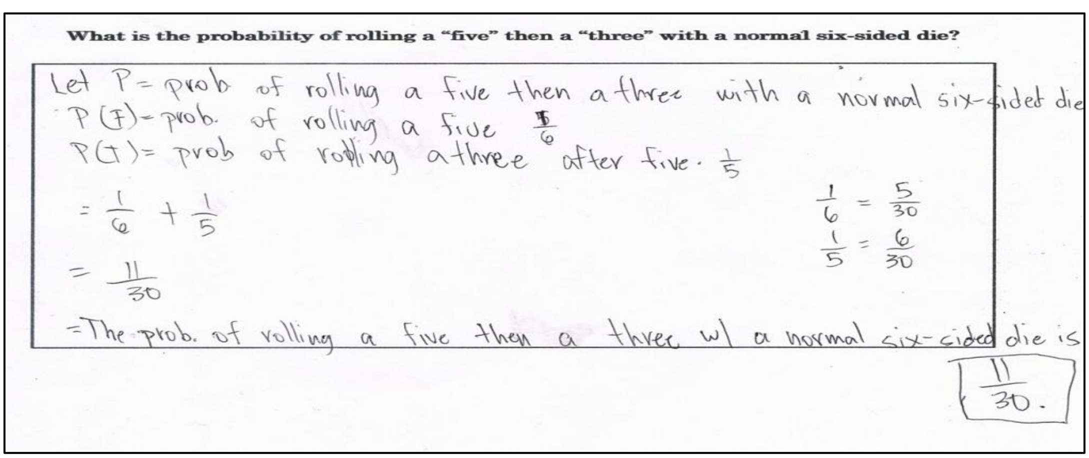
Figure 23. Sample of Comprehension Error in Item no. 3

This Comprehension error made by the student was confirmed in the interview using NEA. The transcribed interview is given below.

I: Please read the problem to me.
S: [The student reads question number 3 correctly]
I: Tell me what the question asked you to do.
S: ***Kung lalabas daw ba itong "five or "three" pag ni-roll yung die.(Error occurred at this stage)***
I: Tell me the method you used to answer the problem.
S: Mutually exclusive event.
I: Can you explain to me your solution?
S: The probability of A – the probability of getting 5. The probability of B – the probability of getting 3. The probability of A is equal to 5/6. The probability of B is equal to 3/6. Yung formula po ay the probability of A union B is probability if A plus the probability of B. So, 5/6 plus 3/6 is 8/6. The probability of getting "five" then "three" is 8/6.
I: Please box your final answer.
S: [The student boxed his final answer]

The result of the interview revealed that the error occurred at the Comprehension level since the student could read the question correctly but failed to give what the problem was asking her to do. It is clear that the problem asked for the probability of rolling a five then three with a normal six-side die when it is rolled twice. However, when the student was asked about what the problem asked him to do, he answered about the occurrence of five or three when the die was rolled. This confirms the contention of Clarkson (1983) that for most errors, students had either not been able to understand the word problems or, if understanding of words had been present, they had not devised appropriate strategies for solving the given problems.

The next figure shows an example of Transformation error in item no. 3.


What is the probability of rolling a "five" then a "three" with a normal six-sided die?

Let P = prob of rolling a five then a three with a normal six-sided die
P(F) = prob. of rolling a five $\frac{1}{6}$
P(T) = prob of rolling a three after five. $\frac{1}{5}$

$= \frac{1}{6} + \frac{1}{5}$ $\qquad \frac{1}{6} = \frac{5}{30}$ $\qquad \frac{1}{5} = \frac{6}{30}$

$= \frac{11}{30}$

= The prob. of rolling a five then a three w/ a normal six-sided die is $\boxed{\frac{11}{30}}$.

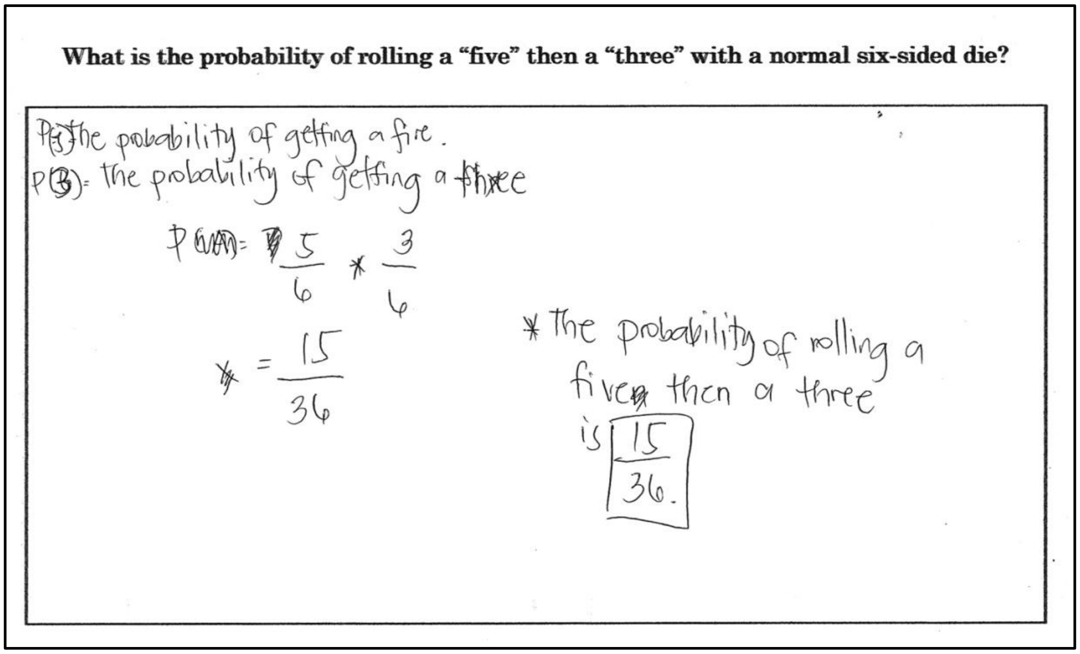
Figure 24.Sample of Transformation Error in Item no. 3

The transformation error was confirmed in the interview using NEA. The transcript of the actual interview was given below:

I: Please read the problem to me.
S: [The student reads question number 3]
I: Tell me what the problem asked you to find.
S: The probability of rolling a "five" then a "three" with a normal six-sided die.
I: Tell me the method you used to solve the problem.
S: Sir, addition rule. ***(The error occurred at this stage)***
I: Can you explain to me your solution?
S: Sir what I have here is the probability of rolling a five 1/6 then the probability of rolling a three after five is 1/5 because I have deducted 1 from six sir. I will add 1/6 and 1/5 but before that sir I have to do the… yung nasa side po… yun po I had 5/30 plus 6/30 is 11/30. So, the probability of rolling a "five" then a "three" with a normal six-sided die is 11/30.
I: Please box your final answer.
S: [The student boxed his final answer]

Based on the interview, the error occurred at the transformation level when the student was asked about the method she used to solve item no. 3. She responded that she used "Addition rule" which is inappropriate for the problem calls for the application of Multiplication rule of probability. Hence, the error belonged to transformation.

The next figure will show us another example of error in item no. 3.


What is the probability of rolling a “five” then a “three” with a normal six-sided die?

P(5) the probability of getting a five.
P(3)= the probability of getting a three

$P(5)(3) = \frac{5}{6} * \frac{3}{6}$

$= \frac{15}{36}$

* The probability of rolling a five then a three is $\boxed{\frac{15}{36}}$.

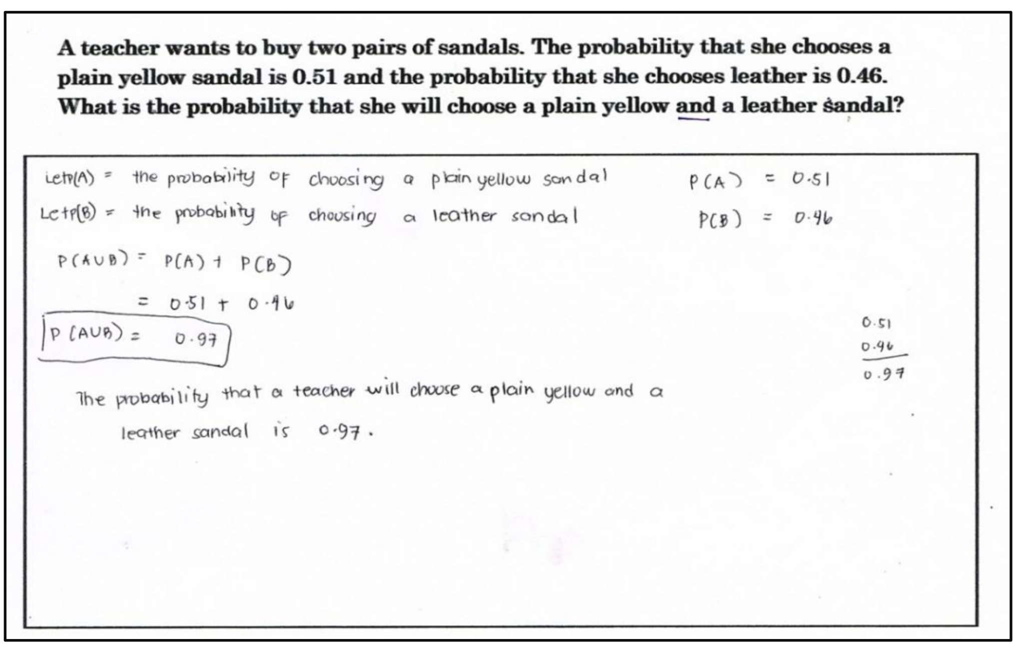
Figure 25. Sample of Process Skill Error in Item no. 3

The Process Skill error was confirmed in the interview. The transcript is given below.

I: Please read the problem to me.
S: [The student reads question number 3]
I: Tell me what the problem asked you to do.
S: The probability of getting a “five” then a “three” with a normal six-sided die.
I: Tell me the method you used to answer the problem.
S: Multiplication method
I: Can you explain to me your solution?
*S: **Ahmm, P(5) is the probability of getting a five and P(3) is the probability of getting a three. It is equal to 5/6 kasi sabi nyo po ay normal six-sided die. Tapos po, ti-nayms ko po sa 3/6 and the answer is 15/36. (Error occur at this stage)***
I: What is your final answer?
S: The probability of rolling a “five” then a “three” is 15/36
I: Please box your final answer.
S: [The student boxed his final answer]

In this interview, the student was able to read and comprehend what the problem asked her to do. She clearly knew that the multiplication rule was the appropriate method to solve the problem. However, she failed to represent the value of the probability of getting a “five” and the probability of getting a “five” in a normal-six sided die which was both equal to 1/6. Thus, the error occurs at the process skill stage. This finding confirms Noraini and Shiney (2005) that when the students lacked a solid conceptual understanding of the basic probability concept they ended up in mindless substitution in formulas.

Figure 26 shows an example of a transformation error in item no. 10.



A teacher wants to buy two pairs of sandals. The probability that she chooses a plain yellow sandal is 0.51 and the probability that she chooses leather is 0.46. What is the probability that she will choose a plain yellow and a leather sandal?

Let P(A) = the probability of choosing a plain yellow sandal $P(A) = 0.51$

Let P(B) = the probability of choosing a leather sandal $P(B) = 0.46$

$P(A \cup B) = P(A) + P(B)$

$= 0.51 + 0.46$

$\boxed{P(A \cup B) = 0.97}$

$$\begin{array}{r} 0.51 \\ \underline{0.46} \\ 0.97 \end{array}$$

The probability that a teacher will choose a plain yellow and a leather sandal is 0.97.

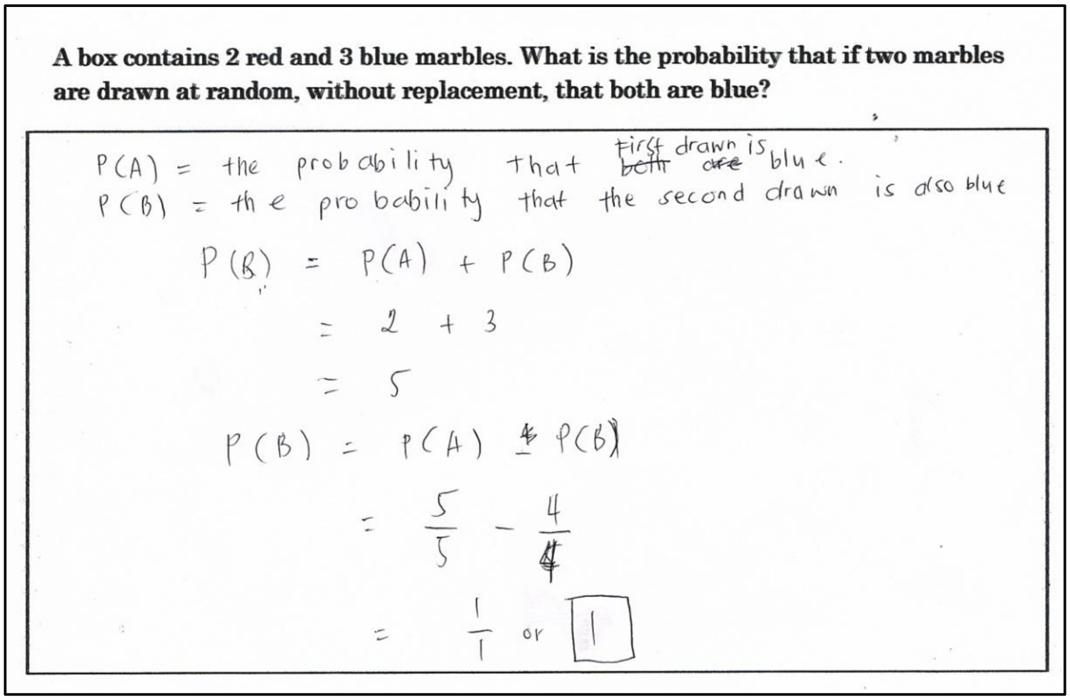
Figure 26. Sample of Transformation Error in Item no. 10

This Transformation error was confirmed in the transcribed interview given below

I: Please read the question to me.
S: [The student read question number 10 correctly]
I: Tell me what the problem asked you to find.
S: The probability of getting a yellow sandal and a leather sandal.
I: What method you used to solve the problem?
S: ***Addition. (Error occurred at this stage)***
I: Can you explain to me your solution?
S: Let the probability of A be the probability of choosing a plain yellow sandal. Let the probability of B be the probability of choosing a leather sandal. So I used the formula, the probability A union B is equal to probability if A plus the probability of B. Substitute the values of probability of A and probability of B, 0.51 plus 0.46 equals 0.97. So the probability that the teacher will choose a plain yellow and a leather sandal is 0.97.
I: Is that your answer to the problem?
S: [student nod her head]
I: Can you box your final answer.
S: [The student boxed her final answer]

The interview revealed that the error occurred at the transformation level since the student was able to read and comprehend the problem but failed to use the appropriate method to solve the problem. The student used Addition rule instead of using the Multiplication rule of probability.

The next figure shows another example of a transformation error in Multiplication Rule of Probability but this time in item no. 12.



A box contains 2 red and 3 blue marbles. What is the probability that if two marbles are drawn at random, without replacement, that both are blue?

P(A) = the probability that first drawn is ~~both~~ ~~are~~ blue.
P(B) = the probability that the second drawn is also blue

$P(B) = P(A) + P(B)$
$= 2 + 3$
$= 5$

$P(B) = P(A) \; \cancel{+} \; - \; P(B)$
$= \frac{5}{5} - \frac{4}{4}$
$= \frac{1}{1}$ or $\boxed{1}$

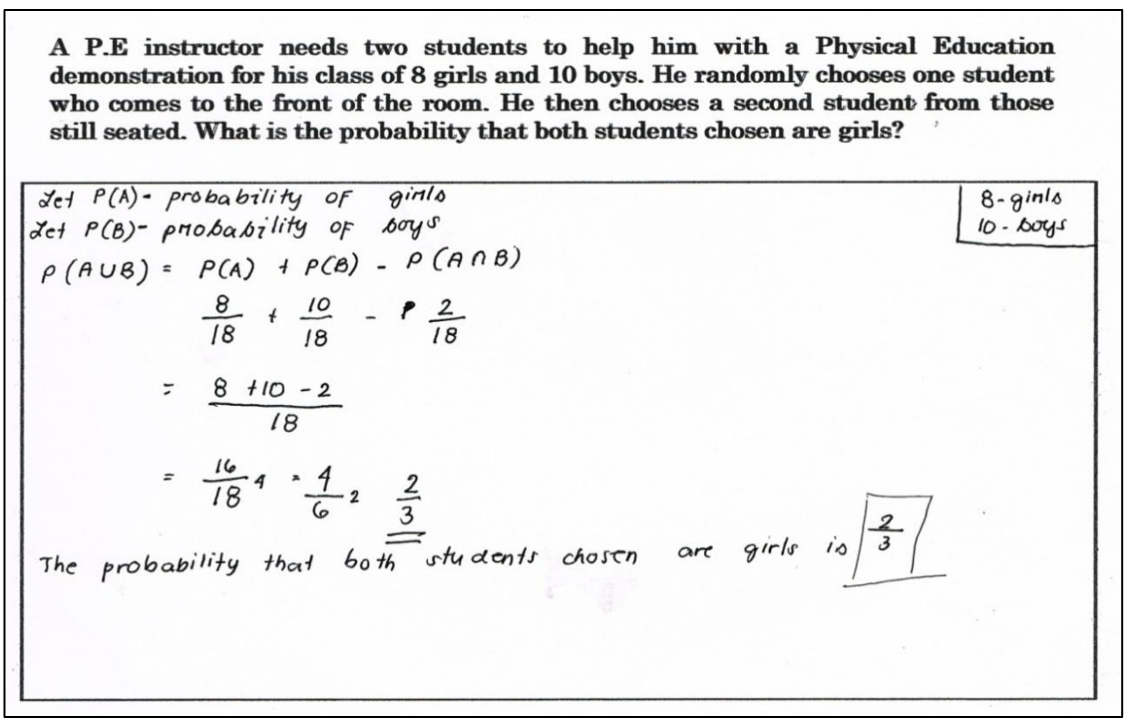
Figure 27. Sample of Transformation Error in Item no. 12

This transformation error is confirmed with the next transcribed interview:

I: Please read the problem to me.
S: [The student read question number 12 correctly]
I: Please tell me what the problem asked you to do.
S: What is the probability that if two marbles are drawn at random, without replacement, that both are blue.
I: What method did you use to solve the problem?
S: Sinulat ko po yung probability na A na probability that first drawn is blue. Tapos, the probability of B, that the second drawn is also blue.
I: Then?
S: ***I add the ano… yung two red at three blue marbles pinag-add ko po. (Error occurred at this stage)***
I: Can you explain to me your solution?
S: The probability of blue marbles… tapos eh pinag add ko po yung probability of A plus sa probability of B. Tapos two plus three equals 5. Yun na po.
I: What is your answer to the problem?
S: Sir eto po 1.
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the transcript, the error occurred at the transformation level since the student correctly read the question and comprehended the problem but used an inappropriate method to answer the problem. The student added her assumed value of probabilities of events A and B instead of multiplying. Since the problem calls for the application of Multiplication Rule, the students answer was incorrect.

The next figure shows an example of transformation error in item no. 13 under Multiplication rule of Probability.



A P.E instructor needs two students to help him with a Physical Education demonstration for his class of 8 girls and 10 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. What is the probability that both students chosen are girls?

Let P(A) - probability of girls
Let P(B) - probability of boys

8 - girls
10 - boys

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{8}{18} + \frac{10}{18} - \frac{2}{18}$

$= \frac{8+10-2}{18}$

$= \frac{16}{18} = \frac{4}{6} = \frac{2}{3}$

The probability that both students chosen are girls is $\boxed{\frac{2}{3}}$

Figure 28. Sample of Transformation Error in Item no. 13

This error was verified in the transcript of the interview below:

I: Please read the problem to me.
S: [The student read question number 13]
I: Please tell me what the problem asked you to find.
S: To find the probability that both student chosen are girls
I: Tell me the method you used to solve the problem.
S: I used the method in addition which is Mutually Exclusive Event. ***(Error occurred at this stage)***
I: Can you explain to me your solution?
S: I let P of A be the probability of girls and let P of B be the probability of boys. And then I used the solution in Mutually Exclusive Event which is the P of A union B is equal to P of A plus P of B minus P of A intersection B. Then, di ba sir ang total po ay you have 8 girls and 10 boys, then ang total po nila ay 18. Tapos po yung P of A is 8 over 18 and yung P of B po ay 10 over 18. Dito po sir nalito po ako kaya po ang eto po yung ginawa ko, di ba po dalawa po yungkukunin kaya po 2 over 18. Kasi hindi ko po alam kung kailangan pa rin pong I-table gaya po ng ginawa natin dati kung meron silang intersection kaya po hindi ko po alam kung paano po kukunin yung intersection ang A and B. Kaya po angginawa ko po ay same denominator nalang po sila. Tapos 8 plus 10 minus 2 is 16 tapos sinimplify ko nalang po.
I: What is your answer to the problem?
S: The probability that both student chosen are girls is 2/3.
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the transcript above, the student was able to read the problem and understand what the problem asked her to find. However, when she was asked to tell the method she used to solve the problem, she stated that it was Addition rule for Mutually Exclusive Events. From her answer, the error occurred at the transformation stage for the appropriate method to use was Multiplication rule for dependent events.

d. Conditional Probability Group

The Conditional Probability Group was composed of 16 students who committed errors in answering problems number 11, 14, 15, and 16. Table 6 shows the types of error committed by the students in answering problem under the conditional probability.

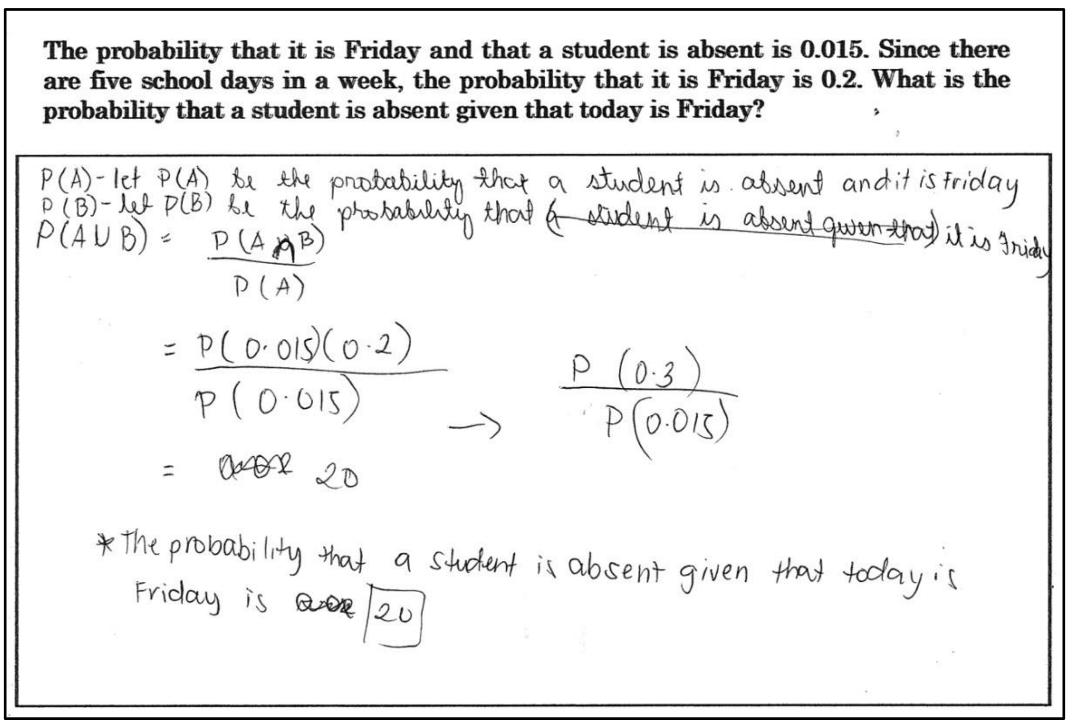
Table 6

Types of Error Committed by the Students in Answering Problem under the Conditional Probability

Item Number	**Newman Classification of Error**							Total
	R	**C**	**T**	**P**	**E**	**X**	**M**	
11	0	0	2	2	0	0	0	**4**
14	0	0	4	0	0	0	0	**4**
15	0	0	3	0	0	0	1	**4**
15	0	0	4	0	0	0	0	**4**
Total	**0**	**0**	**13**	**2**	**0**	**0**	**1**	**16**

The data show that out of sixteen errors made by the students, thirteen (81.25%) were classified as Transformation (T), two (12.5%) were classified as Process Skill (P), and the last one (6.25%) was classified as Motivation (M).

Figure 29 shows us an example of Process Skill error on item no. 11.



The probability that it is Friday and that a student is absent is 0.015. Since there are five school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

P(A)- let P(A) be the probability that a student is absent and it is Friday

P(B)- let P(B) be the probability that ~~a student is absent given that~~ it is Friday

$$P(A \cup B) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{P(0.015)(0.2)}{P(0.015)} \rightarrow \frac{P(0.3)}{P(0.015)}$$

$$= 20$$

* The probability that a student is absent given that today is Friday is $\boxed{20}$

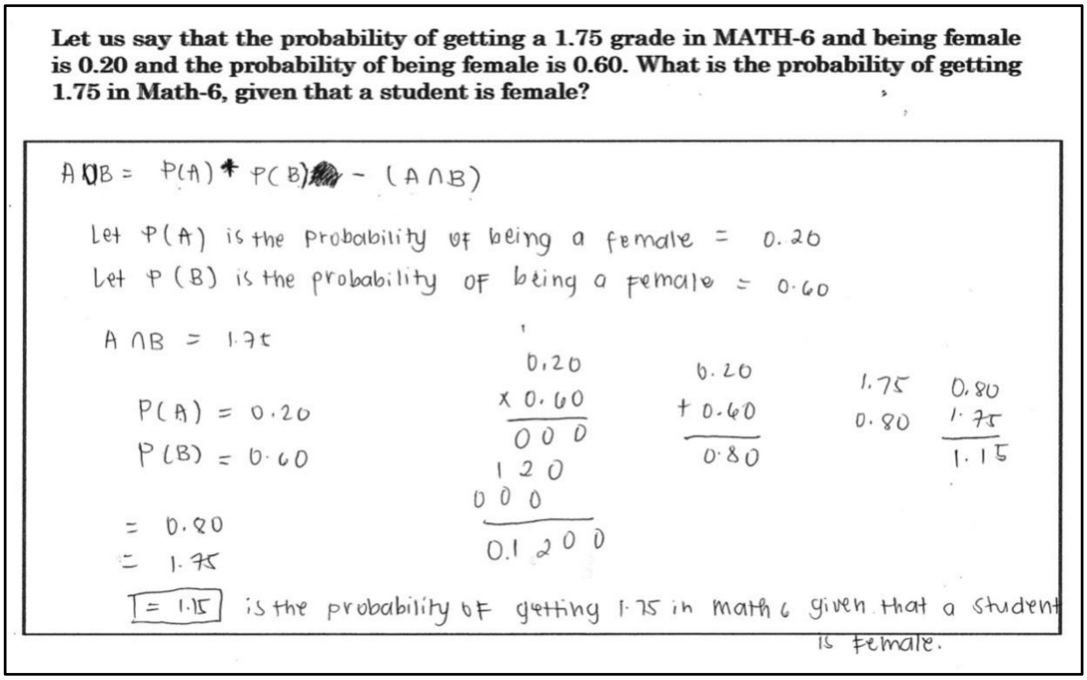
Figure 29. Sample of Process Skill Error in Item n0. 11

The Process Skill error was verified in the interview with the student. The transcript is given below.

I: Please read the question to me.
S: [The student read question number 11]
I: Please tell me what the problem asked you to find.
S: What is the probability that that a student is absent given that today is Friday.
I: Tell me what method you used to solve the problem.
S: Conditional
I: Can you explain to me your solution?
S: ***Let the probability of A be the probability that a student is absent and it is Friday. Let the probability of B be the probability that it is Friday. The probability of A union B is equal to the probability of A intersection B over the probability of A. It is equal to the 0.015 times 0.2 over 0.015. Then 0.015 times 0.2 is equal to 0.3 over 0.015 equals 20. (Error occurred at this stage)***
I: What is your answer to the problem?
S: The probability that a student is absent given that today is Friday is 20.
I: Please box your final answer.
S: [The student boxed her final answer]

Based on the interview, the student was able to read and comprehend what the problem asked him to find. He also knew the appropriate method to solve the problem. However, he failed to apply the conditional probability correctly. He even used a wrong symbol to represent the conditional probability.

Figure 30 gives an example of a Transformation error in item no. 14 under conditional probability



Let us say that the probability of getting a 1.75 grade in MATH-6 and being female is 0.20 and the probability of being female is 0.60. What is the probability of getting 1.75 in Math-6, given that a student is female?

$A \cup B = P(A) + P(B) - (A \cap B)$

Let $P(A)$ is the probability of being a female = 0.20

Let $P(B)$ is the probability of being a female = 0.60

$A \cap B = 1.75$

$P(A) = 0.20$

$P(B) = 0.60$

$= 0.80$

$= 1.75$

$\boxed{= 1.15}$ is the probability of getting 1.75 in math 6 given that a student is female.

$0.20 \times 0.60 = 0.1200$ $0.20 + 0.60 = 0.80$ 1.75 0.80 $1.75 - 0.80 = 1.15$

Figure 30. Sample of Transformation Error in Item no. 14

The transformation error was confirmed in the next transcribed interview.

I: Please read the problem to me.
S: [The student read question number 14 correctly]
I: Please tell me what the problem asked you to do.
S: To get the probability getting 1.75, given that a student is female.
I: Tell me the method you used to answer the problem.
*S: **Addition rule of probability. (Error occurred at this stage)***
I: Please explain to me your solution.
S: A union B is equal to P of A plus P of B minus P of A intersection B. Let P of A be the probability of being a female which is 0.20. Let P of A be the probability of being a female also which is 0.60. Tapos eto pong A "U" B is 1.75. P(A) is equal to 0.20 and P of B is equal to 0.60. I nadd ko po sya sir.. Tama po ba? Then nung ma-add ko po eto po yung sagot, 1.15 is the probability of getting 1.75 in Math 6 given that a student is female.
I: Is that your final answer?
S: yes

In the transcript, the student was able to read the problem correctly. He was also able to comprehend what the problem asked him to do. However, he failed to use an appropriate method to solve the problem. Based on the interview, he added the probabilities of the two events which were inappropriate since the problem called for the application of conditional probability. Thus the error was in Transformation.

The next figure shows another of a Transformation Error in answering a problem related to the Conditional Probability.

At a certain city college, the probability that a student takes Analytic Geometry and Biostatistics is 0.092. The probability that a student takes Biostatistics is 0.53. What is the probability that a student chosen takes Analytic Geometry given that the student is taking Biostatistics?

Let P(A) = Be the probability that a student takes biostatistics
Let P(B) = Be the probability that a students chosen takes analytic Geometry, given that the student is taking Biostatistics.

= P(A) = 0.53
P(B) = 0.092

$$P(A \cup B) = P(A) + P(B)$$
$$= 0.53 + 0.092$$
$$= 0.622$$

therefore, the probability that a student chosen takes analytic geometry given that the student is taking biostatistics is $\boxed{0.622}$

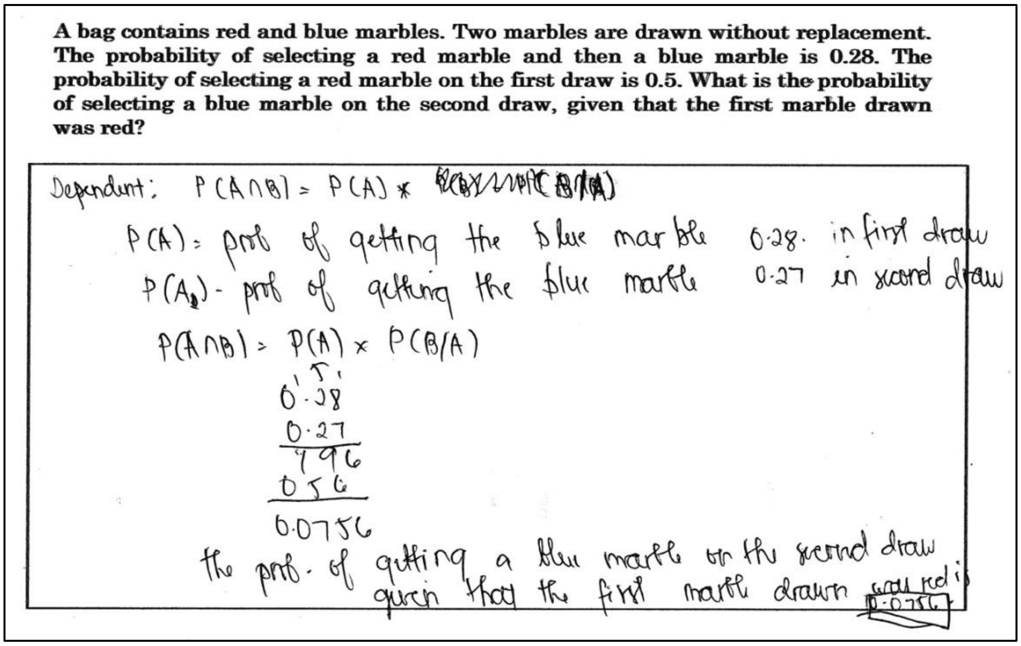
Figure 31. Sample of Transformation Error in Item no. 15

The transformation error was confirmed in the next transcript of the interview.

I: Please read the problem to me. *S: [The student read question number 15]* *I: Please tell me what the problem asked you to find.* *S: The probability that a student chosen takes Analytic Geometry given that the student is taking Biostatistics.* *I: Tell me the method you used to solve the problem.* *S:* ***The formula I used is the probability of A union B is equal to the probability A plus the probability of B. (ERROR occurs at this stage)*** *I: Please explain to me your solution on how you get your answer.* *S: Given that the probability of A is 0.53 and the probability of B is equal to 0.092. Then I substitute the values to the probability of A union B is equal to the probability of A plus the probability of B then I get 0.622* *I: What is your answer to the problem?* *S: Therefore, probability that a student chosen takes Analytic Geometry given that the student is taking Biostatistics is 0.622* *I: Please box your final answer.* *S: [The student boxed her final answer]*

Based on the transcript, the student was able to read and comprehend the problem. However, when asked about the method he used to solve the problem, he answered that he added the probability. With the student's response, the error occurred at the transformation stage when he failed to use the appropriate method to solve the problem.

The next figure shows us another example of Transformation error. But this time it is on item no. 16.


A bag contains red and blue marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and then a blue marble is 0.28. The probability of selecting a red marble on the first draw is 0.5. What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was red?

Dependent: $P(A \cap B) = P(A) * P(B/A)$

$P(A)$ = prob of getting the blue marble 0.28. in first draw

$P(A_2)$ - prob of getting the blue marble 0.27 in second draw

$P(A \cap B) = P(A) \times P(B/A)$

0.28
0.27
196
056
0.0756

the prob. of getting a blue marble on the second draw given that the first marble drawn was red is $\boxed{0.0756}$

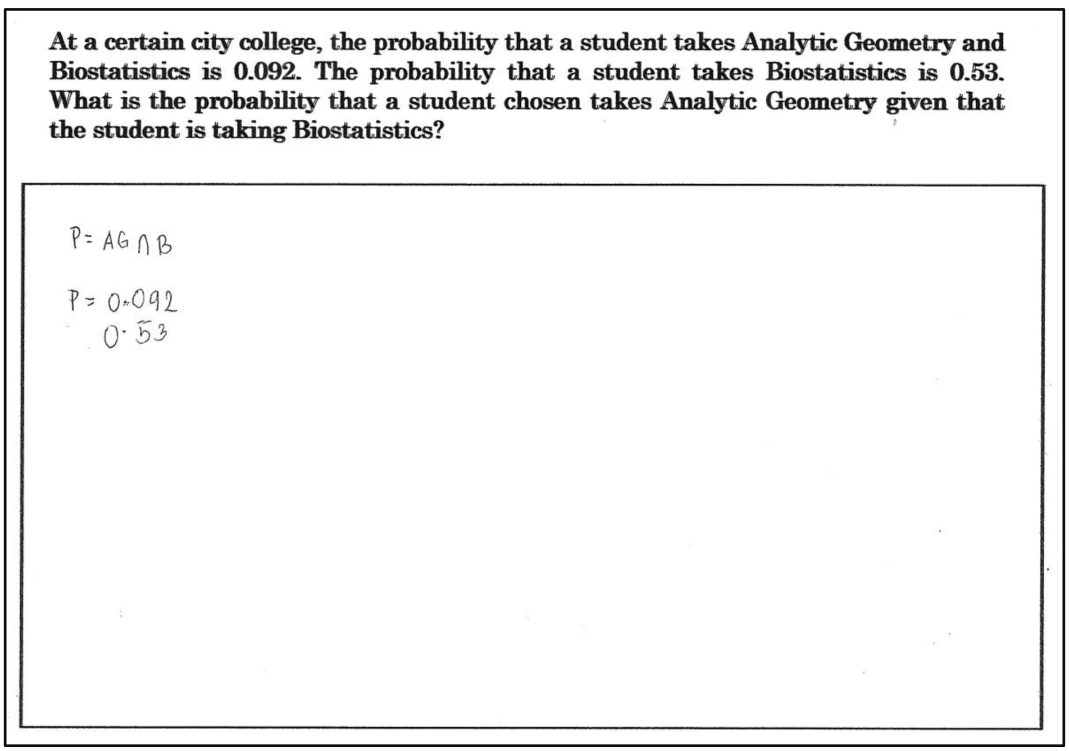
Figure 32. Sample of Transformation Error in Item no. 16

This Transformation error under item no. 16 was verified in the next transcript of the interview with the student.

I: *Please read the problem to me.*
S: *[The student read question number 16]*
I: *Tell me what the problem asked you to find.*
S: *The probability of selecting a blue marble on the second draw, given that the first marble drawn was red?*
I: *Tell me the method you used to solve the problem.*
S: *Dependent.* ***(Error occurred at this stage)***
I: *Please explain to me your solution.*
S: *Ang formula ko pong ginamit is yung probability of A intersect B is equal to probability of A times the probability of B given A. P of A is the probability of getting the blue marble is 0.28 in first draw. Tapos PA_2 is the probability of getting the blue marble is 0.27 in second draw. Tapos P of A intersection B is equal to P of A times P of B given A is 0.0756. So the probability of selecting a blue marble on the second draw, given that the first marble drawn was red 0.0756.*
I: *Is that your final answer?*
S: *Yes*
I: *Please box your final answer.*
S: *[The student boxed her final answer]*

Based on the transcript of the interview, the error occurred at the transformation level when the student responded that the method he used to answer the problem was "dependent". He multiplied the probability which was inappropriate to solve the problem since the problem called for the application of Conditional Probability rule. Thus, the error was considered as Transformation error.

The last figure shows an example of a solution of a student which was classified as a Motivation Error.



At a certain city college, the probability that a student takes Analytic Geometry and Biostatistics is 0.092. The probability that a student takes Biostatistics is 0.53. What is the probability that a student chosen takes Analytic Geometry given that the student is taking Biostatistics?

$P = AG \cap B$

$P = \frac{0.092}{0.53}$

Figure 33. Sample of Student Motivation Error

The solution was inadequate to establish a Motivation error, not until the student underwent the Newman interview.

> ***I:*** *Please read the problem to me.*
> ***S:*** *[The student read question number 15 correctly]*
> ***I:*** *Please tell me what the problem asked you to find.*
> ***S:*** *What is the probability that a student chosen takes Analytic Geometry given that the student is taking Biostatistics?*
> ***I:*** *Tell me the method you used to solve the problem.*
> ***S:*** *[The student is silent for 47 seconds]*
> ***I:*** *So what is the method?*
> ***S:*** *[The student silent again for 25 seconds]*
> ***I:*** *Ano nalang ang unang pumasok sa isip mo as you read the problem?*
> *S: Sir wala talaga…*

With the student’s response, the error was under Motivation.

The next table summarizes the types of error that students committed in solving open-ended problem in probability.

Table 7

Summary of Types of Error that Students Committed in Solving Open-Ended Problem in Probability

STRATA	**Newman Classification of Error**							Total
	R	**C**	**T**	**P**	**E**	**X**	**M**	
Complement Group	0	0	7	6	0	3	0	**16**
Addition Group	0	0	11	2	0	3	0	**16**
Multiplication Group	0	2	13	1	0	0	0	**16**
Conditional Probability	0	0	13	2	0	0	1	**16**
Total	**0**	**2**	**44**	**11**	**0**	**6**	**1**	**64**

Based on the table, out of 64 solutions analyzed using the Newman procedure, forty-four (68.75%) were classified as Transformation errors (T), eleven (17.19%) were Process Skill errors (P), six (9.37%) were Carelessness errors (X), two (3.13%) were Comprehension errors and the last one (1.56%) was classified as Motivation error (M).

The results of this study agree with the findings of Jha (2012) when he discovered that students' error occurred mostly at the transformation level.

The findings also confirm the study of Ellerton and Clements (1992) that that many Asians have considerable difficulty in developing appropriate representation of word problems. The probable reason for this is because English is not the primary language of the Filipinos. Thus, students find it difficult to translate problems into equations and relate them to their own experiences.

However, the results also contradict the findings of Clarkson (1992) that comprehension errors make up a high proportion of the errors made when students attempt to solve mathematical word problems. The probable cause may be caused by the fact that his study focused on elementary children as his subject while this study focused on college students.

Research Question # 2. What remedial intervention can be prepared to address errors committed by the students based on Newman's Error Analysis?

An analysis of students' solution shows that the majority of errors in solving open-ended problems in Probability using the Newman's procedure are Transformation (68.75%) and Process Skill errors (17.19%). With these findings, a Remedial Intervention was prepared to address the Transformation and Process Skill errors committed by the students. The said Remedial Intervention is presented in Appendix G.

CHAPTER V

Summary of Findings, Conclusion, and Recommendation

This chapter presents the summary of findings based on the data analyzed in the previous chapter, significant conclusions about the results, and the recommendations that will be useful to the readers and future researches.

This study aimed to describe and analyze the errors encountered by the students in solving open-ended problems in Probability using the Newman Error Analysis. This study used the descriptive-qualitative method to analyze the answers given by the students. The researcher developed and validated a diagnostic test in Probability composed of 16 items and divided into four subtopics namely: (1) Addition Rule, (2) Complement Rule, (3) Multiplication Rule and (4) Conditional Probability. The participants of the study were the 219 freshmen students taking up Bachelor of Elementary Education and Bachelor of Secondary Education of Cavite State University who were enrolled in the subject MATH 6 – Contemporary Mathematics. To determine the representative samples, simple random sampling was used.

Summary of Findings

The following are the findings on the errors committed by the students in solving open-ended problems in Probability:

1. Most of the students show evidence of Transformation errors in solving problems in Probability.

2. Many students also manifest a Process Skill error. They know the appropriate method to solve the problem but cannot correctly perform the mathematical procedure of the application of Addition Rule, Complement Rule, Multiplication Rule, or Conditional Probability to solve the given problem. Most of the students failed to add and subtract fractions correctly. Some even misused some data and applied inappropriate formulas.

3. A very small percentage of students shows errors in Comprehension. They can read the problem but did not grasp the over-all meaning or what the problem asked him/her to do.

4. Some students show a Carelessness error in solving problems in Probability. They were able to answer the same question correctly for the second time in which they were previously incorrect.

5. There is no evidence of reading error was committed by the students in solving problems in Probability.

Conclusions

Based on the findings, the following conclusions are drawn:

1. Applying appropriate probability rules is the most difficult concept for the students in solving Probability problems.

2. Even if the students know the appropriate method to solve a Probability problem, they commit error because of lack of mastery in addition, subtraction, multiplication, and division of fractions and decimals.

3. Comprehension of Probability problem is not a difficulty for the students.

4. The carelessness error may be caused by time pressure during the examination.

5. The following are the new suggested types of error under Process Skill stage which are not part of Newman Hierarchy of Errors:

- Data identification error
- Application of inappropriate formula
- Computational error

Recommendations

In view of the findings and conclusions, the following recommendations are hereby stated:

1. A definite need for a more comprehensive study of error patterns in Probability with larger sample should be made for more conclusive and accurate result.

2. An experimental research on effective teaching method on how to minimize student's error in solving Probability problem is highly recommended.

3. A case study on the students who committed comprehension error in solving problems in Probability is highly recommended to further explain why they commit such kind of error.

4. An experimental research should be done to test the hypothesis that a student will commit less carelessness error if the exam is given in a longer period of time.

5. A need for an exploratory study for the best teaching method to lessen the student's motivation error in answering open-ended problems in probability must be addressed.

6. An error analysis on addition and subtraction of fraction is highly recommended.

7. Further research on the students' Response to Intervention (RTI) should be made to determine the effectiveness of the remedial intervention.

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A THREE (3) DAY REMEDIAL INTERVENTION

For students struggling in solving problems in Probability

Prepared by Andrew J. Siducon

Rationale:

The study of Probability as a subject is very important because of its applicability to real-life scenario. The main purpose of studying probability is for the decision-making process; in business, in economics, in weather forecasting and even in education. Thus, proper solving of real-life problems should be one of the main focuses in teaching Probability in school.

Noraini and Shiney (2005) stated that the study of probability is often deceptive; on the surface, it seems close to everyday experience where terms such as *randomness* and *chance* are used by laypeople, the media, rating boards, and athletes to justify one action over another. They further state that solutions to probability problems are shaped by the interaction between a general intellectual schema, accepted intuitively by the student, and certain specific constrain to the problem. Probability does not consist of mere technical information and procedures leading to solutions. Rather, it requires a way of thinking that is genuinely different from that required by the most school Mathematics.

This remedial intervention was the product of the writer's careful analysis of students' error using the Newman Error Analysis (NEA) procedure in solving open-ended problems in Probability. As an upshot, this remedial intervention aims to remediate Transformation and Process Skill errors which are found to be the most common error of the students in solving problems in probability. This was specifically designed so that the instructor/teacher can provide additional instructional support to correct the said errors through this three-day remedial intervention (RI). In general, the main goals of this RI are (1) to give one-on-one/group intensive remedial instruction in the application of rules of probability and (2) to improve the problem solving performance of students with learning difficulties in rules of Probability.

The prepared remedial intervention is divided into three sessions:

- Session 1 aims to remediate student's ***transformation error*** in solving open-ended problems in probability. It includes activities of determining the appropriate method to use to solve a particular problem in Probability.
- Session 2 aims to improve the student's critical thinking skill in solving problems in Probability. It includes posting a sample problem with a solution and then the student(s) will try to justify whether the solution is correct or incorrect.
- Session 3 aims to remediate student's ***process skill*** error in solving open-ended problems in Probability. It includes an instructor - guided practice to answer some set of problems in Probability.

Lastly, each remedial session will be composed of the following:

1. Focus of remedial discussion.
2. Remedial students expected outcome.
3. Resources, materials and supplies needed
4. Procedure
5. Evaluation

DAY 1

I. Focus: Student's transformation error in solving-problem in Probability

II. Expected Outcome:

At the end of the remedial session the students are expected to:

1. determine the appropriate method in solving a particular problem in probability

III. Resources, materials and supplies needed:

pens, papers, board and a handout with a sample probability problem

IV. Procedure:

	Activities	**Time**
A. Preliminaries	The instructor will established a positive atmosphere by explaining to the student(s) the purpose and overview of the remedial session.	5 mins.
B. Instructor's Demonstration	The instructor will present some real-life probability problems where in the words ***"or", "either", "not", "except", "and", "then",*** and ***"given that"*** are purposively emphasized to help the students understand the linkage between these context clues and the appropriate mathematical method of their use. For instance, (a) the words "*or*" and "*either*" in the problem are associated to union of events and therefore addition of probability should be done; (b) the words "*not*" and "*except*" are usually used for the complement of an event; (c) the words "*and*" and "*then*" are typically used for intersection of events hence multiplication of probability should be done; and finally (d) the words "*given that*" in the problem suggest that conditional probability should be applied.	15 mins.

	The following are the sample problems for instructor's demonstration: *1. In rolling a die, what is the probability of **NOT** getting an even number?* ***Ans. (Complement rule)*** *2. In an experiment of choosing a card at random from a standard deck of 52 playing cards, what is the probability of choosing an ace **OR** a heart?* ***Ans. (Addition rule for not MEE)*** *3. A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit **EXCEPT** face cards?* ***Ans. (Complement rule)*** *4. At a certain Medical school, the probability that a student takes Human Anatomy and Biochemistry is 27%. The probability that a student takes Biochemistry is 42%. What is the probability that a student takes Human Anatomy **GIVEN THAT** the student is taking Biochemistry?* ***Ans. (Conditional Probability)***	

	*5. In an exam, 4 out of 12 students got a failing grade. If two students are chosen at random without replacement, what is the probability that **BOTH** students got a failing grade in the test?* ***Ans. (Multiplication rule for Dependent Events)*** *6. A single 6-sided die is rolled. What is the probability of rolling a 5 **OR** an odd number?* ***Ans. (Addition rule for not MEE)*** *7. At the Engineering Department, 15% of the female students play volleyball and softball. If 35% of the female students play volleyball, what is the probability that a student plays softball **GIVEN THAT** she plays volleyball?* ***Ans. (Conditional Probability)*** *8. Suppose you have a box with 3 blue marbles, 2 red marbles, and 4 yellow marbles. You are going to pull out one marble, record its color, put it back in the box and draw another marble. What is the probability of pulling out a red marble **FOLLOWED BY** a blue marble?* ***Ans.(Multiplication rule for Independent Events)*** ***9.*** A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit **EXCEPT** diamonds.	

Ans. (Complement rule)

10. Amanda is shopping for a new printer. The probability that she will buy the HP brand is 15/32 and the probability that she will buy Brother brand is 7/16. What is the probability that she will buy **EITHER** of these two brands?

Ans. (Addition rule for MEE)

11. A bank manager wants to buy two pairs of pants. The probability that he chooses black pants is 0.55 and the probability that he chooses Khaki pants is 0.13. What is the probability that he will choose a Black **AND** a Khaki pants?

Ans. (Multiplication rule for Independent Events)

12. Let us say that the probability of getting a 95 grade in History and being male is 0.16 and the probability of being male is 0.54. What is the probability of getting 95 in History, **GIVEN THAT** a student is male?

Ans. ***(Conditional Probability)***

13. There is a group of college instructors composed of 3 Mathematics major, 4 English major and 2 Biology major. What is the probability of

	randomly selecting one person who is **EITHER** Mathematics major *OR* biology major? ***Ans. (Addition rule for MEE)*** *14.* There are 11 marbles in a bag. Two are yellow, five are pink, and four are green. Suppose you pull out one marble, record its color, put it back in the bag and then pull out another marble. Find the probability that the first marble drawn was pink **FOLLOWED BY** green. ***Ans. (Multiplication rule for Independent Events)*** *15.* A veterinarian tells you that if you breed two cream-colored guinea pigs, the probability that an offspring will be pure white is 0.27. What is the probability that it will **NOT** be pure white? ***Ans. (Complement rule)***	

C. Guided Practice	The instructor will ask the student(s) to determine the appropriate method to use in solving the following problems: 1. A ball is drawn at random from a box containing 7 blue balls, 3 white balls, and 3 red balls. What is the probability that it is **NOT** blue? ***Ans. (Complement rule)*** 2. Out of a pack of cards, what is the probability of a Red card **OR** a Queen of Hearts being drawn? ***Ans. (Addition rule for not MEE)*** ***3.*** What is the probability of rolling a "five" **THEN** rolling a "two" with a normal six-sided die? ***Ans. (Multiplication rule for Independent Events)*** ***4.*** In a certain university, the probability that a student takes Biology and Physics is 0.46. The probability that a student takes Physics is 0.80. What is the probability that a student takes Biology **PROVIDED THAT** the student is taking Physics? ***Ans. (Conditional Probability)*** *5.* There are 25 candidates for student council presidency. Out of 12 male candidates, 6 are sophomores and 6 are non-sophomores. Out of 13 female candidates, 3 are sophomores and 10 are non- sophomores. What	15 mins

	is the probability of choosing a male **OR** a non-sophomore? ***Ans. (Addition rule for not MEE)*** ***6.*** What is the probability of **NOT** getting a "five" when you roll a six sided die? ***Ans. (Complement rule)*** ***7.*** An urn contains red and white marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and then a white marble is 8/25. The probability of selecting a red marble on the first draw is 21/50. What is the probability of selecting a white marble on the second draw, ***GIVEN THAT*** the first marble drawn was red? ***Ans. (Conditional Probability)*** 8. A small drawer contains 3 red paperclips, 4 green paperclips, and 5 blue paperclips. One paperclip is taken from the drawer without replacement. **THEN** another paperclip is taken from the drawer. What is the probability that the first paperclip is red **AND** the second paperclip is blue? ***Ans.(Multiplication rule for Dependent Events)*** *After ten (10) minutes, the instructor will show and explain the correct answers.*	

V. Evaluation:

Directions: Determine the appropriate method to solve the following probability problems:

1. If a card is drawn from a well shuffled standard 52 deck of cards, find the probability that the card is not a spade.
2. A single 6-sided die is rolled. What is the probability of rolling a 5 or an odd number?
3. Suppose you are going to draw two cards without replacement from a standard deck. What is the probability that the first card is an ace and the second card is a jack?
4. In a certain canteen, the probability that a student buys a soda is 54%. The probability that the student buys a soda and bread is 39%. What is the probability that the student buys bread, provided that he already bought a soda?
5. In an exam, 4 out of 12 students got a failing grade. If two students are chosen at random without replacement, what is the probability that both students got a failing grade in the test?
6. A cat-lover told you that if you breed a gray and a black cat, the probability that the offspring is a pure black is 72.56%. What is the probability that the offspring is not a pure black?
7. There are 11 marbles in a bag. Two are yellow, five are pink, and four are green. Suppose you pull out one marble, record its color, put it back in the bag and then pull out another marble. Find the probability that the first marble drawn was pink followed by green.
8. A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results are shown in the table:

Gender	*Yes*	*No*	*total*
Male	*32*	*18*	*50*
Female	*8*	*42*	*50*
total	*40*	*60*	*100*

   Find the probability that the respondent answered "yes" given that the respondent was a female.
9. What is the probability of getting a "three" when a six-sided die is rolled twice?
10. In an experiment of choosing a card at random from a standard deck of 52 playing cards, what is the probability of choosing a 5 or a king?

Answer:

1. Complement rule
2. Addition rule for not MEE
3. Multiplication rule for Dependent Events
4. Conditional Probability rule
5. Multiplication rule for Dependent Events
6. Complement rule
7. Multiplication rule for Independent Events
8. Conditional Probability rule
9. Multiplication rule for Independent Events
10. Addition rule for MEE

DAY 2

I. Focus: Improvement of student's critical thinking skill in answering open –ended problems in Probability.

II. Expected Outcome:

At the end of the remedial session the students are expected to:

1. justify whether the solution shown to answer a given probability problem is correct or incorrect.

III. Resources, materials and supplies needed:

pens, papers, board and a handout with a sample probability problem

IV. Procedure:

	Activities	Time
A. Preliminaries	The instructor will establish a positive atmosphere by explaining to the student(s) the purpose and overview of the remedial session.	5 mins.
B. Instructor's Demonstration	The instructor will post the following problems with their corresponding solution. The task for the instructor here is to explain to the students the justification whether the solution to the problem shown is correct or incorrect. Here are the problems with their corresponding solution: 1. ***A man is to choose a card from an ordinary deck of 52 playing cards. What is the probability of not getting an ace card***? **Solution:**	15 mins.

The rule applicable to solve the problem is the Complement rule.

Let P(A) be the probability of getting an ace card.

So, $P(A) = 4/52$

$P(A') = 1 - 4/52$
$= 48/52$ *or* $12/13$

Thus, the probability of not getting an ace card is 48/52 or 12/13. (end of solution).

Question: Is the solution shown correct? Why or why not?

Justification:

The method used is appropriate to solve the problem. The Compliment rule was carried out correctly. One minus 4/52 is equal to 48/52 or 12/13. Thus, the solution and answer given in the problem above is correct.

2. ***Suppose you are to draw two cards from standard deck of 52 playing cards. You are going to draw out one card, record it, put it back in the deck and draw another one. What is the probability that the first draw is a "five" and the second draw is a queen of hearts?***

Solution:

Let P(F) be the probability that the first draw is a "five".
P(Q|F) be the probability that the second draw is a queen of

	hearts given that the first draw is a "five". *So, P(F) = 4/52 and P(Q) = 1/51* *Apply multiplication rule for dependent events.* $P(F \cap Q) = P(F)*P(Q\|F)$ $= 4/52 * 1/51$ $= 1/13 * 1/51$ $= 1/663$ *Therefore, the probability that the first draw is a "five" and the second draw is a queen of hearts is 1/633.* *(end of solution)* *Is the solution shown is correct? Why or why not?* **<u>Justification</u>**: *The solution shown is incorrect for the problem calls for the application of multiplication rule of probability for INDEPENDENT events. The words "Put it back" suggest that the probability of the second event is not affected given that the first event happened. Thus, considering it as independent events.* ***3. In an experiment of choosing a card at random from a standard deck of 52 playing cards, what is the probability of choosing an spade or a king?*** **<u>Solution:</u>**	

	Let P(S) be the probability of choosing a Spade card. *P(K) be the probability of choosing a King card.* *So,* $P(S) = 13/52$ *and* $P(K) = 4/52$ $P(S \cup K) = P(S) + P(K)$ $= 13/52 + 4/52$ $= 17/52$ *Thus, the probability of choosing a spade or a king is 17/52.* *(end of solution)* **<u>Justification</u>***:* *The solution shown is incorrect since addition rule for mutually exclusive events is used. It should have been addition rule for NOT mutually exclusive events since we can choose a card which is a king and a spade at the same time.*	

C. Guided Practice	Ask the student to justify whether the solution shown after each problem below is correct or incorrect: ***1. A bag contains 18 colored marbles: 4 are red, 8 are yellow and 6 are green. A marble is selected at random. What is the probability that the ball chosen is either red or green?*** **<u>Solution:</u>** *Let P(R) be the probability of choosing red marble* *P(G) be the probability of choosing green marble* *So P(R) = 4/18 and P(G) = 6/18* *P(RUG) = P(R) + P(G)* *= 4/18 + 6/18* *= 10/18* *= 5/9* *Hence, the probability that the ball chosen is either red or green is 5/9.* *(end of solution)* *Is the solution shown is correct? Why or why not?* ***Student expected answer:*** *The solution and the answer shown are* ***correct*** *since the appropriate method was used which is addition rule for mutually exclusive events. Furthermore, there is no error in the addition of fraction in the solution shown.*	20 mins

	2. A box contains 4 black socks, 6 white socks and 2 blue socks. What is the probability that if two socks are drawn at random, with replacement, that both are white? **Solution:** *Let $P(W_1)$ be the probability that the first draw is a white sock.* *Let $P(W_2 \mid W_1)$ be the probability that the second draw is also a white sock.* *So, $P(W_1)= 6/12$ and $P(W_2 \mid W_1) = 5/11$.* *Apply the multiplication rule dependent events.* $P(W_1 \cap W_1) = P(W_1)*P(W_2 \mid W_1)$ $= 6/12 * 5/11$ $= 1/2 * 5/11$ $= 5/22$ *Therefore, the probability that if two socks are drawn at random, without replacement, that both are white is 5/22.* *(end of solution)* *Is the solution shown is correct? Why or why not?* ***Student expected answer**:* *The solution shown is **incorrect** since the problem calls for the application of multiplication rule for independent events instead of applying multiplication rule for dependent events. The word "with replacement" in the problem suggest that the probability of the first and*	

second event are independent with each other, meaning to say, the probability of the second event is not affected by the occurrence of the first event.

3. ***There are 25 candidates for student council presidency. Out of 12 male candidates, 6 are sophomores and 6 are non-sophomores. Out of 13 female candidates, 3 are sophomores and 10 are non- sophomores. What is the probability of choosing a male or a non-sophomore?***

Solution:

	Sophomore	*Non-sophomore*	*Total*
Male	*6*	*6*	*12*
Female	*3*	*10*	*13*
Total	*9*	*16*	*25*

	Let P(M) be the probability of choosing a male candidate. *Let P(N) be the probability of choosing a non- sophomore candidate.* *Let P(M∩N) be the probability of choosing a male and a non sophomore.* *So* $P(M) = 12/25$ $P(N) = 16/25$ $P(M \cap N) = 6/25$ *Apply the addition rule of probability for not mutually exclusive events* $P(M \cup N) = P(M) + P(N) - P(M \cap N)$ $= 12/25 + 16/25 - 6/25$ $= 22/25$	

	Thus, the probability of choosing a male ***or*** *a non-sophomore candidate is 22/25.* ***Student expected answer:*** *The solution and the answer shown is* ***correct*** *because appropriate method was used to solve the problem which is addition rule of probability for not mutually exclusive events. Moreover there is no sign of error in the operation of fraction involved.* 4. ***What is the probability of not getting a "five" when you roll a six sided die?*** **<u>Solution:</u>** *Let P(5) be the probability of getting "five" when you roll a six sided die.* *Thus P(5) = 5/6* *Apply the complement rule:* *P(5') = 1 – 5/6* *= 6/6 – 5/6*	

	= 1/6 *Hence the probability of NOT getting a "five" when you roll a six sided die is 1/6.* *(end of solution)* *Is the solution shown is correct? Why or why not?* ***Student expected answer:*** *The solution and answer shown is* ***incorrect*** *since the probability of getting "five" when you roll a six sided die is 1/6 and not 5/6. Further, even if an appropriate method is used and there is no sign of error or inconsistency in performing subtraction of fraction, the answer is considered incorrect due to inappropriate value of the probability of the event given in the problem.*	

V. Evaluation:

Directions: Justify whether the solution and answer shown after each problem presented is correct or incorrect. Write your answer in the box provided.

1. ***A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit except diamonds.***

<u>Solution:</u>

Let P(D) be the probability of drawing a diamond.

Since there are 52 cards and there are 13 diamond cards. Thus P (D) is 13/52.

Apply the complement rule:

$$
\begin{aligned}
P(D') &= 1 - P(D) \\
&= 1 - 13/52 \\
&= 52/52 - 13/52 \\
&= 39/52
\end{aligned}
$$

Hence, the probability that it is of any suit except diamonds is 39/52.

2. ***A bag contains black and white marbles. Two marbles are drawn without replacement. The probability of selecting a white marble on the first draw is 0.42. The probability of selecting a white marble and then a black marble is 0.19. What is the probability of selecting a black marble on the second draw, given that the first marble drawn was white?***

<u>Solution:</u>

Let P(B|W) be the probability of selecting a black marble on the second draw given that the first draw was white.
Let P(W) be the probability of selecting a white marble on the first draw.
Let P(W∩B) the probability of selecting a white marble and then a black marble.

Apply the conditional probability rule:

$$P(B|W) = \frac{P(W \cap B)}{P(W)}$$

$$= \frac{0.19}{0.42}$$

$$= 0.4524$$

3. *What is the probability of getting a "three" when a six-sided die is rolled twice?*

<u>Solution:</u>

Let P(3) be the probability getting a "three when a six-sided die is rolled.

So, P(3) = 1/6

The probability of getting a "three" when a six-sided die is rolled twice is:

> *2 x P(3)*
> *2 x (1/6)*
> *2/6 or 1/3*

Hence, the answer is 2/6 or 1/3.

4. ***Out of a pack of cards, what is the probability of a Red card or a Queen of Hearts being drawn?***

Solution:

Let $P(R)$ be the probability of drawing a red card
$P(Q)$ be the probability of drawing a drawing a queen.
$P(R \cap Q)$ be the probability of choosing a Red or a Queen card.

$P(R) = 26/52$
$P(Q) = 4/52$
$P(R \cap Q) = 2/52$

Apply the addition rule of probability for not mutually exclusive events.

$$P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$$
$$= 26/52 + 4/52 - 2/52$$
$$= 30/52 - 2/52$$
$$= 28/52 \text{ or } 7/13$$

Thus, the probability of a red card or a Queen of Hearts is being drawn is 28/52 or 7/13.

Answer:

1. Correct
2. Correct
3. Incorrect
4. Correct

DAY 3

I. Focus: Student's Process Skill error in solving-problem in Probability

II. Expected Outcome:

At the end of the remedial session the students are expected to:

1. solve probability problems correctly and independently.

III. Resources, materials and supplies needed:

pens, papers, board and a handout with a sample probability problem

IV. Procedure:

	Activities	Time
A. Preliminaries	The instructor will establish a positive atmosphere by explaining to the student(s) the purpose and overview of the remedial session.	5 mins.
B. Instructor's Demonstration	The instructor will present and carefully solve the following problems: ***1. In rolling a die, what is the probability of not getting an even number?*** **<u>Solution:</u>** *Use* ***Complement Rule*** *of Probability.*	20 mins.

A.
Let P(A) = probability of getting an even number.

$P(A') = 1 - P(A)$
$= 1 - \frac{1}{2}$
$= \frac{1}{2}$

B.
Let P(A) = probability of getting an even number.

$P(A') = 1 - P(A)$
$= 1 - 3/6$
$= 3/6$

C.
Let P(A) = probability of getting an even number.

$P(A') = 1 - P(A)$
$= 1 - 0.5$
$= 0.5$

2. What is the probability of rolling a "five" then a "three" with a normal six-sided die?

<u>Solution:</u>

Use ***Multiplication Rule for Independent Events****.*

A.
P(A) = probability of rolling a "5" = ***1/6***
P(B) = probability of rolling a "3" = ***1/6***

$P(A \cap B) = P(A) * P(B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$

B.
P(A) = probability of rolling a 5 = ***0.167***
P(B) = probability of rolling a 3 = ***0.167***

$P(A \cap B) = P(A) * P(B) = 0.167 * 0.167 = 0.028$

3. There is a group of high school teachers composed of 3 Mathematics major, 4 English major and 2 Biology major. What is the probability of randomly selecting a teacher who is either Mathematics major or Biology major?

Solution:

Use ***Addition Rule for Mutually Exclusive Events.***

A.
P(M) = probability of choosing a Mathematics teacher
P(B) = probability of choosing a Biology teacher

P(M) = 3/9 or 1/3
P(B) = 2/9

$$P\ (A \cup B) = P(M) + P(B) = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$$

B.
P(M) = probability of choosing a Mathematics teacher
P(B) = probability of choosing a Biology teacher

P(M) = 0.33
P(B) = 0.22

$$P\ (A \cup B) = P(M) + P(B) = 0.33 + 0.22 = 0.55$$

4. There are 90 college students. Out of 30 males, 5 are freshmen and 25 are non-freshmen. Out of 60 females, 10 are freshmen and 50 are non-freshmen. What is the probability of choosing a female or a freshman?

Solution:

Use ***Addition Rule for Not Mutually Exclusive Events.***

	A. *P(A) = probability of choosing a female =* ***60/90*** *P(B) = probability of choosing a freshman =* ***15/90*** *P(A∩B) = probability of choosing a female and a freshman)* = ***10/90*** $P\ (A\ \cup B) = \ P(A) + P(B) - \ P(A \cap B) = \frac{60}{90} + \frac{15}{90} - \frac{10}{90} = \frac{65}{90}\ or\ \frac{13}{18}$ *B.* *P(A) = probability of choosing a female =* ***0.67*** *P(B) = probability of choosing a freshman =* ***0.17*** *P(A∩B) = probability of choosing a female and a freshman)* = ***0.11*** $P(A\ \cup B) = P(A) + P(B) - \ P(A \cap B) = 0.67 + 0.17 - 0.11 = 0.73$ ***5. The probability that it is Friday and that a student is absent is 0.015. Since there are five school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?*** **<u>Solution:</u>** *Use* ***Conditional Rule of Probability.*** *Let P(F) = probability that it is Friday =* ***0.2***	

	P(F∩A)= probability that it is Friday and that a student is absent = ***0.015*** $P(A\|F) = \frac{P(F \cap A)}{P(F)} = \frac{0.015}{0.2} = 0.075$ ***6. A box contains 2 red and 3 blue marbles. What is the probability that if two marbles are drawn at random, without replacement, that both are blue?*** **<u>Solution:</u>** *Use* ***Multiplication Rule for Dependent Events.*** *A.* *P(A) = probability that* 1^{st} *draw is blue =* ***3/5*** *P(B|A)= probability that* 2^{nd} *draw is blue given that the first draw is also blue =* ***2/4*** $P\ (A\ \cap B) = \ P(A)\ *\ P(B\|A) = \frac{3}{5} * \frac{2}{4} = \frac{6}{20}\ or\ \frac{3}{10}$ *B.* *P(A) = probability that* 1^{st} *draw is blue =* ***0.6***	

	P(B∣A)= probability that 2^{nd} draw is blue given that the first draw is also blue = ***0.5*** $P\ (A\ \cap B) =\ \ P(A)\ *\ P(B\vert A)$ $= 0.6\ * 0.5 =$ 0.3	
C. Guided Practice	Let the student answer the following problem individually or by small group. Encourage them to show their complete solution in the space provided after each problem. ***1. A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is of any suit except hearts.***	20 mins

	Students expected answer: A. Let P(H) = probability of drawing HEARTS = **13/52 or 1/4** P(H') = 1 – P(H) = 1 – 13/52 = 39/52 Or P(H') = 1 – P(H) = 1 - ¼ = ¾ B. Let P(H) = probability of drawing HEARTS = **0.25** P(H') = 1 – P(H) = 1 – 0.25 = 0.75 ***2. A P.E instructor needs two students to help him with a Physical Education demonstration for his class of 8 girls and 10 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. What is the probability that both students chosen are girls?***	

Students expected answer:

A.
Let P(G1) = probability that the first student he choose is a girl = **8/18.**
P(G2 | G1)= probability that second student he choose is a girl given that the first is also a girl = **7/17.**

$$P\ (G1 \cap G2) = \ P(G1)\ *\ P(G2|G1) = \frac{8}{18} * \frac{7}{17} = \frac{56}{306}\ or\ \frac{28}{153}$$

B.
Let P(G1) = probability that the first student he choose is a girl = **0.44.**
P(G2 | G1)= probability that second student he choose is a girl given that the first is also a girl = **0.41.**

$$P\ (G1 \cap G2) = P(G1) * P(G2|G1) = 0.44 * 0.41 = 0.1804$$

3. A costumer is shopping around for a calculator. The probability that he will buy the Casio model is 1/10 and the probability that he will buy Sharp model is 3/8. What is the probability that he will buy either of these two models?

Students expected answer:

A.
Let P(C) = probability of buying Casio model.
P(S) = probability of buying Sharp model.

P(C) = **1/10**
P(S) = **3/8**

$$P\ (C \cup S) = \ P(C) + \ P(S) = \frac{1}{10} + \frac{3}{8} = \frac{4+15}{40} = \ \frac{19}{40}$$

B.
Let P(C) = probability of buying Casio model.
P(S) = probability of buying Sharp model.

P(C) = 1/10
P(S) = 3/8

$$P\ (C \cup S) = P(C) + P(S) = \frac{1}{10} + \frac{3}{8} = \frac{8+30}{80} = \frac{38}{80} = \frac{19}{40}$$

C.
Let P(C) = probability of buying Casio model.
P(S) = probability of buying Sharp model.

P(C) = **0.1**
P(S) = **0.375**

$$P\ (C \cup S) = P(C) + P(S) = 0.1 + 0.375 = 0.475$$

4. ***A bag contains red and blue marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and then a blue marble is 0.28. The probability of selecting a red marble on the first draw is 0.5. What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was red?***

Students expected answer:

Let P(A) = probability of selecting a red marble on the first draw = **0.5**
Let P(A∩B) = probability of selecting a red marble and then a blue marble= **0.28**

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.28}{0.50} = 0.56$$

After 20 minutes, have the student(s) demonstrate the way to solve the problem on the board. If the student chooses an appropriate method but failed to carry out the process in using

	the formula, label his solution as "old-way" and show the student the proper way to solve the problem and label it as the "new-way". With this, student will be aware of their error since they can differentiate the erroneous process to the correct process.	

V. Evaluation:

<u>Directions</u>: *Read each item carefully. You are required to show your complete solution in the space provided after each problem.*

1. What is the probability on getting any number except 2 when you roll an ordinary six-sided die?

2. Find the probability when a day of the week is chosen at random using the following conditions:
    a. choosing Monday or choosing Tuesday
    b. choosing a Weekend day or choosing Saturday

3. There are 11 marbles in a bag. Two are yellow, five are pink and four are green. Suppose you pull out one marble, record its color, put it back in the bag and then pull out another marble. Find the probability that the first marble is pink then followed by green.

4. There is a group of retired Philippine Navy. The probability that a retired navy is a female and licensed doctor is 0.0023. The probability that a retired navy is a female is 0.09. Find the probability that a retired navy is a licensed doctor given that she is a female.

Answer:

1. *Use **Complement Rule of Probability.***

   Let P(A) be the probability of getting "2" when a six-sided die is rolled.

   So, P(A) = 1/6

   P(A') = 1 – P(A)
   = 1 – 1/6
   = 5/6

   *The probability on getting any number except 2 when you roll an ordinary six-sided die is **5/6.***

2. *A) Use **Addition Rule for Mutually Exclusive Events.***

   Let P(A) be the probability of choosing Monday
   Let P(B) be the probability of choosing Tuesday

   $$P(A \cup B) = P(A) + P(B) = \frac{1}{7} + \frac{1}{7} = \mathbf{\frac{2}{7}}$$

   *B) Use **Addition Rule for not Mutually Exclusive Events.***

   Let P(A) be the probability of choosing a weekend
   Let P(B) be the probability of choosing Saturday
   Let P(A∩B) be the probability of choosing a weekend and Saturday

   So, P(A) = 2/7, P(B) = 1/7 and P(A∩B) = 1/7

   $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
   $$= \frac{2}{7} + \frac{1}{7} - \frac{1}{7} = \mathbf{\frac{2}{7}}$$

3. *Use **Multiplication rule for Independent Events.***

   Let P(A) be the probability that the first draw is pink.
   Let P(B) be the probability that the second draw is green.

   So P(A) = 5/11 and P(B) = 4/11

   $$P(A \cap B) = P(A) * P(B)$$
   $$= \frac{5}{11} * \frac{4}{11}$$
   $$= \mathbf{\frac{20}{121}}$$

4. *Use **Conditional Probability.***

   Let P(A) be the probability that a retired navy is a female.
   Let P(A ∩ B) be the probability that a retired navy is a female and a licensed doctor.

P(A) = 0.09 and P(A ∩ B) = 0.0023

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.0023}{0.09}$$

$$= \mathbf{0.256}$$